Abstract

We have developed a new holographic associative memory (HAM) based on an adaptive learning which uses learning pattern method (LPM). The LPM utilizes the simple optical implementation of outer-product learning, however we have obtained the better performance of adaptive learning. Results of optical experiment and computer simulation are represented.

I. Introduction

Neural networks are characterized by massive parallelism, dense interconnection of processing elements (neurons), and information storage in distributed manners. Optical information processing using a hologram inherently has such properties. Recently, optical implementations of Hopfield model and various holographic associative memory (HAM) have been developed by many authors. Another important property of neural network is the adaptive learning that is the ability to learn dynamically the interconnection (synaptic) weight through iterative adaptation.

In the associative memory such as Hopfield model, the interconnection matrix is obtained by adding all outer products of memory patterns, and it has a merit that the optical implementation can be done easily with vector-matrix multiplication devices or by holographic systems. However, there exist some limitations such that the memory patterns should be pseudo-orthogonal (statistically independent) and have an approximately equal number of ones and zeros in order to have maximum performance. Undesired stable states may also exist when the number of stored patterns is larger.

In this paper, we propose a new HAM using learning patterns derived from the interconnection matrix obtained as a result of an adaptive learning rule. We note that all memory patterns are stored in stable state when we store those learning patterns in a holographic recording medium. In Sec.II, the principle of the learning pattern method (LPM) is introduced. The derivation of learning patterns to be used in experiment, the experimental setup and the experimental results for recognizing the various inputs are presented, and finally the merit and applications of the present HAM are discussed. The result of computer simulation of present HAM is compared with that of Hopfield Model.

II. Principle

A. Review of the Delta Rule

For the input pattern $i_j$ we expect the desired output pattern $t_i$, and the actually
obtained output pattern \( o_i \), then we define the measure of the error by

\[
E = (1/2)\sum(t_i - o_i)^2,
\]

(1)

where \( \Sigma_i \) denotes summation over \( i \). According to the well known adaptive learning rule,\(^{17}\) the change of the interconnection weight \( \Delta W_{ij} \) through an iteration is related to the gradient descent in the error function \( E \) as following:

\[
\Delta W_{ij} \propto -\frac{\partial E}{\partial W_{ij}},
\]

(2)

which may be written by using the chain rule as the product of two parts,

\[
\frac{\partial E}{\partial W_{ij}} = \frac{\partial E}{\partial \alpha_i} \times \frac{\partial \alpha_i}{\partial W_{ij}}
\]

(3)

The first part in the right hand side of Eq.(3) is the derivative of the error with respect to the output and the second part is the derivative of the output with respect to the interconnection weight. From Eq.(1) and as \( \alpha_i = \Sigma_j W_{ij} \), we get

\[
\frac{\partial E}{\partial \alpha_i} = -(t_i - o_i) = -\delta_i,
\]

(4)

\[
\frac{\partial \alpha_i}{\partial W_{ij}} = i_j,
\]

(5)

\[
-\frac{\partial E}{\partial W_{ij}} = \delta_i \delta_j,
\]

(6)

and by giving the learning rate \( \eta \), we obtain

\[
\Delta W_{ij} = \eta(t_i - o_i)i_j = \eta \delta_i \delta_j.
\]

(7)

Therefore, the interconnection matrix \( W_{ij} \) can be changed by this rule in the way the error is minimized.

B. Definition of Learning Patterns

In the optical implementation of HAM, the direct application of the adaptive learning rule Eq.(7) is not easy compared to that of the conventional outer-product learning.\(^4\) The LPM is a method which uses the adaptive learning and the solutions of the inverse problem of outer-product algebra. The principle of the proposed LPM is as follows.

Let \( W^* \) be an interconnection matrix having discrete and all positive elements obtained from Eq.(7) with proper modification.(see Sec.III-A) This matrix can be expressed as the linear combination of other matrices \( T^m \) defined as following:

\[
W^* = \Sigma_m \alpha_m T^m = \Sigma_m \alpha_m u^m (v^m)^T,
\]

(8)

where \( T \) denotes transpose operation, \( \alpha_m \) is the constant coefficient, and the matrix \( T^m \) is the outer-product between new vectors \( u^m \) and \( v^m \) with elements of binary values \((1,0)\). We define \( u^m \) and \( v^m \) as the learning patterns. The derivation of learning patterns from \( W^* \) is the inverse problem of the outer-product. Although the way of it is not unique, we try to get a less number of the learning patterns for the convenience of the present experiment. The roles of learning patterns in the experiment is described in Sec. III.

C. Retrieval of the Stored Information

Recognition algorithm for an input \( b^{in} \) is similar to that of the Hopfield model:

\[
b^{out}_i = \begin{cases} 1, & \text{if } \sum_j W_{ij} b^{in}_j > \theta_i, \\ 0, & \text{otherwise}, \end{cases}
\]

(9)

where \( \theta_i \) is the input dependent threshold level\(^7\) and it is written by

\[
\theta_i = \lambda \sum_j b^{in}_j,
\]

(10)

where \( N_0 \) is the number of 1's in the input \( b^{in} \) and \( \lambda \) is a constant scale factor which determines the slope of threshold level with respect to \( N_0 \). The energy function \( U \) is defined as usually as following;

\[
U = -(1/2) \Sigma_i \Sigma_j W_{ij} b_i^{in} b_j^{in} + \Sigma \theta_i b_i^{in},
\]

\[
= -(1/2) \Sigma_i \Sigma_j W_{ij} v_i^{in} v_j^{in} + \lambda (N_0)^2.
\]

(11)
It is important to get the appropriate value of \( \lambda \), because the stable state of energy in Eq.(11) is depends on \( \lambda \).

III. Experiment

A. Memory Patterns

The three 2-D binary patterns \( b^1 \), \( b^2 \), and \( b^3 \) are the three Korean alphabets (Hangul) as shown in Fig.1(a). They are selected as the memory patterns for the present experiment and simulation work. We note that two of them, \( b^1 \) and \( b^2 \), are contained in \( b^3 \). In the form of 25 bits vector (Fig.1(b)), they are written as follow:

\[
\begin{align*}
  b^1 &= (11111100001000010000100001)^T, \\
  b^2 &= (11111110000100001000011111)^T, \\
  b^3 &= (1111111100011100011100011111)^T.
\end{align*}
\] (12)

![Fig.1. (a). Three 2-D memory patterns with 25 (5×5) binary (transparent/opaque) pixels. (b). Pattern basis for vector representation.](image)

The algorithm for obtaining the interconnection matrix of discrete and positive elements is as follows;

1. \( W_{ij}^* = 0 \), \( \eta = 1 \).
2. Give threshold level (TL).
3. \( m = 1 \).
4. \( i = \ t_i = b_i^m \).
5. Make output as binary pattern:
   \[
   o_i = 1, \quad \text{if} \quad \sum_j W_{ij}^* i_j > \text{TL}, \\
   = 0, \quad \text{otherwise}.
   \]
6. Apply Eq.(7) with \( W_{ij}^* = 0 \) if \( W_{ij}^* < 0 \).
7. Repeat (5), (6) until \( t_i = o_i \) for all \( i \).
8. Repeat (4) ~ (7) for all \( m \).
9. Repeat (2) ~ (8) by changing TL slowly until \( \Delta W_{ij}^* = 0 \).

Figure 2 illustrates the 25×25 interconnection matrix \( W^* \) obtained by using above algorithm. The threshold level is an important factor to get the interconnection matrix of which all memory patterns to be stored in stable state.

![Fig.2. The 25×25 interconnection matrix W* obtained from the delta rule for the memory patterns b1, b2, and b3.](image)

B. Derivation of Learning Patterns

The interconnection matrix of Fig.2 can be represented as the sum of four matrices \( T^1 \), \( T^2 \), \( T^3 \), and \( T^4 \) with constant coefficients \( \alpha_1 = \alpha_2 = 2 \), \( \alpha_3 = 5 \), and \( \alpha_4 = 1 \), respectively.

\[
W_{ij}^* = \sum_m \alpha_i^m T_{ij}^m,
\]

\[
= 2T_{ij}^1 + 2T_{ij}^2 + 5T_{ij}^3 + T_{ij}^4,
\]

\[
= 2c_1^1 c_1^1 + 2c_2^1 c_2^1 + 5c_3^1 c_3^1 + c_4^1 c_4^1,
\] (13)

where \( T^1 \), \( T^2 \), and \( T^3 \) are three symmetric
metrics consisted in the outer-products of \( \eta^1, \eta^2, \) and \( \eta^3 \) with themselves, and \( T^4 \) is the outer-product between \( \eta^3 \) and \( \eta^4 \). The learning patterns \( \eta^1, \eta^2, \eta^3, \) and \( \eta^4 \) are written in the vector form as following;

\[
\begin{align*}
\eta^1 &= (111110000000000000000001)^T, \\
\eta^2 &= (000001000010000100001111)^T, \\
\eta^3 &= (000000000100001000010000)^T, \\
\eta^4 &= (000001000110001100011111)^T.
\end{align*}
\] (14)

Fig.3. Four learning patterns \( \eta^1, \eta^2, \eta^3, \) and \( \eta^4 \) extracted from the interconnection matrix.

Those four learning patterns are shown in Fig.3. An interesting fact is that the learning patterns can be represented as the mixed states of the memory patterns \( b^1, b^2, \) and \( b^3 \) as they follow;

\[
\begin{align*}
\eta^1 &= b^1 \Lambda b^2, \\
\eta^2 &= b^2 \Lambda \eta^1, \\
\eta^3 &= b^3 \Lambda \eta^1, \\
\eta^4 &= b^3 \Lambda (b^1 \Lambda b^2),
\end{align*}
\] (15)

where \( \Lambda \) and \( \Lambda \) stand for AND and NOT logic operations, respectively. We point out that three memory patterns \( b^1, b^2, \) and \( b^3 \) are stored in the present HAM as the consequence of recording those four learning patterns \( \eta^1, \eta^2, \eta^3, \) and \( \eta^4 \).

C. Threshold Level

To illustrate the dependence of energy on the \( \lambda \) in Eq.(11), and to know the appropriate value of \( \lambda \) which determines the threshold level in Eq.(10), we constructed an energy values diagram along two paths for various values of \( \lambda \) as shown in Fig.4. Both paths begin from the memory pattern \( b^1, \) and are constraint to pass other memory patterns \( b^2 \) and \( b^3 \), respectively. When the value of \( \lambda \) is small, the memory pattern \( b^3 \), which has comparably large number of 1's, is most stably stored. On the other hand, the pattern \( b^1 \), which has small number of 1's, is stably stored for larger value of \( \lambda \). We note that all memory patterns \( b^1, b^2, \) and \( b^3 \) can be in stable state for \( 0.5 \leq \lambda \leq 0.6 \).

For the input \( b^1 \), as an example, the output is obtained from the Eq.(12)~Eq.(14) as followings;

\[
\begin{align*}
\sum_\mu W_{\mu} b^1_\mu &= \Sigma [2c_i^1c_i^1 + 2c_i^2c_i^2 + 5c_i^3c_i^3 + c_i^1c_i^2]b^1_i, \\
&= 2c_1^1 \Sigma c_i^1b^1_i + 2c_1^2 \Sigma c_i^2b^1_i + 5c_1^3 \Sigma c_i^3b^1_i + c_1^1 \Sigma c_i^1b^1_i + c_1^3 \Sigma c_i^3b^1_i, \\
&= 15(c_1^1 + c_1^3), \\
&= 15b_1^1.
\end{align*}
\] (16)

Eq.(16) shows that the output is the same as the input multiplied by a factor of 15. Therefore, we obtain the output which is exactly the same state of \( b^1 \) after threshold
operation of Eq.(9) and Eq.(10), i.e.,

\[ \theta_i = \lambda N_0, \]
\[ = 0.6 \times 9 = 5.4, \]
\[ < 15. \]

(17)

Fig.5. Schematic diagram of recording the outer-product of learning patterns.

D. Optical Implementation

Figure 5 shows schematically the lensless diffuser system used for recording the outer-products of the learning patterns. The interference patterns, between the collimated coherent beam passing through the input plane \( I_1 \) and the scattered beam from the input plane \( I_2 \), construct holographically the outer-product of the learning patterns \( c^m \) and \( c^n \) placed at \( I_1 \) and \( I_2 \), respectively. The recording medium is Kodak High Resolution Plate-Type 1A and it is exposed four times successively by recording the four pairs of learning patterns \( [c^1, c^1], [c^2, c^2], [c^3, c^3], \) and \( [c^n, c^n] \) with the exposure times proportional to the ratio of 2:2:5:1, which are the constant coefficients \( \alpha_m (m=1,2,3, \text{and} 4) \) in Eq.(13). The 514.5(nm) Ar* laser beam is used for exposure. The mask \( (3cm \times 3cm) \) with 25 \( (5 \times 5) \) transparent/opaque pixels \( (2mm \times 2mm) \) is used throughout the experiment. The diffuser is an optically flat glass ground with silicon carbide powder of grain size \( 80 \sim 100(\mu m) \) and the same diffuser is used throughout the experiment. The limitations of pixel size due to the diffraction effect have been discussed in other paper.8

Fig.6. Experimental setup of reconstruction process.

Figure 6 represents the recognizing system for the various inputs of the memory patterns \( b^n (m=1,2, \text{and} 3) \). The 1st order diffracted beam from hologram arriving at the screen gives the reconstructed output. A CCD-camera is used to detect the reconstructed output. Threshold operation is carried out by using an electronic processor connected to the CCD-camera. The input dependent threshold level is controlled by detecting the focused intensity of the undiffracted beam (0th order diffraction) passing through the hologram at the photo-diode detector, which is proportional to the number of transparent pixels of the input pattern \( N_0 \) in Eq.(10). The thresholded output is displayed on the monitor. Feedback is made by using the thresholded output as the new input pattern of next iteration. Figure 7 illustrates the outputs (reconstructed and thresholded) for the various inputs with differing HDISTs. The correct output \( b^i \) is obtained for the inputs of (a) and (b). Similarly \( b^2 \) is obtained for the inputs of (c) and (d), and finally \( b^3 \) is obtained for the inputs of (e) and (f).

In order to see the overall performance of the system quantitatively, computer simulations are carried out for our system and compared with that of the Hopfield model. Figure 8 represents the results of computer simulation in which the recognition ratio, defined as the number of correct recognitions
per the total number of the inputs. Simulations are repeated for 100 random generated inputs of each HDIST (0 to 25), and the results are averaged for the three memory vectors $b^1$, $b^2$, and $b^3$.

| INPUT | OUTPUT  \\
|-------|---------  \\
|       | Reconstructed | Thresholded |
| (a)   |          |             |
| (b)   |          |             |
| (c)   |          |             |
| (d)   |          |             |
| (e)   |          |             |
| (f)   |          |             |

![Diagram](image)

Fig. 8. Results of computer simulation of present model(+) with $\lambda=0.6$ in Eq.(10) and Hopfield model(□).

Fig. 7. Experimental results of recognizing the various inputs (a)~(f).

IV. Discussion and Conclusion

LPM utilizes both the simple implementation of outer-product learning and the better performance of adaptive learning. The interconnection matrix having elements all positive values, does not reduce basically the performance of the system, because it has an effect only in raising the bias level of the matrix, and it needs input dependent threshold level. In the experiment, learning patterns are recorded in HAM instead of the memory patterns. There is a considerable error correction capability in our HAM. Results of the computer simulation show that more than 90% recognition ability is obtained within the HDIST of 4 which corresponding to 16% error in pattern, which have been shown to be better than those of Hopfield model. Our system will be a good solution for recognizing the characters or images which have similar or common parts in the others, for example [E,F], [c,o,e], and human faces.

In conclusion, we have proposed a new HAM based on the the adaptive learning and the optical implementation is simply done by outer-product of learning patterns. We have shown that the memory patterns are stored inherently in stable state when these learning patterns are recorded in the hologram. We have presented the results of the optical experiment and computer simulation.

References