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Dynamics of multi-photon resonances in a driven Jaynes–Cummings system

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Fock-state is a highly non-classical radiation-field state. So if one can generate a Fock-state it is possible to study many interesting quantum-mechanical aspects. But in spite of its attraction, it is very difficult to generate a Fock-state experimentally although there have been many theoretical and experimental efforts to do it. Recently Chough et. al.\(^{(1)}\) proposed a feasible scheme to achieve quasi number states. The key is to exploit the multi-photon resonances occurring in a driven Jaynes–Cummings system, so it is important to understand the processes at multi-photon resonances. In the present work we study the dynamics of multi-photon resonances in the driven Jaynes–Cummings system.

In our model a two-level atom resonantly coupled with a single-mode cavity is driven by a classical field of arbitrary-frequency as described by a Hamiltonian

\[
H = \frac{1}{2} \hbar \omega_A \sigma_z + \hbar \omega_c a^+ a + i \hbar g (a^+ \sigma_+ - a \sigma_-) + i \hbar \epsilon (\sigma_+ e^{-i \omega_c} - \sigma_- e^{i \omega_c}),
\]

where \(\omega_A, \omega_c, \omega_L\) are the atomic transition, the cavity resonance, the driving-field frequency respectively. And \(g\) is the atom–cavity coupling strength, \(\epsilon\) the coupling strength of the driving field and the atom proportional to the driving-field amplitude. Without the external driving-field, the atom–cavity system has the energy eigenstates (dressed states) and eigenenergies as follows.

\[
|\Psi^+\rangle = \cos \theta |n-1, e\rangle + i \sin \theta |n, g\rangle,
\]

\[
|\Psi^-\rangle = \sin \theta |n-1, e\rangle + i \cos \theta |n, g\rangle,
\]

\[
E_{\pm}^N = n \hbar \omega_c - \frac{\hbar \delta}{2} \pm \hbar \sqrt{g^2 n + \frac{\delta^2}{4}},
\]

where \(\tan 2\theta = -\frac{2g\sqrt{n}}{\delta}, \quad \delta = \omega_c - \omega_A\).

When the frequency of the driving-fields is such that \(\hbar \omega_L = E_0^N\), then \(N\)-photon resonance appears. Moreover, if we observe the time-evolution of the mean photon-number and the excited-state population of the atom, it is inferred that the atom–cavity system evolves sinusoidally between \(|0, g\rangle\) and \(|\Psi^+_N\rangle\).

In the figure below, we draw the dynamics for instance at two-photon resonance as a function of the normalized time \(gt\). It shows the intra-cavity mean photon number and the excited-state population which coincide with each other. The atom–cavity system evolves adiabatically between the two states \(|0, g\rangle\) and \(|\Psi^+_2\rangle\). Of course, there is no direct coupling between \(|0, g\rangle\) and \(|\Psi^+_2\rangle\).
which can be easily seen in the above Hamiltonian. Therefore, the intermediate states $|\Psi^+_1\rangle$, $|\Psi^-_1\rangle$ should intervene in the interaction.

When the two-photon resonance condition is satisfied ($2\hbar \omega_L = E^+_1$), the foregoing Hamiltonian in the interaction picture with four truncated bases $|0, g\rangle$, $|\Psi^+_1\rangle$, $|\Psi^-_1\rangle$, $|\Psi^+_2\rangle$ is given as

$$
H' = \begin{pmatrix}
0 & -iV^-_1 & -iV^+_1 & 0 \\
iV^-_1 & A^+_1 & 0 & V^+_2 \\
iV^+_1 & 0 & A^-_1 & -V^-_2 \\
0 & V^-_2 & -V^+_2 & 0
\end{pmatrix}
$$

where

$$
A^+_1 = \frac{E^+_1}{\hbar} - \omega_L
$$

$$
V^+_1 = \epsilon \cos \theta_1, \quad V^-_1 = \epsilon \sin \theta_1
$$

$$
V^+_2 = \epsilon \cos \theta_2 \sin \theta_1, \quad V^-_2 = \epsilon \cos \theta_2 \cos \theta_1
$$

If $A^+_1$ are rather larger than the off-diagonal elements, then two eigenvalues ($\lambda_1, \lambda_2$) are very small and the corresponding eigenstates are the superposition of nearly only two states $|0, g\rangle$ and $|\Psi^+_2\rangle$. Thus if the initial state is $|0, g\rangle$ then the system evolves adiabatically between $|0, g\rangle$ and $|\Psi^+_2\rangle$ with the periodicity $2\pi/|\lambda_1 - \lambda_2|$.

This adiabaticity is very similar to the STIRAP used in the coherent population transfer. But in this case we cannot transfer the system from $|0, g\rangle$ to $|\Psi^+_2\rangle$ since $V^+_1, V^-_1, V^+_2, V^-_2$ are not controllable independently.

![Graph](image)

We will also discuss the anti-damping effect occurring at two-photon resonance. The mean photon number may be increased even if the damping constants are increased.

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