Squeezing by damping in a driven coupled-oscillator system

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A quantum-mechanical harmonic oscillator undergoes continuous amplitude fluctuation even in its ground state. This fluctuation, also known as the vacuum fluctuation, arises from the nonvanishing commutation relation, \([a, a^+] = 1\), where \(a(a^+)\) is the annihilation (creation) operator of the harmonic oscillator. One can make, however, the fluctuation of one quadrature amplitude decrease below the vacuum-state (or the coherent state) level at the cost of that of the other quadrature.

This phenomenon, known as squeezing, is most commonly generated by the inherent nonlinearity of the interaction involved, which is seen from the definition of the squeezing operator

\[ S(r) = \exp\left[ r/2(a^+ a - a^+ a^+ - a a^+) \right], \]

\( r \) being the squeezing parameter. Squeezing mechanism may thus be interpreted as making pairs of correlated oscillator quanta. In most cases, the oscillator is inevitably damped by its coupling to the reservoir, and the squeezing is degraded since the coupling introduces the reservoir fluctuations to the oscillator. For instance, the known schemes for generating the squeezed light employ the nonlinear processes such as the parametric amplification, or the four-wave mixing. Usually, the interacting medium is put inside an optical cavity in order to build up the field by increasing the interaction time. But the cavity damping destroys the quantum correlation of the field via loss of the correlated photon pairs.

In the conventional squeezing schemes, therefore, damping has been considered to play a negative role in squeezing. In this work, by contrast, we show that squeezing is manifested by damping when a harmonic oscillator, \( a \), is coupled to its reservoir, \( R_a \), under indirect pumping. The coupling strengths to each mode of the reservoir oscillators are related to the damping rate, \( \Gamma_a \), of the oscillator by the fluctuation-dissipation theorem. We will show that the squeezing effect is enhanced as the damping rate is increased to some degree. Thus, we call the mechanism squeezing by damping.

In our model, the target oscillator \( a \) is, instead of being pumped directly, coupled to an auxiliary saturable oscillator \( b \) driven by a classical field. Although the system has the inherent nonlinear property due to the saturability of the oscillator \( b \), the nonlinearity producing the squeezing effect does not reveal itself without the damping. The fundamental cause of the squeezing is the
correlation of the oscillator $b$ and the reservoir $R_a$. This nontrivial correlation is, of course, brought by the coupling chain $R_a \leftrightarrow a \leftrightarrow b$. Since the amplitude of the oscillator $a$ is contributed from both the oscillator $b$ and the reservoir $R_a$, the (negative) correlation between $b$ and $R_a$ can make one quadrature-fluctuation of the oscillator $a$ decrease below the vacuum level.

The unconventional features of the squeezing by damping can be seen from its characteristic condition $2 \Omega \Gamma_a/g^2 \sim 1$, where $g$ is the coupling strength between the oscillators $a$ and $b$, and $\Omega$ is the pumping strength. This condition shows that the pumping strength should be inversely proportional to the damping rate for the optimal squeezing. This relation is in a striking contrast to those of the conventional schemes, where the pumping intensity should be proportional to the damping rate.

We employ the quantum-Langevin approach to investigate the correlation between $a$ and $b$, and the equivalent master equation to calculate numerically the dynamics of the system. In particular, the effective Hamiltonian describing the novel mechanism of the squeezing by damping is derived from the master equation.

![Graphs showing mean excitation number, Mandel-Q value, and phase fluctuation](image)

Fig. 1 (a) Mean excitation number (divided by 4), the Mandel–Q value, and the phase fluctuation $\Delta \cos \phi$. (b) $\Delta X_1$ and $\Delta X_2$ as a function of $2 \Gamma_a/g$ for $\Omega/g = 2$, $\Gamma_b/g = 0.02$.