A Line-to-ground Cable Fault Location Method for Underground Distribution System

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Abstract - This paper proposes a line-to-ground cable fault location method for underground distribution system. The researched cable is composed of core and sheath. And underground cable system has been analyzed using Distributed Parameter Circuit. The effectiveness of proposed algorithm has been verified through EMTDC simulations.

1. Introduction

Modern power systems demand larger capacity transmission and higher quality electric power than ever. Due to the increased environmental concern, electrical cables are present in a very large number of industrial and residential areas. One of the main applications of underground circuits is for underground residential distribution. But when there are some faults in underground cable system, finding the failure is harder, and fixing the damage or replacing the equipment would take longer at great costs. So cable fault location must be estimated as accurately as possible. So far many techniques and methods of cable fault location have been reported. For instance, one of the fault location methods is using Traveling Wave [1].

This paper proposes a new algorithm calculating the fault distance for one-phase to ground fault on an underground power cable. Proposed algorithm has been tested with various fault distances and fault impedances.

2. Proposed Algorithm

2.1 Distributed Parameter Circuit (DPC) [2]

Fig 1 Relationship of Voltage and Current in the line

Cable impedance $Z_{dc}$ and admittance $Y_{dc}$ are distributed parameter on every $dx$ of the cable line as shown in Fig.1. The basic equation of distributed parameter circuit is:

$$dE = -Idx$$

And then (1) can be expressed as follows:

$$-dE/dx = I$$

Combining (1) and (2), it is easy to get the result:

$$d^2E/dx^2 = Z_{dc}E$$

Assuming $\gamma = \sqrt{Z_{dc}}$, $I$ and $E$ can be solved as follows:

$$I = A_1 e^{-\gamma x} - A_2 e^{\gamma x}$$

$$E = \gamma (A_1 e^{-\gamma x} + A_2 e^{\gamma x})$$

Coefficient method.

Since an exponential function can be transformed into hyperbolic function via the formula which is expressed as follows:

$$\gamma = \cosh \gamma x \pm \sinh \gamma x$$

Finally the solution of $E$ and $I$ can be expressed by hyperbolic function.

$$E_x = E_c \cosh \gamma x - \frac{Z_{dc}}{\gamma} I_s \sinh \gamma x$$

$$I_x = -\frac{Z_{dc}}{\gamma} E_s \sinh \gamma x + I_c \cosh \gamma x$$

That means, as long as the voltage and current of sending-end is known, the voltage and current of some point in the line can be calculated through the above-mentioned solution (6).

2.2 Proposed Algorithm

2.2.1 Equations based on DPC

A simplified model of cable system is shown in Fig.2. In comparison with overhead line, underground cable takes on significant characteristics such as a little smaller inductance and quite larger capacitance.

Due to the analysis of Distributed Parameter Circuit, it is easy to get the following equations:

$$-\frac{\partial V_{dc}}{\partial x} = Z_{dc} V_{dc} + Z_{sdc} I_{dc}$$

$$-\frac{\partial V_{sc}}{\partial x} = Z_{sc} I_{dc} + Z_{sdc} I_{sc}$$

$$-\frac{\partial I_{dc}}{\partial x} = Y_{dc} V_{dc} + Y_{sdc} V_{sc}$$

$$-\frac{\partial I_{sc}}{\partial x} = Y_{sc} V_{sc} + Y_{sdc} V_{dc}$$

where

$Z_{dc}$: core impedance for three phase

$Z_{sdc}$: sheath impedance for three phase

$Z_{sc}$: core-to-sheath mutual impedance for three phase

$Y_{dc}$: core impedance for three phase

$Y_{sdc}$: sheath impedance for three phase

$Y_{sc}$: core-to-sheath mutual impedance for three phase

Through symmetrical conversion, (7) to (10) can be expressed in matrix form as follows:
Zero-sequence, positive-sequence and negative-sequence are independent in (11). In order to explain this proposed algorithm more clearly, the equation of positive-sequence would be chosen to analyze below.

In positive-sequence, the eigenvalues of coefficient matrix in (11) can be calculated as $\alpha_1$ and $\beta_1$. So according to the analysis of Distributed Parameter Circuit, (11) can be solved. The solutions would be expressed in hyperbolic function, which are illustrated as follows:

$$
V_{x0}(x) = A_1 \cosh \alpha_1 x + B_1 \sinh \alpha_1 x + C_1 \cosh \beta_1 x + D_1 \sinh \beta_1 x \\
V_{y0}(x) = A_1 \cosh \alpha_1 x + B_1 \sinh \alpha_1 x + C_1 \cosh \beta_1 x + D_1 \sinh \beta_1 x \\
I_{x0}(x) = A_1 \cosh \alpha_1 x + B_1 \sinh \alpha_1 x + C_1 \cosh \beta_1 x + D_1 \sinh \beta_1 x \\
I_{y0}(x) = A_1 \cosh \alpha_1 x + B_1 \sinh \alpha_1 x + C_1 \cosh \beta_1 x + D_1 \sinh \beta_1 x
$$

Through undetermined coefficient method, all coefficients can be solved as listed below:

$$
A_1 = C_1 \alpha_1 | A_1 |, B_1 = C_1 \beta_1 | A_1 |, C_1 = C_1, D_1 = C_1, D_1 = C_1 C_1
$$

So 16 coefficients can be decreased to 4 coefficients. And (12) to (15) are expressed in matrix form as follows:

$$
\begin{bmatrix}
V_{x0} \(a\) \\
V_{y0} \(a\) \\
I_{x0} \(a\) \\
I_{y0} \(a\)
\end{bmatrix} =
\begin{bmatrix}
\cosh \alpha_1 x + \cosh \beta_1 x \\
\cosh \alpha_1 x + \cosh \beta_1 x \\
\cosh \alpha_1 x + \cosh \beta_1 x \\
\cosh \alpha_1 x + \cosh \beta_1 x
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1
\end{bmatrix}
$$

Also through undetermined coefficient method, all constants can be solved as shown below:

$$
C_{ii} = \frac{\alpha_1 - 2 \alpha_1 Z_a Y_{ac} Z_a Y_{bc}}{Z_a Y_{ac} Z_a Y_{bc}}, \quad C_{ii} = \frac{Y_a + Y_c C_{ii}}{\alpha_1}
$$

By the same way, it is easy to get the equations and constants of zero-sequence and negative-sequence.

2.2.2 Analysis of fault conditions

Assuming that core-to-sheath to ground fault occurs in phase a, the equivalent circuit model is shown in Fig.3.

The cable system is divided into two sections which are Section A and B. Section A is from sending-end to fault point, and Section B is from fault point to receiving-end. According to (16), it is easy to establish the equations of Section B. For example, the equations of positive-sequence is expressed as follows:

$$
V_{e0} = L_x I_{x0} + Z_x Y_{x0} I_{x0} = - (La_2 a_2 - La_1 a_1) R_0 I_{x0} = 0
$$

Assuming the actual fault location is $p$ and $x=p$ is same point with $y=0$.

In order to calculate unknown parameters, all conditions in the whole system would be summarized.

(a) Conditions for zero-sequence, positive-sequence and negative-sequence

The following conditions are analyzed at the sending-end:

(i) Core voltage is equal to source voltage.

$$
V_{x0}(0) = V_d, \quad V_{y0}(0) = V_d, \quad V_{z0}(0) = V_d
$$

(ii) Core current is equal to source current.

$$
I_{x0}(0) = I_d, \quad I_{y0}(0) = I_d, \quad I_{z0}(0) = I_d
$$

(iii) Sheath voltage of zero-sequence and negative-sequence is equal to the multiplication of the grounding resistance of zero-sequence and sheath current of zero-sequence; but sheath voltage of positive-sequence is equal to zero.

$$
V_{x0}(0) = R_{g0} I_{x0}(0), \quad V_{y0}(0) = 0, \quad V_{z0}(0) = 0
$$

The following conditions is analyzed at the fault point:

(iv) Core voltage of Section A is equal to that of Section B.

$$
V_{x0}(p) = V_{x0}(0), \quad V_{y0}(p) = V_{y0}(0), \quad V_{z0}(p) = V_{z0}(0)
$$

(v) Core voltage is equal to the multiplication of the load impedance and core current.

$$
V_{x0}(p) = Z_{l0} I_{x0}(p), \quad V_{y0}(p) = Z_{l0} I_{y0}(p), \quad V_{z0}(p) = Z_{l0} I_{z0}(p)
$$

(vi) Same as Condition (iii).

$$
V_{x0}(0) = R_{g0} I_{x0}(0), \quad V_{y0}(0) = 0, \quad V_{z0}(0) = 0
$$

(b) Conditions in phase a, b and c

The following conditions are analyzed at the fault point:

(vii) Sheath voltage of the faulted phase a in both Section A and B are equal to zero.

$$
V_{x0}(p) = 0, \quad V_{y0}(p) = 0
$$

(viii) In the non-faulted phase b, core current of Section A is equal to that of Section B. Also sheath current of Section A is equal to that of Section B.

$$
I_{x0}(b) = I_{x0}(b), \quad I_{y0}(b) = I_{y0}(b)
$$

(ix) In the non-faulted phase c, the conditions are same with condition (viii).

$$
I_{x0}(c) = I_{x0}(c), \quad I_{y0}(c) = I_{y0}(c)
$$

From all conditions above, it is easy to establish 24 equations based on proposed algorithm to calculate all parameters, such as $A_1, B_1, C_1, D_1, E_1, F_1, G_1, H_1$ for positive-sequence. At the same time, other parameters for zero-sequence and negative-sequence also can be calculated easily.

2.2.3 Analysis of the solution

In Fig.4, it is easy to get the fault equation $V_j = I_j R_j$ and the following equation can be made.

$$
R_j p, R_j = V_{x0} = -(La_2 a_2 - La_1 a_1) R_0
$$

And then (27) can be divided as follows:

$$
R_j p, R_j = f(p, R_j) + jf(p, R_j) = 0
$$
That means, \( f(p, R_p) = 0 \), \( f(p, R_r) = 0 \) \( \text{(29)} \)

At last, Newton-Raphson iteration method is dedicated in getting the value of fault distance \( p \).

2.2.4 Equivalent method for one-section
For simplification, Assuming that there are two sections in cable system, which is shown in Fig.5.

![Circuit diagram](image)

Fig.5 Circuit of cable system with two sections

According to Fig.5, equivalent method is depicted as follows:

\[
Z_{eq} = \begin{bmatrix} Z_{11} & 0 \\ 0 & R_{eq} \end{bmatrix}
\]

i) The admittance \( Y_{eq} = Y_{eq} \)

ii) 1st equivalence: \( Z_{eq, ab} = Z_{eq} \cdot (E + Y_{eq, e} \cdot Z_{eq})^{-1} \), \( E \) is e.

iii) 2nd equivalence: \( Z_{eq, ab} = Z_{eq, ab} + Z_{eq} \)

iv) 3rd equivalence: \( Z_{eq, ab} = Z_{eq, ab} \cdot (E + Y_{eq, e} \cdot Z_{eq, ab})^{-1} \), so finally it would be expressed as follows:

\[
Z_{eq, ab} = \begin{bmatrix} Z_1 & Z_2 \\ Z_3 & Z_4 \end{bmatrix}
\]

As tested in EMTDC, \( z_2 \) is much smaller than \( z_1 \) that means the influence of sheath-to-core is very smaller when using equivalent method to calculate the equivalent impedance.

Due to the analysis of impedance equivalent method, Condition (v) is changed as:

\[
\begin{align*}
V_{eq}(l-p) &= Z_{eq} \cdot I_{eq}(l-p) \\
V_{eq}(l-p) &= Z_{eq} \cdot I_{eq}(l-p)
\end{align*}
\]

\( \text{(30)} \)

Condition (vi) is changed as:

\[
\begin{align*}
V_{eq}(l-p) &= Z_{10} \cdot I_{eq}(l-p) + Z_{10} \cdot I_{eq}(l-p) \\
V_{eq}(l-p) &= Z_{20} \cdot I_{eq}(l-p)
\end{align*}
\]

\( \text{(31)} \)

As discussion above, the analysis of all conditions are suitable for cable system with one section. In order to make this algorithm fit other cable system with two or more than two sections, some conditions have to be changed. So equivalent method regards a complex cable system as a simple cable system with one section. So the above-mentioned conditions can fit it without any change. If fault occurs in the second section in Fig.5, Condition (i) and (ii) have to be changed. That is, the values of source voltage and current would be changed into that of the beginning of the second section. Also, Considering that sheath voltage and current are very sensitive, Condition (iii), the sheath current at the sending-end would be assumed, which makes proposed algorithm more precisely.

If there are more than two sections in cable system, it would be done by the same way.

3. Case Study
3.1 Underground cable system model
The cable type is SC coaxial cable consisting of core and sheath (of 2000 mmp², kraft) [3], and the model system is composed of four sections as shown in Fig.6. The voltage level is 154 kV and cable total length is 1200m.

All grounding resistance are 10[Ω]. The parameters of the tested cable impedance and admittance are obtained from EMTDC simulation.

![Simulation model](image)

Fig.6 Simulation model system in EMTDC

3.2 Test results
In this test, the fault type is a core-to-sheath to ground fault in phase A. In each case, four different fault distances varying from 150[m] to 1080[m] by 300[m] step and three fault resistances of 0.1[Ω], 15[Ω], 30[Ω] have been considered. The phasors of the core voltages and currents at the sending-end are obtained by the Discrete Fourier Transform (DFT) having one cycle data window. The error of the fault location is calculated by the following equation.

\[\text{Error}(\%) = \frac{\text{Estimated distance} - \text{Actual distance}}{\text{cable total length}} \times 100 \]

(32)

Figure 7 shows the estimation error with three different fault resistances. The maximum error of 0.9[%] is observed for a fault resistance less than 15[Ω], especially the maximum error of 1.1[%] is observed for a fault resistance of 30[Ω]. Therefore, proposed algorithm is efficient to estimate fault location for underground cable system.

![Estimation Error](image)

Fig.7 Estimation Error of the proposed algorithm

4. Conclusion
This paper proposes a line-to-ground cable fault location method for underground distribution system. Test results verify that proposed algorithm is useful to estimate cable fault location for underground distribution system. Circuit equivalent method makes it more effective and flexible. That means proposed algorithm is suitable for more complicated cable system. The further research is going to deal with real cable system which combines underground cable and overhead line.

Reference