A Decision Theoretic Approach to Source Direction Finding Based on the Hopfield Neural Network

Hopfield 신경회로망에 바탕을 둔 음원 방향 탐지의 결정 이론적 접근

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Abstract

A decision theoretic concept is introduced to investigate whether targets of interest are present or not, at some steering direction. The solutions to this problem are described as a set of discrete numbers 0 or 1, corresponding to the direction under consideration. This coded number representation is transplanted in the optimisation technique based on the Hopfield neural network, which may provide an easy understanding of determining the direction of arrival (DOA) of sources. Difficulties encountered in using the conventional state schemes of Hopfield neural network models are addressed and their related issues are raised. To deal with them, the idea that a neuron that decreases more energy difference for its state change of 0 to 1 can have higher priority in the order of state transition than others is introduced. This does not only lead to a new state update scheme but also opens a different story in comparison to previous work. To cast the perspectives of the proposed approach and illustrate its effectiveness in source direction finding in array sensor systems, simulation results and related discussions are presented in this paper.

요 약

다중배치 시스템의 본질적, 특화 감지의 방향에 대한 목표물의 유무를 판단하는 문제에 접근하기 위하여 결정이론의 소개

Abstract

A decision theoretic concept is introduced to investigate whether targets of interest are present or not, at some steering direction. The solutions to this problem are described as a set of discrete numbers 0 or 1, corresponding to the direction under consideration. This coded number representation is transplanted in the optimisation technique based on the Hopfield neural network, which may provide an easy understanding of determining the direction of arrival (DOA) of sources. Difficulties encountered in using the conventional state schemes of Hopfield neural network models are addressed and their related issues are raised. To deal with them, the idea that a neuron that decreases more energy difference for its state change of 0 to 1 can have higher priority in the order of state transition than others is introduced. This does not only lead to a new state update scheme but also opens a different story in comparison to previous work. To cast the perspectives of the proposed approach and illustrate its effectiveness in source direction finding in array sensor systems, simulation results and related discussions are presented in this paper.

I.Introduction

In the last decade, high resolution array signal processing methods have appeared that are com-
mosty based on the eigenstructure of the correlation matrix. These methods [1-6] consist of estimating the correlation matrix matrix form the measurements of equi-spaced array sensors and decomposing the matrix into the signal subspace and the noise subspace. The orthogonality between the subspaces is exploited to achieve the high resolution spectral distribution over the steering angle. Those techniques have provided fundamentals for approaching acoustic radar or sonar systems problems—the early direction finding of stationary or moving sources (aircraft or sea vessels). They are in general based on linear algebra, i.e., singular value decomposition [7], which enables us to estimate the “best-fitted” direction of arrival of radiating or reflecting sources in the least squares sense.

However, we have observed another aspect arising from the array sensor system, that is the early warning system. It involves a classical problem of deciding whether sources are present at the steering direction or not (in details discussed in Chapter 2 of reference [8]). When a source at the steering angle is present, the solution corresponding to the position is one, and otherwise it becomes zero. Obviously, the solutions to the problem are the set of simple numbers, 0’s or 1’s. It is indeed a decision-theoretic problem that accompanies a “nonlinear” mapping of processed information about the DOA onto the decision space whose state is generally described as a set of binary numbers. The above eigenstructure methods may provide the basis for obtaining related information, but do not lead to any lead to any logical approach to the nonlinear decision mapping problem. This fact implies the possibility of approaching the direction finding problem from different viewpoints.

This paper exploits the optimisation technique using the Hopfield neural network models [9,10], which have proven to be very successful in combinatorial (NP-completeness) optimisation problems: the traveling salesman problem of finding the shortest route connecting multiple cities [11] and the Hitchcock problem of distributing goods from several sources to numerous locations in such a way to minimise the transportation cost [12]. One of the important properties of the Hopfield model-based optimisation method is the ability to simultaneously consider a large number of alternative hypotheses and at remarkable speed make adequate decisions on them for given data. This feature has provided the major motivation of investigating the effectiveness of the Hopfield model-based optimisation in source direction finding. In Section II, basic ideas behind the Hopfield models are described and linked to the above decision problem in the array sensor systems. In Section III, we map this decision problem onto the Lyapunov candidate function of the Hopfield model and make modifications for improving the possibility of convergence to the better solutions. Simulation results and discussions are presented in Section IV. Finally, concluding remarks are summarised in Section V.

II. Fundamentals in Hopfield Neural Network Models

1. Hopfield Models

The Hopfield models [9,10] consist of a number of mutually interconnected computation units, called the neurons, whose states are defined by their outputs \(v_i\). Each neuron state can be described as a discrete value, i.e., 0 or 1. Fig. 1

![Fig. 1 Schematic setup of Hopfield neural networks](image-url)
shows the schematic setup of Hopfield neural network models.

Each neuron receives multiple inputs, denoted by the vector \( v = (v_1, v_2, \ldots, v_N) \), projects them onto its interconnection weight vector \( w = (w_1, v_2, \ldots, v_N) \), and then adds an externally supplied bias input \( b \) to the weighted value. This result represents the internal potential \( u \) of neuron \( i \)

\[
\tag{1} u_i = \sum_{j=1}^N w_{ij} v_j + b_i
\]

where \( N \) is the number of neurons. When switches \( s_k \) in Fig.1 turn on at some discrete time, the sampled internal potentials \( u \), are sent to the nonlinear activation units to change or leave neuron outputs according to a threshold performed by the nonlinear units \( N \), that is

\[
\tag{2} v_i(n) = N(u_i(n)) = \text{step}(u_i(n))
\]

where \( n \) is discrete time index and \( \text{step}(u) \) denotes a unit step function which is 1 for \( u > 0 \) and 0 for \( u \leq 0 \). Thus neurons take binary values 0 or 1. These binary outputs are feed back to the input function of interconnected weights so that neurons gradually evolve to one of stable states in \( N \)-dimensional discrete space.

Hopfield [9,10] showed that if neuron weights \( w \) are symmetric \( (w \equiv w^T) \) then neurons in the network model evolve to one of stable states in such a way of minimising a Lyapunov candidate function, called the energy function,

\[
\tag{3} F = -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N w_{ij} v_i v_j - \sum_{i=1}^N b_i v_i.
\]

In fact, the evolution neurons give in (1) is seen to be identical to the negative gradient of the energy function (3) with respect to the neuron states \( \{v_i\} \). Here, one point is clear that only when we define a cost function of interest in our problems that is equivalent to the energy function (3) we can find their solutions by updating the neuron states according to (1) and (2). This aspect is well illustrate in previous work such as the traveling salesman problem [11] and the Hitchcock problem [12]. A major issue in defining the energy function for direction finding will be examined in Section III, and the rest of this section will address the neuron state transition scheme.

2. Asynchronous State Transition Mode

In Fig.1, the transition of neuron states is shown to depend on the ways of operating the switch/sample unit. As noted by Takeda and Goodman [12], there are several possible ways of doing it. When we turn on all the switches synchronously at some discrete time \( n \), we can simultaneously update all the neurons. This state transition scheme, referred to as the synchronous state transition, seems to be a "normal" one. By contrary in the stability proof in [9,10], we have experienced "unexpected" results of this scheme, i.e. "oscillatory or wandering" behaviour of the neuron states around the minima of the energy function, and moreover have failed to implement the state transition in a stable manner. A clear understanding of the reason for these unfavourable still remains as an open question in the neural network community. However, it should be noted that the unfavorable features can arise from the ways of updating all the neuron states, i.e. the ways of operating the switches in Fig.1.

We choose the asynchronous neuron state transition scheme [12] so as to reduce the above unfavorable features as much as possible. The scheme is as follows: The switches in this asynchronous mode are turned on and off with random delay between each switch such that neurons change their states

\[
\tag{4} u_i(n + \Delta t) = \sum_{j=1}^N w_{ij} v_j(n + \Delta t) + b_i
\]

and

\[
\tag{5} v_i(n + \Delta t + \epsilon) = N(u_i(n + \Delta t))
\]

where \( \Delta t \) is random time delays and \( \epsilon \) is a small positive constant (more details are found in [12]).
In the applications of Hopfield neural networks, another important question is how to draw logical clues from our practical problems and then link them to our problems. First, let us consider K equi-spaced array sensors to monitor M plane wave sources located at angles \( \{\theta_m : m = 1, \ldots, M\} \). Using the quadrature demodulation/modulation-based preprocessing unit as in Fig. 2, monitored signals are obtained in an analytic form of sampled narrowband signals for \( n = 1, \ldots, N \).

\[
y(n) = [y_1(n), y_2(n), \ldots, y_M(n)]^T = S \cdot c(n) + n_p(n) \quad (5)
\]

In (5), the matrix \( S = [s_1, s_2, \ldots, s_M] \) consists of the steering vectors \( s_m = [1, e^{-j\theta_1}, e^{-j\theta_2}, \ldots, e^{-j\theta_M}] \) \( (t_m = \kappa \cdot d \cdot \sin(\theta_m)) \) denotes a phase difference, \( \kappa \) and \( d \) are the wave number and the gap between equi-spaced array sensors, respectively, \( c(n) = [c_1(n), c_2(n), \ldots, c_M(n)]^T \) is the complex amplitude vector of \( M \) sources, and \( n_p(n) = [n_{p1}(n), n_{p2}(n), \ldots, n_{pM}(n)]^T \) denotes the preprocessed complex noise vector.

**III. Hopfield Model-Based Direction Finding**

A major problem of interest in this paper is to decide whether a source at some direction \( \theta \) is present or not. To approach this classical decision
problem, we may choose the range of a limited steering angle to some desired resolution. For example, given the range between the steering and final directions $\theta_i$ and $\theta_j(\theta_i, \theta_j)$, our interest is concerned with $N_0 + 1$ directions $\theta_i = \theta_0 + i \Delta \theta$ $i = 0, 1, \ldots, N_0$ with direction resolution $\Delta \theta = (\theta_i - \theta_j)/N_0$, $N_0$ the direction index. If a source at the direction $\theta$, present, then the decision result may be described as a value of 1 and, if not, it may be a value of 0. One bit may be sufficient to describe the decision state at direction $\theta$. Thus each neuron state $v_j$ is related to the decision state of direction $\theta_i$. For $N_0 + 1$ directions, $(N_0+1)$ neurons participate to solve the direction finding problem.

To approach the direction finding problem, we should define an adequate cost function that is of the quadratic form similar to (3). As introduced by Rastogi et al. [13], the orthogonal projection matrix $P_j = u_j \cdot u_j^H$ (the super script $H$ denotes the Hermitian operator), which is constructed by the unit steering vector $u_j = [1, e^{-i\Delta \theta}, e^{-2i\Delta \theta}, \ldots, e^{-(N_0+1)i\Delta \theta}]$ $\sqrt{N}$ ($\tau = \kappa \cdot \sin(\theta)$) of direction $\theta_i$, can be exploited to examine the presence of a source at the direction. By projecting time series $y(n)$ onto the orthogonal matrices $\{P_j\}$, we obtain $(N_0+1)$ direction components

$$d_j(n) = [u_j \cdot u_j^H] \cdot y(n) = a_j(n) \cdot u_j,$$  \hspace{1cm} (6)

where $a_j(n) = (u_j^H \cdot y(n))$ is the direction cosine of $y(n)$ in reference to the projection matrix $P_j$. The direction components include useful information to judge the existence of source at direction $\theta_i$, so that they may be related to the decision result described as the neuron states $\{v_j\}$. It is readily seen that if $v_j$ is close to 1 then much ‘weighting’ value is assigned to the unit vector $u_j$ (n) while in case of $v_j = 0$ no significance is given to it. Let the projected direction matrix $D(n) = [d_0(n), d_1(n), \ldots, d_{N_0}(n)] = [P_0 \cdot y(n), P_1 \cdot y(n), \ldots, P_{N_0} \cdot y(n)]$ at time index n and the decision state $v = [v_0, v_1, \ldots, v_N]^T$, Then we can construct a K dimensional signal $y(n) = D(n) \cdot v$ and compare it to the sampled signal $y(n)$. Here we can define the cost function as the mean squared error between the sampled and constructed signals for the N0 samples

$$E = \frac{1}{N_0} \sum_{n=1}^{N_0} |y(n) - y(n)|^2 + \frac{1}{N_0} \sum_{n=1}^{N_0} |y(n)|^2$$

$$= \sum_{n=1}^{N_0} \sum_{n=1}^{N_0} Re \left[ -\frac{1}{N_0} \sum_{n=1}^{N_0} a_j(n)^* a_i(n) \cdot \delta_{ij} \cdot v_j \cdot v_i \right]$$

$$- \sum_{n=1}^{N_0} \sum_{n=1}^{N_0} |a_j(n)|^2 \cdot v_i$$  \hspace{1cm} (7)

where $Re \cdot v_j$ denotes the real part of complex number, the symbol $\delta_{ij}$ is the complex conjugate, and a scalar value $\delta_{ij} = u_i^H \cdot u_j$ is the direction cosine between two unit vectors. As explained in [13], the cost function is of the quadratic form respect to the decision states $\{v_j\}$ and is also similar to the Hopfield energy function (3). This implies the possibility of solving the direction finding problem using the Hopfield neural network based optimization technique.

To remove the instability arising from the non-zero diagonal terms $w_{ii} \neq 0$ of the coefficients of $v_i, v_j$ in (7) as noted by Hopfield [9,10], we may add to the cost function (7) another cost term

$$\frac{1}{N_0} \sum_{n=1}^{N_0} \sum_{n=1}^{N_0} \sum_{n=1}^{N_0} a_j(n)^2 \cdot v_i (1 - v_i)$$  \hspace{1cm} (8)

whose minimisation constrains the decision results $v_i$ to lie in 0 or 1. Therefore, it is straightforward to obtain the expressions of the interconnection weights and the bias terms as in (3) by computing the negative gradient of the added cost function of (7) and (8) with respect to the decision states $\{v_j\}$

$$w_{ji} = -\frac{2}{N_0} \cdot Re \left[ \frac{1}{N_0} \sum_{n=1}^{N_0} a_j(n)^* a_i(n) \cdot \delta_{ij} \right]$$

for $i \neq j$  \hspace{1cm} (9)

and $w_{ii} = 0$. 
The interconnection weights \( \mathbf{w}_i \) are shown to be computed from the mean of real parts of inner product of two direction vectors \( a_i(n)u_i \) and \( a_i(n)u_i \), and the bias terms to be the mean squared value of direction vectors \( a_i(n)u_i \). The weights and the bias terms in (9) and (10), which appear similar to a time averaged version of the corresponding terms in [13-15], are used for the state transition scheme (4) in Section II.2.

IV. Simulation Results and Discussions

Computer simulations were carried out to examine the decision results by applying the Hopfield model-based optimisation technique to the direction finding problem in the array sensor system that consists of 16 equi-spaced sensors \( K = 16 \). Other simulation parameters were as follows: the space \( d \) was chosen to satisfy \( k \cdot d = \pi \cdot c \cdot \tau \) (wave number \( 2\pi /k \), \( c \) = considered frequency and \( \tau \) = wave speed), a sampling time \( \Delta T = 128 / f_s \) (128 words per period), and the normalised bandwidth of low-pass filter \( \Delta B = 0.1 \) (pass-bandwidth \( \times 2 \cdot \Delta T \)). We considered four sources \( M = 4 \) located at \( 46.0°, 94.0°, 132.0°, 312.0° \) whose relative amplitudes were \( [1.0, 0.7, 1.1, 1.0] \). Given the signal-to-noise ratio (SNR) and the bandwidth ratio of the low-pass filter \( \Delta B = \) the pass bandwidth/the Nyquist frequency of sampled signals in the quadrature demodulation/modulation unit, the Gaussian random variables with zero mean and variance

\[
\sigma^2 = \frac{1}{N} \sum_{n=1}^{N} |c_{mn}|^2 / 2 \cdot 10^{\text{SNR} / 10} \times \Delta B^2
\]  

were added to the original source signals and then the preprocessing unit shown in Fig.2 was employed to generate 'noisy' analytic signals as given in (5). It should be noted that SNR in (11) indicates the amount of contaminated level within the pass bandwidth of the analytical signals, not in the full bandwidth of array sensor outputs. The initial neuron potentials \( h_{n,m} \) were set to 0.0 and the outputs \( h_{n,m} \) were initialised by the small random variables uniformly distributed in \( 10^{-3} \times [0, 1] \). The asynchronous state transition scheme proposed in Section II.2 were used to update the neuron outputs.

In this simulation, the range of direction-of-arrival (DOA) from 0° to 50° degrees was discretised at intervals of 0.5° degrees such that a Hopfield network model of 101 neurons was considered to examine the decision theoretic problem of source direction finding. For SNR = -20 [dB] (equivalent SNR = 0 [dB] after quadrature demodulation/modulation-based preprocessing), we first computed the weights and bias terms in (9) and (10), evaluated the energy difference levels of 101 neurons by changing each neuron state from 0 to 1, and then sorted them in the ascending order. Fig.3 shows a typical example for the evaluated energy difference levels of 101 neurons.

![Fig. 3. Energy difference level for state transition of 101 neurons from 0 to 1.](image)

In this figure, four distinctive energy difference levels (marked by 'I', 'II', 'III', and 'IV') are shown. The state transition of neurons whose difference is less than that of level II can minimise the energy (3) more than other neurons can do so that they are first of all updated. Neurons whose energy difference level is between level II and III are next updated. According to the ascending order of the difference energy levels, the rest of neurons are updated one by one.

Fig.4(a) illustrates the decision state (solid line), that
is the neuron outputs, after updating 101 neurons. Note that the dashed line is the 'normalised' mean squared value of projected direction vectors \( \mathbf{v}_1 \), \( \mathbf{v}_2 \) and the four vertically dashed lines denote the position of sources. The final neuron states at the directions \( 6.0°, 15.0°, 31.0°, 44.0° \) are shown to be set to 1 and the other states to be 0. These decision results indicate some evidence that the decision-theoretic approach addressed in this paper leads to the possibility of solving the direction finding problem of array sensor systems.

Fig. 4. Decision result and trend of energy level after 101 neuron state updates.

Fig. 4(b) shows the trend of the network energy defined in (7) during 701 during 101 neuron state updates. A series of four staged energy transitions are seen to occur, which is well matched with the four distinctive energy difference levels shown in Fig. 3. Each 'steep' energy transition is observed only when neuron updates are moved from the region of one distinctive energy difference level to another. But, no obvious change of the energy level is seen for other neuron state updates. Specifically, after first 47 neurons has been updated, the energy level is shown to remain constant. This means that the decision state has been settled down to the final steady solution.

We further examined independent samples to see their final decision results. Table 1 shows the final results, their mean and variance. Even in case of 20 dB SNR, no mistake in deciding the directions of three sources at \( 16.0°, 31.0°, 44.0° \) are made. But, for the source direction \( 15.0° \) whose strength is below 5 dB in comparison to that of others, the decision state is shown to be a little biased in TABLE 1.

TABLE 1. Decision results for seven independent samples.

<table>
<thead>
<tr>
<th>Samples</th>
<th>Source Locations [deg]:</th>
<th>Energy E</th>
</tr>
</thead>
<tbody>
<tr>
<td>#1</td>
<td>( \theta_1 = 6.0°, \theta_2 = 15.0°, \theta_3 = 31.0°, \theta_4 = 44.0° )</td>
<td>-5662.68</td>
</tr>
<tr>
<td>#2</td>
<td>( \theta_1 = 6.0°, \theta_2 = 31.0°, \theta_3 = 44.0° )</td>
<td>-5551.28</td>
</tr>
<tr>
<td>#3</td>
<td>( \theta_1 = 6.0°, \theta_2 = 15.0°, \theta_3 = 31.0°, \theta_4 = 44.0° )</td>
<td>-5655.23</td>
</tr>
<tr>
<td>#4</td>
<td>( \theta_1 = 6.0°, \theta_2 = 15.0°, \theta_3 = 31.0°, \theta_4 = 44.0° )</td>
<td>-5673.64</td>
</tr>
<tr>
<td>#5</td>
<td>( \theta_1 = 6.0°, \theta_2 = 31.0°, \theta_3 = 44.0° )</td>
<td>-5689.15</td>
</tr>
<tr>
<td>#6</td>
<td>( \theta_1 = 6.0°, \theta_2 = 15.0°, \theta_3 = 31.0°, \theta_4 = 44.0° )</td>
<td>-5699.52</td>
</tr>
<tr>
<td>#7</td>
<td>( \theta_1 = 6.0°, \theta_2 = 31.0°, \theta_3 = 44.0° )</td>
<td>-5726.94</td>
</tr>
<tr>
<td>Mean</td>
<td>( \theta_1 = 6.0°, \theta_2 = 15.0°, \theta_3 = 31.0°, \theta_4 = 44.0° )</td>
<td>-5652.64</td>
</tr>
<tr>
<td>Variance</td>
<td>( 0.00, 0.04, 0.00, 0.00 )</td>
<td>1962.15</td>
</tr>
</tbody>
</table>

At the beginning of this study, we had chosen the state transition scheme suggested in previous work \([12]\), referred to as the 'sequential' state transition scheme, which based on the order of time delays as \( 0 \leq \Delta t_1 \leq \Delta t_2 \leq \cdots \leq \Delta t_n \). Fig. 3 shows the final decision state and the trend of the network energy respectively, which is the best results among seven independent runs. We observed three interesting phenomena from the results: the 'decision splitting', the 'biased' decision and the 'misjudgement' in decision state. First, two paired peaks at \( 14.5°, 9.5° \) and \( 28.5°, 35.5° \) are seen near the regions of two sources \( 6.0°, 31.0° \) (the first and third vertically dashed lines). Each decision corresponding to both sources is splitted into two components located at the left and right hand sides, which referred to the decision splitting. Second, the decision results for the source direction \( 15.0° \) (the second and forth vertically dashed lines) are shown to be biased to the directions of \( 16.0°, 46.5° \) respectively. Finally, one peak at the direction of \( 122.0° \) is seen in
Fig. 5: One possibility of this 'misjudgement' may be understood from the fact that a neuron corresponding to the much smaller energy difference level as shown in Fig. 3 can take on 1 because its decision state of 0 or 1 does make little difference in the energy level.

To the contrary of the previous results shown in Fig.1, the trend of the energy level in Fig.5 does not provide any clear understanding about the logical relationship between the neuron state transition and the minimization of the cost function (5). These observations have allowed us to develop the proposed update scheme in Section II.2. That is the earlier update for the higher significance in the network energy (3), and furthermore to achieve the better performance of the Hopfield neural network-based decision-theoretic approach to the direction finding problem.

V. Concluding Remarks

We introduced another aspect for source direction finding in array sensor systems and fundamentals to approach that problem in a sense of classical theory. The mapping of decision states over the considered DOA range onto the outputs of the discrete Hopfield neural network are found to play a central role in this paper. A new state transition scheme according to the ascending order of the energy level is proposed. The simulation results at least may indicate that the proposed scheme is more successful in source direction finding than the previous one. Further decision-theoretic study on two issues-the closely located sources direction finding and the wideband sources direction finding problem—is in progress.

References


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