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STABLE DISCRETE-TIME ADAPTIVE CONTROL FOR PERIODIC SYSTEMS

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Abstract: This paper presents a discrete-time model reference adaptive control technique for periodically time-varying plants. It is shown that the identification problem for periodic parameters can be reduced to that of constant unknown parameters case. The global stability of the resulting closed-loop system is established using the key technical lemma of Goodwin, Ramadge and Caines.

1. Introduction

A number of techniques now exist for designing adaptive control systems when the unknown plants are linear and time-invariant. Some of the techniques with proven convergence properties for the time-invariant case are suitable, in principle, for slowly time-varying systems. However, many of them do not work well for rapidly time-varying systems. Therefore, much interest now exists in adaptive control for time-varying systems.

In the practically significant situation of unknown time-varying parameter, there has been some progress on the analysis of existing algorithms when used in time-varying case. Anderson and Johnstone [1] have shown that exponential convergence of the algorithm in time-invariant case will guarantee tracking error and parameter error boundedness when the plant parameters are actually slowly time-varying. However, they have not presented an algorithm that can cause the tracking error to zero for time-varying systems. Goodwin and Teoh [2], and Xianya and Evans [3] have proposed adaptive control strategies which are based on the covariance matrix resetting. Their schemes are able to successfully control unknown systems with quite rapidly time-varying parameters. However, They have not proved convergence properties of their algorithms in the general case where the parameters are varying in an arbitrary fashion. Xianya and Evans have given a convergence proof only when the system is restricted to the linearly time-varying parameter case. Ohkawa [4,5] has presented stable model reference adaptive control systems for time-varying plants. Though the asymptotic stability of the tracking error is established, it is assumed that plants have time-varying parameters which are described by a sine function of time or series of time.

This paper presents a stable discrete-time model reference adaptive control system (MRACS) for periodic plants. Periodic systems frequently appear in engineering fields as, for instance, Hill's equations [6]. Here, the period of time-varying parameters is assumed to be known. It is shown that the identification problem is reduced to that of constant unknown parameters case. The stability, therefore, can be established using the key technical lemma of Goodwin, Ramadge and Caines.

In Section 2, an alternative representation of periodically time-varying parameters is given. In Section 3, the control problem is formulated. In Section 4, the model reference adaptive control is considered. A convergence theorem for the adaptive algorithm is presented in Section 5. Finally, simulation results are displayed in Section 6.

2. Representation of Periodically Time-Varying Parameters

A periodic unknown parameter $\theta(t)$ with period N can be represented by the discrete-time Fourier series [7]

$$\theta(t)=\sum_{k=0}^{N-1}[\alpha_k\cos(k\omega_0 t)+\beta_k\sin(k\omega_0 t)] \tag{1}$$

where the α_k and β_k are the unknown Fourier coefficients

$$\alpha_k=\frac{1}{N}\sum_{i=1}^{N-1}\theta(i)\cos(ik\omega_0),\quad \alpha_0=\frac{1}{N} \tag{2a}$$

$$\beta_k=\frac{1}{N}\sum_{i=1}^{N-1}\theta(i)\sin(ik\omega_0) \tag{2b}$$

and $\omega_0=2\pi/N$ is the fundamental frequency. Since the coefficients α_k and β_k are constant, a well-known identification scheme for constant parameters can be applied.

3. Problem Statement

Let the system be described by the following time-varying difference equation

$$A(t,q^{-1})y(t)=B(t,q^{-1})q^{-d}u(t) \tag{3}$$

where $y(t)$ and $u(t)$ are the plant output and input, respectively, and

$$A(t,q^{-1})=1+a_1(t)q^{-1}+\ldots+a_n(t)q^{-n} \quad (4a)$$

$$B(t,q^{-1})=b_0(t)+\ldots+b_m(t)q^{-m},\ b_0(t)\neq 0 \quad (4b)$$

in which $q^{-i}y(t)=y(t-i)$. The unknown time-varying parameters a_i $(i=1,\ldots,n)$ and b_j $(j=1,\ldots,m)$ are periodic with period N. Thus

$$a_i(t+N)=a_i(t),\ b_j(t+N)=b_j(t). \quad (5)$$

We assume that the period N is known. Further, it is assumed that the system is stably invertible and the degree n, m and the time delay d are known.

The objective of the controller design is to make the plant output $y(t)$ follow the reference model output $y_M(t)$. The reference model output $y_M(t+d)$ is assumed to be known at time t.

4. Model Reference Adaptive Control System

For the moment, we will restrict attention to the case of known parameters. There exist unique polynomials

$$R(t,q^{-1})=1+r_1(t)q^{-1}+\ldots+r_{d-1}(t)q^{-d+1} \quad (6a)$$

$$S(t,q^{-1})=s_0(t)+\ldots+s_{n-1}(t)q^{-n+1} \quad (6b)$$

such that the following polynomial identity holds

$$R(t,q^{-1})A(t,q^{-1})+S(t,q^{-1})q^{-d}=1. \quad (7)$$

Using this polynomial identity and eqn.(3), one can write

$$\begin{aligned} y(t+d) &= R(t+d,q^{-1})A(t+d,q^{-1})y(t+d) \\ &\quad +S(t+d,q^{-1})q^{-d}y(t+d) \\ &= R(t+d,q^{-1})B(t+d,q^{-1})q^{-d}u(t+d) \\ &\quad +S(t+d,q^{-1})q^{-d}y(t+d) \\ &= R(t+d,q^{-1})B(t+d,q^{-1})u(t) \\ &\quad +S(t+d,q^{-1})y(t). \end{aligned} \quad (8)$$

From eqn.(7), the coefficients of the polynomials R and S are periodic with period N as well as the coefficients a_i and b_j. Hence, we obtain

$$y(t+d)=\theta_0(t-h)u(t)+\boldsymbol{\theta}^T(t-h)\mathbf{z}(t) \quad (9)$$

where

$$\mathbf{z}^T(t)=[u(t-1),\ldots,u(t-m-d+1), y(t),\ldots,y(t-n+1)]$$

$$\boldsymbol{\theta}^T(t-h)=[r_1'(t-h),\ldots,r_{m+d-1}'(t-h), s_0(t-h),\ldots,s_{n-1}(t-h)]$$

$$\begin{aligned} R'(t,q^{-1}) &= R(t,q^{-1})B(t,q^{-1}) \\ &= r_0'(t)+\ldots+r_{m+d-1}'(t)q^{-m-d+1} \end{aligned}$$

$$\theta_0(t-h)=r_0'(t-h)$$

with $h=N-d'$ and d' denotes the residue of d divided by N. The time-varying parameters θ_0 and $\boldsymbol{\theta}$ are periodic with period N. Now, applying the representation (1) of periodic parameters to eqn.(9), we have

$$\begin{aligned} y(t+d) &= [\sum_{k=0}^{N-1}(\alpha_{0k}\cos(k\omega_0(t-h)) \\ &\quad +\beta_{0k}\sin(k\omega_0(t-h)))]u(t) \\ &\quad +\sum_{i=1}^{n+m+d-1}\sum_{k=0}^{N-1}[\alpha_{ik}\cos(k\omega_0(t-h)) \\ &\quad +\beta_{ik}\sin(k\omega_0(t-h))]z_i(t) \\ &= \sum_{i=0}^{n+m+d-1}\sum_{k=0}^{N-1}[\alpha_{ik}\zeta_{ik}(t)+\beta_{ik}\eta_{ik}(t)] \\ &\qquad\qquad \beta_{i0}=0 \\ &= \gamma^T\psi(t) \end{aligned} \quad (10)$$

where

$$\zeta_{ik}(t)=\cos(k\omega_0(t-h))z_i(t)$$

$$\eta_{ik}(t)=\sin(k\omega_0(t-h))z_i(t)$$

$$z_0(t)=u(t)$$

$$\gamma^T=(\alpha_{00},\ldots,\alpha_{0,N-1},\beta_{01},\ldots,\beta_{0,N-1}, \alpha_{10},\ldots,\beta_{n+m+d-1,N-1})$$

$$\psi^T=(\zeta_{00},\ldots,\zeta_{0,N-1},\eta_{01},\ldots,\eta_{0,N-1}, \zeta_{10},\ldots,\eta_{n+m+d-1,N-1}).$$

It is clear that the control objective will be achieved if the control input is computed so that the right-hand side of eqn.(10) is equal to $y_M(t+d)$.

Actually, the system parameters a_i and b_i are unknown. Hence, it is natural to replace the unknown controller parameters α_{ik} and β_{ik} in eqn.(10) by adjustable parameters $\hat{\alpha}_{ik}(t-h)$ and $\hat{\beta}_{ik}(t-h)$ in which $\hat{\beta}_{i0}=0$. The adjustable parameters will be updated by an adaptation mechanism. It readily follows that the control input must be

$$u(t)=-[\sum_{k=0}^{N-1}(\hat{\alpha}_{0k}(t-h)\cos(k\omega_0(t-h))+\hat{\beta}_{0k}(t-h)\sin(k\omega_0(t-h)))]^{-1}$$

$$\times[\sum_{i=1}^{n+m+d-1}\sum_{k=0}^{N-1}(\hat{\alpha}_{ik}(t-h)\zeta_{ik}(t)+\hat{\beta}_{ik}(t-h)\eta_{ik}(t))-y_M(t+d)]. \quad (11)$$

The parameter vector $\hat{\gamma}^T(t)=(\hat{\alpha}_{00}(t),..,\hat{\beta}_{n+m+d-1,N-1}(t))$ is updated by the following recursive least squares algorithm

$$P(t)=P(t-1)+\psi(t-d)\psi^T(t-d),\ P(0)>0 \quad (12)$$

$$\hat{\gamma}(t)=\hat{\gamma}(t-1)-P^{-1}(t)\psi(t-d) \times[\hat{\gamma}^T(t-1)\psi(t-d)-y(t)] \quad (13)$$

5. Stability Analysis

We can now establish the following global convergence result.

Theorem 1. The adaptive control algorithm (11)-(13) when applied to the system (3) yields

(i) $y(t)$ and $u(t)$ are bounded, (14)

(ii) $\lim_{t\to\infty}(y_M(t)-y(t))=0.$ (15)

Proof. We first recall elementary properties of the recursive squares algorithm [8]:

$$\lim_{t\to\infty}\frac{e^2(t)}{1+c_1\psi^T(t-d)\psi(t-d)}=0 \quad (16)$$

$$\lim_{t\to\infty}\|\hat{\gamma}(t)-\hat{\gamma}(t-k)\|=0 \text{ for any finite } k \quad (17)$$

where

$$e(t)=\hat{\gamma}^T(t-1)\psi(t-d)-y(t)$$

$$c_1=\lambda_{max}P(0).$$

If we define the tracking error $\varepsilon(t)$ as

$$\varepsilon(t)=y_M(t)-y(t) \quad (18)$$

then, from eqn.(10) and eqn.(11)

$$\varepsilon(t)=\tilde{\gamma}^T(t-d-h)\psi(t-d) \quad (19)$$

where

$$\tilde{\gamma}(t)=\hat{\gamma}(t)-\gamma.$$

Thus

$$\frac{|\varepsilon(t)|}{[1+c_1\psi^T(t-d)\psi(t-d)]^{1/2}}=\frac{|\tilde{\gamma}^T(t-d-h)\psi(t-d)|}{[1+c_1\psi^T(t-d)\psi(t-d)]^{1/2}}=\frac{|(\hat{\gamma}^T(t-d-h)-\hat{\gamma}^T(t-1))\psi(t-d)+e(t)|}{[1+c_1\psi^T(t-d)\psi(t-d)]^{1/2}}. \quad (20)$$

Now the limit of the right-hand side is clearly zero from eqn.(16) and eqn.(17).

Hence

$$\lim_{t\to\infty}\frac{|\varepsilon(t)|}{[1+c_1\psi^T(t-d)\psi(t-d)]^{1/2}}=0. \quad (21)$$

We proceed to use the key technical lemma [9].

From the assumption of the inverse stability of the system, there exist constants $0\le c_2<\infty$ and $0<c_3<\infty$ such that

$$|u(k-d)|\le c_2+c_3\max_{t'\le t}(|y(t')|) \quad \text{for all } 1\le k\le t. \quad (22)$$

Therefore, using the definition of $\psi(t-d)$,

$$\|\psi(t-d)\|\le c_4+c_5\max_{t'\le t}(|y(t')|) \quad (23)$$

for constants $0\le c_4<\infty$ and $0<c_5<\infty$.

Here,

$$|\varepsilon(t)|=|y(t)-y_M(t)| \ge|y(t)|-|y_M(t)| \ge|y(t)|-c_6;\quad 0\le c_6<\infty. \quad (24)$$

Thus

$$\|\psi(t-d)\|\le c_4+c_5\max_{t'\le t}(|\varepsilon(t')|+c_6) =c_7+c_8\max_{t'\le t}(|\varepsilon(t')|);\quad 0\le c_7<\infty,\ 0<c_8<\infty. \quad (25)$$

Hence, applying eqn.(21) and eqn.(25) to the key technical lemma [9], we conclude (i) and (ii). (Q.E.D.)

6. Simulation Results

Computer simulation was carried out for a second-order periodically time-varying system with period N=3,

$$(1+a_1(t)q^{-1}+a_2(t)q^{-2})y(t)=b_0(t)q^{-1}u(t) \quad (26)$$

$$a_1(t)=-1+0.2a_{10}(t),$$

$$a_{10}(t)=\text{iteration of } (0,1,-1)$$

$$a_2(t)=0.25+0.05a_{20}(t),$$

$$a_{20}(t)=\text{iteration of } (1,-1,-1)$$

$$b_0(t)=0.5+0.1b_{00}(t),$$

$$b_{00}(t)=\text{iteration of } (0,-1,1)$$

The reference output is given by

$$(1-0.5q^{-1})y_M(t)=q^{-1}r(t) \qquad (27)$$

where r(t) is a rectangular signal with amplitude 0.1 and the period 20. The initial values of estimated parameters were taken to be zero except for $\hat{\alpha}_{00}(0)=0.5$, $\hat{\alpha}_{10}(0)=1.0$, $\hat{\alpha}_{20}(0)=-0.25$. The value of other constant occurring in the design was chosen as follows

$$P(0)=10^{-3}I \qquad (28)$$

where I is the identity matrix.

The output, input and identified parameters are shown in Fig.1. It can be seen that the objective is achieved.

7. Conclusions

A stable discrete-time MRACS for periodically time-varying plants has been presented. The identification problem is redued to that of constant parameters which correspond to the Fourier coefficients of periodic parameters of the plant. The resulting closed-loop system is shown to be globally stable by using the key technical lemma of Goodwin, Ramadge and Caines.

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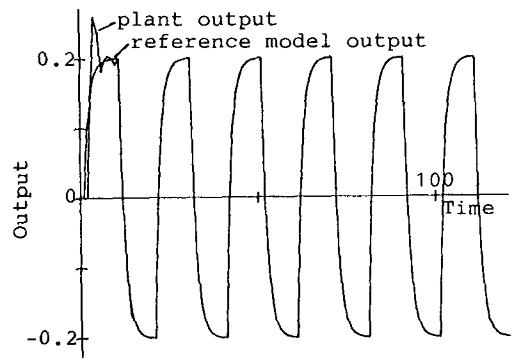

Fig.1 (a) Output

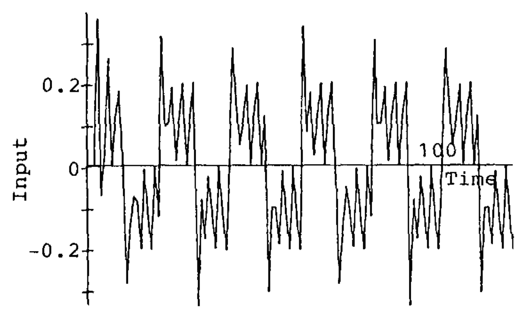

Fig.1 (b) Input

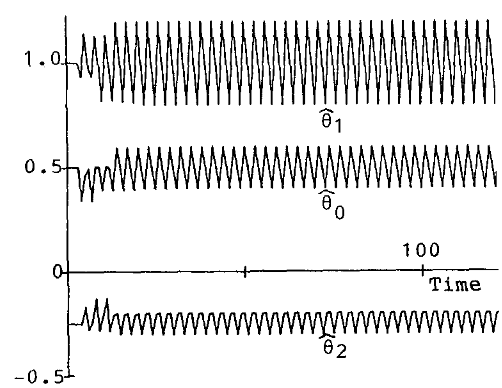

Fig.1 (c) Identified parameters