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Predictive Control : A survey and Some new stability results

*DaeGyu Byun and WookHyun Kwon

Department of Control and Instrumentation Eng.
Seoul National Univ.
Seoul 151, Korea

ABSTRACT

Various kinds of predictive control design methods such as MAC(Model Algorithmic Control), DMC(Dynamic Matrix control), EHAC(Extended Horizon Adaptive Control), GPC(Generalized Predictive Control), RHTC(Receding Horizon Tracking Controller), and PVC(PreView Controller) are surveyed and compared in this paper. In addition, stability properties of these control laws known to date are summarized and some new stability results are presented.

1. Introduction

Currently, several similar computer control techniques based on a socalled "Predictive" strategy have been introduced and their successful applications to industrial multivariable processes have been reported [9,12,36,49]. The main idea of the strategy is to predict the effect of potential control actions on the future values of the process output and to find the best control actions which minimize the deviation of the predicted outputs from the desired outputs. The prediction horizon extends over the significant part of the process response to the current control signal. Generally, the horizon covers the deadtime part and the nonminimum phase response part. This idea enables the predictive control to be well suited to typical processes where there may be variable dead time and nonminimum phase responses.

It is assumed in the predictive control that the finite future values of the desired trajectory is known a priori. This information about the future trajectory may provide the feedforward control action which is believed to be useful to the improvement of the transient response.

Various predictive control laws and the used models known to date are listed below:

(1) Model Algorithmic Control (MAC)
; Impulse response model [1,2,3,6,9,10]

(2) Dynamic Matrix Control (DMC)
; Step response model [11,13,14]

(3) Extended Horizon Adaptive Control (EHAC)
; ARMAX model [18,19,21]

(4) Generalized Predictive Control (GPC)
; CARIMA model [23,24,25,26]

(5) Receding Horizon Tracking Controller (RHTC)
; State space model [27,31]

(6) PreView controller (PVC)
; State space model [33]

(7) Extended Horizon Self Adaptive Control (EHSAC)
; CARIMA model [38,39,40,41,42]

(8) Predictive Control Algorithm (PCA)
; Impulse response model [45,46,47,48]

(9) MUltiStep Multivariable Adaptive Regulator (MUSMAR)
; ARMAX model [43,44]

The above algorithms may have some common properties due to their use of the same predictive strategy, but they have different properties as well due to the different forms of the underlying models, quadratic costs, and basic assumptions.

In this paper, we focus our attention on the control law, and so the identification feature which some of predictive control laws possess will not be considered. Predictive Control laws (1) - (6) are outlined and compared here. We exclude (7) and (8) since they can be easily cast into the framework of the other control laws such as GPC and MAC, and exclude (9) since it takes the form of the implicit self tuning controller which makes direct comparisons difficult. Some comparisons are made in [46,48,50] but they focus on the predictive control laws based on the weighting sequence models such as MAC, DMC, and PCA. We believe that our survey is not exhausted. Only the recent papers are listed and there may be some papers left uncited.

It should be noted that the stability properties of the predictive control have not yet fully investigated. This is partly because most of the predictive control laws assume the input/output models which have difficulty in studying the internal properties of the control system. The stability results proven in RHTC [27] are instructive since RHTC takes a state space model as the underlying plant model. Hence we propose that the stability properties for the predictive control laws be studied in the state space framework. In section 3, we will show how this is possible and present some new stability results for the predictive control laws based on the input/output models.

2. Controller design strategy

The strategy for predictive control laws can be summarized as follows:

1) it is assumed that the finite future values of the desired trajectory are available at each instant of time, which is an acceptible assumption in many practical control problems.
2) at the present moment t, the future outputs of the plant are predicted based on the given input and output data.
3) the control vector is computed to minimize the given performance index. In most cases, this control vector minimizes the deviations of the predicted output from the desired trajectory.
4) the first control signal u(t) is actually applied to the plant and the whole procedure is repeated.

Predictive control laws using this strategy, however, differ from each other depending on <1> the assumed plant model, <2> the form of the future trajectoty, <3> the form of the cost function, <4> constraints on control input <5> prediction equation of the plant output. We outline the predictive control laws centered around these points. In addition to this, we present the related research results and compare with each other.

MODEL ALGORITHMIC CONTROL (MAC)

<1> MAC assumes an impulse response model as the undering plant model.

$$y_M(t) = \sum_{j=1}^{\infty} h_j u(t-j) \quad (1)$$

In practice, however, an open loop stable system is assumed in which case the FIR type model is given.

$$y_M(t) = \sum_{j=1}^{n_m} h_j u(t-j) \quad (2)$$

When a state space model is given, a control law is obtained after it is transformed into an impulse response model [4,5].
<2> In most cases, a simple first order exponential trajectory is used as the reference trajectory which is peculiar to MAC:

$$y_r(t+k) = a^K y_r(t) + (1-\alpha^K) C$$
$$k = 1,2, \ldots, \quad |\alpha| < 1$$
$$y_r(t) = y(t) \quad (3)$$

where C is a set point and $y(t)$ is an actual plant output.
<3> The cost function is given as

$$J = \sum_{k=1}^{T} (y_p(t+k) - y_r(t+k))^2 \quad (4)$$

where $y_p(t+k)$ is a predicted output.
<4> If the input is assumed to be free of constraints (T=1), the cost functon (4) becomes

$$J = (y_p(t+1) - y_r(t+1))^2 \quad (5)$$

Little and Edgar [8] solve the linear quadratic control problem with linear inequality constraints.
<5> Since one step ahead prediction is of the form

$$y_p(t+1) = \sum_{j=1}^{n_m} h_j u(t+1-j) + (y(t)-y_M(t))$$
$$= u(t) + \sum_{j=2}^{n_m} h_j u(t+1-j) + (y(t)-y_M(t)) \quad (6)$$

a control input mimimizing the cost function (5) is:

$$u(t) = y_r(t+1) - (y(t)-y_M(t)) - \sum_{j=2}^{n_m} h_j u(t+1-j)$$
$$= (1-\alpha)(C-y(t)) + y_M(t) - \sum_{j=2}^{n_m} h_j u(t+1-j) \quad (7)$$

Though MAC reports some successful applications to real plants [9,49], it is shown to be inferior to other weighting-sequence-model based predictive control laws such as DMC and PCA [48]. Rouhani and Mehra [3,7] show that the the control law (7) is unstable for nonminimum phase systems and that some curious method should be used for maintaining stability which is seen to bo unrealistic.

Dynamic MATRIX CONTROL (DMC)

<1> DMC assumes a step response model as the underlying plant model.

$$y(t) = \sum_{j=1}^{\infty} S_j \Delta u(t-j) \quad (8)$$

Unlike the impulse response sequence $\{h_i\}$, $\{S_i\}$ does not converge to zero for large i, but the open loop stability assumption gives $S_j = S_{n_m}$ for $j \geq n_m$. A step response sequence $\{S_i\}$ is simply related to $\{h_i\}$ by

$$S_K = \sum_{i=1}^{K} h_i, \quad k = 1,2, \ldots \quad (9)$$

and

$$h_K = S_K - S_{K-1} \quad (10)$$

<2> There is no assumption given a priori for the future trajectory.
<3> The cost function is given as

$$J = \sum_{k=1}^{T} (y(t+k) - y_r(t+k))^2 \quad (11)$$

<4> In DMC, however, future increments in control $\Delta u(t+k)$ are taken to be zero for k≥pu . In other words, the strategy is to say that pu future control actions are allowed to be freely chosen, after which the control signals will be held constant. With a

nonminimum phase plant, for example, the unconstrained future control grows in amplitude, but the strategy inhibits such growth from occuring [48]. This idea, in addition, renders the reduction in the computation of the control law. This strategy is used as well in GPC [23,24] and EPSAC [38,39].

<5> The prediction equation is given by

$$Y = S\Delta U + \bar{Y} \tag{12}$$

where

$$Y = [y(t+1),\ y(t+2),\ \ldots,\ y(t+T)]' \tag{13}$$

$$\Delta U = [\Delta u(t),\ \Delta u(t+1),\ \ldots,\ \Delta u(t+pu-1)]' \tag{14}$$

$$\bar{Y} = [\bar{y}(t+1),\ \bar{y}(t+2),\ \ldots,\ \bar{y}(t+T)]' \tag{15}$$

$$S = \begin{bmatrix} s_1 & & 0 \\ s_2 & s_1 & \\ & & \ddots \; s_1 \\ \vdots & & \vdots \\ s_T & \cdots & s_{T-pu+1} \end{bmatrix} \tag{16}$$

$\bar{y}(t+k)$ is the prediction of the process output when there is no future control action taken, giving

$$\bar{y}(t+k) = \sum_{j=0}^{t-1} s_{t+k-j}\Delta u(j) \tag{17}$$

Utilizing the above results, the performance index (11) can be written as

$$J = (Y - Y_r)'(Y - Y_r)$$

where

$$Y_r = [y_r(t+1),\ y_r(t+2),\ \ldots,\ y_r(t+T)] \tag{18}$$

The resulting control law is [11] :

$$\Delta U = (S'S)^{-1}S'(Y_r - \bar{Y}) \tag{19}$$

where only the first element of ΔU is actually implemented. It should be noticed, however, that when S matrix is nearly singular, the computation of the solution is ill-conditioned. To deal with this problem move suppression factor f is incorporated [14,15,16,17], which adds a positive constant f to the diagonal elements of $S'S$ matrix such that

$$\Delta U = (S'S + fI)^{-1}S'(Y_r - \bar{Y}) \tag{20}$$

Equation (20) can be obtained as well from minimizing the following performance index:

$$J' = (Y - Y_r)'(Y - Y_r) + f\Delta U'\Delta U \tag{21}$$

which has the effects to suppress the variations of the control effort. A different way of handling the ill-conditioned matrix problem is approached by principal components analysis [17].

Cutler [52] has stated that a stability analysis is not required for DMC because it is so robust. However, this statement is not always the case. For example, when pu=T, all future controls are free to move, and the solution corresponds to pure model following which can be unstable [48]. As it is, the stability of DMC depends on the tuning parameters T, pu, and f. Maurath et al. [14] show that the stability property is a complex function of these parameters.

EXTENDED HORIZON ADAPTIVE CONTROL (EHAC)

<1> EHAC assumes an ARMAX model as the underlying plant model.

$$A(q^{-1})y(t) = B(q^{-1})u(t) + e(t) \tag{22}$$

where $A(q^{-1})$ and $B(q^{-1})$ are polynomials in the backward shift operator q^{-1}:

$$A(q^{-1}) = 1 + a_1q^{-1} + \ldots + a_nq^{-n} \tag{23}$$

$$B(q^{-1}) = b_0 + b_1q^{-1} + \ldots + b_mq^{-m} \quad (n > m) \tag{24}$$

$e(t)$ represent errors made in the modelling and disturbances acting on the process and assumed to be a constant e.

<2> There is no assumption given a priori for the future trajectory.

<3> The cost function is given as

$$J = \sum_{k=0}^{T-1} u(t+k)^2 \tag{25a}$$

with the following constraint

$$y(t+T) = y_r(t+T) \tag{25b}$$

Deadbeat controllers set the d-step ahead predicton equal to the desired value where d is the system dead time. The above cost function, however, allows the process longer time to reach this objective when $T \geq d$ with minimum efforts.

<4> The input is assumed to be free of constraints.

<5> The prediction equation can be written as

$$y(t+T) = G(q^{-1})y(t) + F(q^{-1})B(q^{-1})u(t+T-1) + F(1)d \tag{26}$$

where $F(q^{-1})$ and $G(q^{-1})$ are polynomials of order T-1 and n-1 that satisfy a Diophantine equation:

$$1 = F(q^{-1})A(q^{-1}) + q^{-T}G(q^{-1}) \tag{27}$$

If we represent $F(q^{-1})B(q^{-1})$ and $G(q^{-1})$ in (26) by

$$F(q^{-1})B(q^{-1}) = \beta_0 + \beta_1q^{-1} + \ldots + \beta_{T+m-1}\, q^{T+m-1} \tag{28}$$

$$G(q^{-1}) = \alpha_0 + \alpha_1q^{-1} + \ldots + \alpha_{n-1}\, q^{-n+1} \tag{29}$$

the resulting control law is given by [19]

$$u(t) = \beta_0\Big(\sum_{j=0}^{T-1} \beta_j^{\,2}\Big)^{-1}(y_r(t+T) - \bar{y}(t+T)) \tag{30}$$

where

$$\bar{y}(t+T) = \sum_{j=0}^{n-1} \alpha_j y(t-j) + \sum_{j=0}^{m} \beta_{T+j}\, u(t-j-1) \tag{31}$$

This result is extended to multivariable control systems in [18,53].

A similar result is developed using state space models by Goodwin and Dugard [53] and using ARMAX models by Dugard et al. [21] for multivariable stochastic system. But these control laws are periodic with period T, and T successive control laws have to be cyclically applied, which distinguishes these control laws from EHAC. Scattolini and Clarke [22] extend the results using CARIMA model for offset rejection. The BIBO stability of the non-adaptive extended horizon controller is proven in [19]. We will present in section 4 a new stability result for the non-adaptive extended horizon controller using the state space approach.

GENERALIZED PREDICTIVE CONTROL (GPC)

<1> GPC assumes a CARIMA(Controlled Autoregressive Integrated Moving Average) model as the underlying plant model.

$$A(q^{-1})\Delta y(t) = B(q^{-1})\Delta u(t-1)+c(q^{-1})\xi(t) \qquad (32)$$

where $A(q^{-1})$ and $B(q^{-1})$ is same as defined in (23) and (24), $\xi(t)$ is an uncorrelated random sequence, and Δ is the differencing operator $1-q^{-1}$.

<2> There is no assumption given a priori for the future trajectory.

<3> The cost function is given as

$$J(D,T)=E\{\sum_{k=D}^{T}[y(t+k)-y_r(t+k)]^2+\sum_{k=1}^{T} r[\Delta u(t+k-1)]^2\} \qquad (33)$$

If the deadtime of the plant is known a priori, then D can be chosen as the deadtime or more but generally it is taken as 1.

<4> GPC uses the same idea as DMC [11] that after an interval $pu < T$ control increments are assumed to be zero, i.e. $\Delta u(t+k-1) = 0$ for $k > pu$.

<5> The prediction equation is obtained using a Diophantine equation as

$$y(t+k)=F_k(q^{-1})B(q^{-1})\Delta u(t+k-1)+G_k(q^{-1})y(t) \qquad (34)$$

where $F_k(q^{-1})$ and $G_k(q^{-1})$ satisfy

$$1 = F_k(q^{-1})A(q^{-1})\Delta+q^{-k}G_k(q^{-1}) \qquad (35)$$

Since the controls are to be determined regardless of the future noise sequence, $\{\xi(t+k)\}$ is ignored in the prediction equation. The prediction equation can be written in the vector form (12) [24] :

$$Y = F\Delta U + \bar{Y} \qquad (36)$$

where

$$F = \begin{bmatrix} \beta_0 & & \underline{0} \\ \beta_1 & \beta_0 & \\ \vdots & & \ddots\ \beta_0 \\ \cdot & & \\ \beta_{T-1} & \beta_{T-2} & \cdots\ \beta_{T-pu} \end{bmatrix} \qquad (37)$$

$\bar{y}(t+k)$ is the component of $y(t+k)$ composed of signals which are known at time t, for example:

$$\bar{y}(t+1) = [F_1(q^{-1})B(q^{-1})-\beta_0]\ \Delta u(t)+G_1(q^{-1})y(t)$$

$$\bar{y}(t+2)=[F_1(q^{-1})B(q^{-1})-\beta_1 q^{-1}-\beta_0]\Delta u(t)+G_2(q^{-1})y(t)$$

etc.

Now, the cost function (33) with D=1 is written as

$$J(1,T)=(F\Delta U+\bar{Y}-Y_r)'(F\Delta U+\bar{Y}-Y_r)+r\Delta U'\Delta U \qquad (38)$$

and the minimization of J(1,T) gives

$$\Delta U = (F'F + rI)^{-1}F'(Y_r-\bar{Y}) \qquad (39)$$

which is the same form as (20). Hence GPC can be thought as the generalization of DMC to more general systems represented by (32).

From the form of the control law, we can see that GPC and DMC give zero offset provided that the vector $\bar{Y}$ involves a unit steady state gain in the feedback path. Clarke et al. [25] attempts to prove the stability of GPC for various settings of pu,D,T, and r. They prove the stability properties in the framework of the state space model. The following table lists the settings for which the stability properties are proved.

Table 1. Special cases of GPC settings

pu	D	T	r	Results
1	1	$\to\infty$	0	Thm. 2 of [25]
T-n1+1	1	$\to\infty$	0	Thm. 1 of [25]
n1	n1	$\geq$ 2n1-1	0	Thm. 3 of [25]

n1 : order of the state space model

We will investigate the stability properties for more general settings of pu,D,T, and r later in this paper.

RECEDING HORIZON TRACKING CONTROLLER (RHTC)

<1> RHTC assumes a state space model as the underlying plant model.

$$x(t+1) = Ax(t) + Bu(t) \qquad (40a)$$

$$y(t) = cx(t) \qquad (40b)$$

where $x(t)\in R^n$, $u(t)\in R^m$, and $y(t)\in R^p$.

<2> There is no assumption given a priori for the future trajectory.

<3> The cost function is given as

$$J=\tfrac{1}{2}\sum_{k=0}^{T-1}\{(y(t+k)-y_r(t+k))'Q(y(t+k)-y_r(t+k)) +u(t+k)'Ru(t+k)\}+\tfrac{1}{2}(y(t+T)-y_r(t+T)) F(y(t+T)-y_r(t+T)) \qquad (41)$$

where Q and F are $p\times p$ real positive matrices, and R is an $n\times n$ real positive matrix.

<4> The input is assumed to be free of constraints.

<5> It should be noted that explicit prediction equations are not used in the minimization of the cost function (41).

Following the standard optimization procedures, the control giving minimum cost is given by

$$u(t)=-[R+B'K(T-1)B]^{-1}B'[K(T-1)Ax(t)+g(t+1)] \qquad (42)$$

where K(T-1) is obtained from the following Riccati equation

$$K(t+1)=A'K(t)A-A'K(t)B[R+B'K(t)B]^{-1}B'K(t)A+c'Qc$$

$$K(0)=c'Fc \qquad (43)$$

and g(t+1) from the following equation

$$g(t+1)=-\Phi'(T,1)c'Fy_r(t+T)-\sum_{i=1}^{T-1}\Phi'(i,1)c'Qy_r(t+i) \qquad (44)$$

$$\phi(k,k_0)=A_c(k-1)A_c(k-2)\ \dots\ A_c(k_0) \quad (45)$$

$$A_c(k)=[I-B(R+B'K(T-k-1)B)^{-1}B'K(T-k-1)]A \quad (46)$$

Similar receding horizon regulation problems are found in [28,30,32] for continuous time systems and in [29] for discrete time systems. As we can see in (42), the state space formulation gives a fixed gain state feedback control law unlike a finite horizon LQ problem where a time varying feedback gain is given.

If the states are not directly accessible a state reconstruction scheme should be used. If a deadbeat observer [54] is used as the state reconstruction scheme, the stste $x(t)$ can be calculated from the finite input/output data, in which case the control law (42) is almost the same form as the other predictive control laws based on input/output models.

The state space approach can be thought to be a more general form compared to input/output model based predictive control laws and gives some advantages in studying the internal properties of the closed loop systems.

It is shown in [27] that the control law (42) has a finite horizon T over which the closed loop system becomes asymptotically stable. Particularly, if a infinite weight is put on a terminal matrix F, which is equivalent to the constraint $y(t+T) = y_r(t+T)$, the closed loop system is shown to be asymptotically stable for $T \geq n$ These stability results can give insights into the stsbility proof of the other predictive control laws. Mohta and Clarke [23] uses the receding horizon regulation problem [29] for the stability proof of GPC.

PREVIEW CONTROLLER (PVC)

<1> PVC assumes a stochastic state space model as the underlying plant model.

$$x(t+1) = Ax(t) + Bu(t) + w(t) \quad (47a)$$

$$y(t) = cx(t) + v(t) \quad (47b)$$

where

x_0, $w(t)$, and $v(t)$ are independent,

x_0 is Gaussian with $E[x_0] = \bar{x}_0$ and $E[(x_0-\bar{x}_0)(x_0-\bar{x}_0)'] = X_0$

$w(t)$ is white with $N(0,W_t)$, and

$v(t)$ is white with $N(0,V_t)(V_t>0)$.

<2> A sequence of reference signals $y_r(t)$ is modeled as the output of a reference signal generator :

$$x_r(t+1) = A_r x_r(t) + B_r w_r(t) \quad (48a)$$

$$z_r(t) = c_r x_r(t) + v_r(t) \quad (48b)$$

<3> The cost function is given as

$$J=E[\tfrac{1}{2}(y(N)-y_r(N))'Q_N(y(N)-y_r(N)) + \tfrac{1}{2}\sum_{i=0}^{N-1}\{(y(i)-y_r(i))'Q(y(i)-y_r(i))+u'(i)Ru(i)\} \quad (49)$$

where Q and Q_N are $p \times p$ real positive matrices, and R is an $n \times n$ real positive matrix. It should be noted that in PVC future observations of $y_r(t+k)$ from k=1 to k=T $(T<N)$ are given a priori by system (48) and thereafter only statistical quantities of $y_r(t)$ are given. The overall cost (49) is, however, fixed with horizon N, which is a little different feature compared to the other predictive control laws where both the overall cost and the future reference signals recedes. For comparisons with other predictive control laws the case where N approachs infinity will be dealt with.

<4> The input is assumed to be free of constraints.

<5> As in RHTC, explicit prediction equations are not used in the minimization of the cost function (49).

If the perfect measurement is assumed the control law minimizing the cost (49) is given by [33] :

$$u(t)=-[R+B'\bar{K}B]^{-1}B'[\bar{K}Ax(t)+g(t+1)] \quad (50)$$

where $\bar{K}$ is the steady state solution of (43), and

$$g(t+1)=-[\bar{\phi}'(T-1)c'Q_N c_r+\bar{\phi}'(T)\{\sum_{k=0}^{\infty}(\bar{\phi}'(k)c'Qc_r(A_r)^k\}A_r]$$
$$x_r(t+T)-\sum_{i=1}^{T-1}\bar{\phi}(i-1)c'Qy_r(t+i)$$
$$\bar{\phi}(k) = [A-B(R+B'\bar{K}B)^{-1}B'\bar{K}A]^k \quad (51)$$

The control law (50) is a similar form to the control law (42) of RHTC. If horizon T is large enough for the Riccati equation solution to converge to a steady state value, then RHTC with an appropriate terminal weighting F can be the same control law as PVC.

Successful applications of PVC to real systems are reported [36]. Tomizuka [37] provides PVC for continuous time systems and PVC problems with integral actions are dealt with in [34,35] for offset free tracking.

It is known that the control law (50) is asymptotically stable if and only if the system is stablizable and detectable [55].

3. New stability results for predictive control laws

There have been some approachs to the analysis of stability properties of input/output model based predictive control laws. Rouhani and Mehra [3,7] show that MAC is unstable for nonminimum phase systems , but it was necessary to assume a one-step ahead implementation for the proof. Maurath et al. [14] deal with the stability properties of DMC and show that they are a complex function of design parameters. Ydstie shows in [19] that extended horizon controllers are BIBO stable under some conditions. All these results rely on transfer function approach based on input/output models, in which framework, however , the stability analysis is restrictive and not easy.

We propose that state space models should be used for the stability analysis of input/output model based predictive control laws. Input/output models can be transformed to state space models. It is shown in [20] that the predictive control laws in their original forms can be represented in terms of state space model parameters, which enables the stability analysis to be done in the state space framework.

The state space analysis of model predictive control is done in [5], where the original system is given in the form of a state space model and the solution is obtained from the transformed impulse response model. Stability properties of GPC is studied in [23,25]. Though they have ideas using state space models for stability analysis they do not use tne idea that the contol laws can be represented by the transformed state space model parameters.

Some new stability results are presented of which details are available in [20].

Results 1. < Stability Property of EHAC >

We transform the model (22) into a state space model {A,B,C} (in observable canonical form). If the system {A,B } is completely controllable and A is invertible , then the control law (31) is a stabilizing law for any $T>n$.

Result 2. < Stability Property of GPC >

Since the disturbance can be ignored as far as the stability properties are concerned, we transform the model (32) without the disturbance term into the following state space form

$$x(t+1) = Ax(t) + b\Delta u(t)$$

$$y(t) = cx(t)$$

Note that

$$A(q^{-1})\Delta = A(q^{-1})(1-q^{-1}) = 1+\tilde{a}_1 q^{-1} + \dots + \tilde{a}_{n+1} q^{-n-1}$$

1) GPC parameters setting

: pu=1, D=1, T=any positive constant, r=0

If all the roots of polynomial (52) lie within the unit circle, then the control law (39) is a stabilizing law.

$$H(z) = \bar{K} z^{n+1} + (\tilde{a}_1 \bar{K} + \bar{K}_1) z^n + \dots + (\tilde{a}_{n+1} \bar{K} + \bar{K}_{n+1}) \quad (52)$$

where

$$\bar{K} = (cb)^2 + (cAb)^2 + \dots + (cA^{T-1}b)^2$$

$$[\bar{K}_1 \ \bar{K}_2 \ \dots \ \bar{K}_{n+1}] = [cbcA + cAb\,cA^2 + \dots + cA^{T-1}b\,cA^T]$$

2) GPC parameters setting

: pu=T=any positive constant, D=1, and r=0

If all the roots of the polynomial (53) lie within the unit circle , then the control law (39) is a stabilizing law.

$$H(z) = b_0 z^m + b_1 z^{m-1} + \dots + b_m \quad (53)$$

This fact says that the control law is unstable for nonminimum phase systems under the given parameter setting. However, this is an expected result since the given parameter setting is equivalent to the parameter setting: pu=T=D=1 and r=0, in which case the control law becomes a one step ahead control law.

4. Conclusions

This paper shows that many different control design methods which have been developed by so many different investigators and so many different approachs can be categorized as one group of control design methods distinguished from the other various control design methods. We call this "Predictive control". Though the predictive control laws have been less known comapred to other design methods, listed references, not complete, say that there are considerable research results related to the predictive control , most of which are developed independently.

Theoretical development, however, are not perfect and much are left to be done. Even stability properties are not known completely. This paper introduces some results on the stability properties. It would be a meaningful work to find out common properties which all the predictive control laws possess and to pick out special properties which are peculiar to each control law. The common properties may be attributable to the adoption of the same strategy and the special properties to the different plant models. It is known that most of the predictive control laws can be deadbeat controllers by proper setting of the tuning parameters [19,23,25,50,56]. We can see in this paper that the solutions of predictive control laws have the almost same form. These facts imply that the predictive control laws can have some common properties and be unified to some degree.

REFERENCES

[1] J. Richalet, A. Rault, J.L. Testud, and J. Papon, "Model Predictive Heuristic Control : Applications to Industrial processes," Automatica, Vol.14, PP. 413-428, 1978.

[2] R.K. Mehra, R. Rouhani, A. Rault, and J.G. Reid, "Model Algorithmic Control : Theoretical Results on Robustness," Proc. JACC, PP.387-392, 1979.

[3] R. Rouhani and R.K. Mehra, "Model Algorithmic Control(MAC) : Basic Theoretical Properties," Automatica, Vol.18, no.4, PP.401-414, 1982.

[4] J.G. Reid, D.E. Chaffin, and J.T.Silverthorn, "Output Predictive Algorithmic Control : Precision Tracking with Application to Terrain Following," J.Guidance and Control, Vol.4, no.5, PP.502-509, Sept.-Oct., 1981.

[5] J.G. Reid and S. Mahmood, "A State Apace Analysis of The Stability and Robustness Properties of Model Predictive Control (MPC)," Proc. ACC, WP1, PP.335-338, 1986.

[6] R.K. Mehra, R. Rouhani, and L. Praly, "New Theoretical Development in Multivariable Predictive Algorithmic Control," Proc. JACC, FA9-B, 1980.

[7] R.K. Mehra and R.Rouhani, "Theoretical Considerations on Model Algorithmic Control for Nonminimum Phase Systems," Proc.JACC, FA8-B, 1980.

[8] D.L. Little and T.F. Edgar, "Predictive Control Using Constrained Optimal Control," Proc.ACC, FA1, PP.1365-1371, 1986.

[9] R.K. Mehra, R. Rouhani, J. Eterno, J. Richalet, and A. Rault, Chemical Process Control 2, T.F.Edgar and D.E.Seborg (Eds.), Eng. Foundation, N.Y.

[10] J. Richalet, "General Principal of Scenario Predictive Control Techniques," Proc. JACC, FA9-A, 1980.

[11] C.R. Cutler and B.L. Ramaker, "Dynamic Matrix Control - A Computer Control Algorithm," Proc. JACC, San Francisco, 1980.

[12] R.H. Luecke, J.C. Lewis, H.Y. Lin, and W.K. Yoon, "Dynamic Matrix Control of A Batch Distillation Column," Proc. ACC, WA7, PP.209-213, 1985.

[13] C.R. Cutler, "Dynamic Matrix Control of Imbalanced Systems," ISA Trans., Vol.21, no.1, PP.1-6, 1982.

[14] P.R. Maurach, D.A. Mellichamp, and D.E. Seborg, "Predictive Controller Design for SISO Systems," Proc. ACC, FP4, PP.1546-1552, 1985.

[15] A.M. Morshedi, C.R.Cutler, and T.A. Skrovanek, "Optimal Solution of Dynamic Matrix Control with Linear Programming Techniques (LDMC)," Proc. ACC, WA7, PP.199-208, 1985.

[16] Pu Yuan and D.E. Seborg, "Predictive Control Using an Observer for Load Estimation," Proc.ACC, TA1, PP.669-675, 1986.

[17] P.R. Maurath, D.E. Seborg, and D.A. Mellichamp, "Predictive Controller Design by Principal Components Analysis," Proc. ACC, TP7, PP.1059-1065, 1985.

[18] B.E. Ydstie and L.K. Liu, "Single and Multivariable Control with Extended Prediction Horizons," Proc. ACC, FA4, PP.1303-1308, 1984.

[19] B.E. Ydstie, "Extended Horizon Adaptive Control," Proc. IFAC World Congress, Budapest, Hungary, 1984.

[20] D.G. Byun and W.H. Kwon, "Stability Properties of Predictive Control Laws," ISL Internal Report, SNU., 1987

[21] L. dugard, G.C. Goodwin, and X. Xianya, "The Role of The Interactor Matrix in Multivariable Stochastic Adaptive Control," Automatica, Vol.20, no.5, pp.701-709, 1984.

[22] R. Scattolini and D.W. Clarke, "Multivariable model following self-tuning control with offset rejection," Int.J.Control, Vol.42, no.6, PP.1309-1322, 1985.

[23] C. Mohtadi and D.W. Clarke, "Generalized Predictive Control, LQ, or Pole-placement : A Unified Approach," Proc. 25th Conf. on Decision and Control, FA-4, PP.1536-1541, Dec. 1986.

[24] D.W. Clarke, C.Mohtadi, and P.S.Tuffs, "Generalized Predictive Control - Part 1. Basic Algorithm," Automatica, Vol.23, no.2, PP.137-148, 1987.

[25] D.W. Clarke, C.Mohtadi, and P.S.Tuffs, "Generalized Predictive Control - Part 2. Extensions and Interpretations," Automatica, Vol.23, no.2, PP.149-160, 1987.

[26] D.W. Clarke, "Generalized Predictive Control : A Robust Self-tuning Algorithm," Proc. ACC, TA7, PP.990-995, 1987.

[27] W.H. Kwon, D.G. Byun, and O.K. Kwon, "Receding Horizon Tracking Controller as a Class of Predictive Controller," submitted to 1987 KACC.

[28] W.H. Kwon and A.E. Pearson, "A Modified Quadratic Cost Problem and Feedback Stabilization of a Linear System," IEEE Trans. AC., Vol.22, pp.838-842, Oct. 1977.

[29] W.H. Kwon and A.E. Pearson, "On Feedback Stabilization of Time Varying Discrete Time Systems," IEEE Trans. AC., Vol.23, PP.479-481, Jun. 1978.

[30] R. Longchamp, "Singular Perturbation Analysis of A Receding Horizon Controller," Automatica, Vol.19, no.3, PP.303-308, 1983.

[31] J. Rodellar, J.M. Martin-Sanches, and A.H. Barbat, "Driver Block Design and Applications to Structural Control," Proc. CDC, FA3, PP.1529-1535, Dec. 1986.

[32] W.H. Kwon, A.M. Bruckstein, and T.Kailath, "Stabilizing State Feedback Design via The Moving Horizon Method," Int.J.Control, Vol.37, no.3, PP.631-643, 1983.

[33] M. Tomizuka and D.E. Whitney, "Optimal Discrete Finite Preview Problems (Why and How is Future Information Important ?)," J. of Dynamic System, Measurement, and Control, PP.310-325, Dec. 1975

[34] M. Tomizuka and D.E. Rosenthal, "On the Optimal Digital State Vector Feedback Controller with Integral and Preview Actions," ASME J. of Dynamic Systems, Measurement, and Control, PP.172-178, June 1979.

[35] T. Katayama, T. Ohki, T. Inoue, and T. Kato, "Design of an Optimal Controller for a Discrete-Time System Subject to Previewable Demand," Int.J.Control, Vol.41, no.3, PP.677-699, 1985.

[36] M. Tomizuka, D. Dornfeld, X.-Q. Bian, and H.-G. Cai, "Experimental Evaluation of The Preview Servo Scheme for a Two-Axis Positioning System," ASME J. of Dynamic Systems, Measurement, and Control, PP.1-5, Mar. 1984.

[37] M. Tomizuka, "Optimal Continuous Finite Preview Problem," IEEE Trans. AC., PP.362-365, Jun. 1975.

[38] R.M.C. De Keyser and A.V. Cauwenberghe, "Towards Robust Adaptive Control with Extended Predictive Control," Proc. CDC, FA4, PP.1548-1549, Dec. 1986.

[39] A.R.V. Cauwenberghe and R.M.C. De Keyser, "Self-Adaptive Long Range Predictive Control," Proc. ACC, TP10, PP.1155-1160, 1985.

[40] R.M.C. De Keyser and A.R.V. Cauwenberghe, "A Self-Tuning Multistep Predictor Application," Automatica, Vol.17, no.1, PP.167-174, 1981.

[41] R.M.C. De Keyser and A.R.V. Cauwenberghe, "Self-Tuning Predictive Control," Journal A., Vol.22, no.4, PP.167-174, 1981.

[42] R.M.C. De Keyser and A.R.V. Cauwenberghe, "Application of Self Tuning Predictive Control," Journal A., Vol.23, no.1, PP.1-10, 1982.

[43] C. Greco, G.Menga, E. Mosca, and G. Zappa, "Performance Improvement of Self-Tuning Controllers by Multistep Horizons : The MUSMAR Approach," Automatica, Vol.20, no.5, PP.681-699, 1984.

[44] E. MOsca, G.Zappa, and C.Manfredi, "Multistep Horizon Self-Tuning Controllers : The MUSMAR Approach," IFAC 9th World Congress, 14.4/F-2, PP.155-159, Budapest, Hungary, 1984.

[45] P.M. Bruijn, H.B. Verbruggen, "Model Algorithmic Control Using Impulse Response Models," Journal A., Vol.25, no.2, PP.69-74, 1984.

[46] P.M. Bruijn, H.B. Verbruggen, and O.V. Appeldoorn, "Predictive Control : A Comparison and Simple Implementation," IFAC Low Cost Automation, PP.271-276, Spain, 1986.

[47] P.M. Bruijn, L.J. Bootsma, and H.B. Verbruggen, "Predictive Control Using Impulse Response Models," IFAC Symposium on Digital Computer Applications to Process Control, Dusseldorf, 1980.

[48] D.W. Clarke and L.Zhang, "Long-Range Predictive Control Using Weighting-Sequence Models," IEE Proc., Vol.34, Pt.D, no.3, PP.187-195, May 1987.

[49] M. Lecrique, A. Rault, M.Tessier, and J.L. Testud, "Multivariable Regulation of A Thermal Power Plant Steam Generator," IFAC World Congress, Helsinki, 1978

[50] G.D. Martin, "Long-Range Predictive Control," AIChE J., Vol.27, no.5, pp.748-753, Sep. 1981.

[51] C.R. Cutler, PH.D. Thesis, University of Houston, Houston, 1983.

[52] G.C. Goodwin and L.Dugard, "Stochastic Adaptive Control with Known and Unknown Interactor Matrices," IFAC Symposium on Adaptive Control, San Francisco, Jun. 1983.

[53] M.P. Golden, L.K.Liu, and B.E. Ydstie, "Multivariable Control Using Extended Prediction Horizons," Proc. CDC, FA4, PP.1550-1551, Dec. 1986.

[54] K.J. Astrom and B. Wittenmark, Computer Controlled Systems - Theory and Design, Prentice-Hall, Inc., Englewood Cliffs, 1984.

[55] H. Kwakernaak and R.Sivan, Linear Optimal Control Systems, Wiley, N.Y., 1972.

[56] E. Yaz and H. Selbuz, "A Note on The Receding-Horizon Control Method," Int.J.Control, Vol.39, no.4, PP.853-855, 1984.