'87 KACC 1987.10.16~17

Mathematical Modelling of Moving Target and Development of Real Time Tracking Method Using Kalman Filter

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Abstract

Some of the initial steps necessary for the application of Kalman filter will be discussed in general. The application of filtering for tracking system will then be illustrated by simple examples. Practical implementation problems as well as hardware synthesis difficulties, are discussed.

1. Introduction

In general, dynamic systems such as aircraft and missile called targets, may be modelled as stochastic dynamic systems. In recent years the Kalman filter techniques have been applied to a wide variety of tracking their real trajectories,because they are powerful to estimate the state of such rapid moving targets.

Most of the major tracking system manufacturers have developed or proposed system with Kalman filtering, and it is being used in several tracking system that are in operational use. Kalman filter has now become an expected part of almost every proposed new tracking system. The reason for the popularity is that Kalman filter in tracking systems is not hard to filed. There are at least three major complementary factors that have come together at proper time and these factors are an increased need, the mathematical tool and the necessary equipments such as digital computer.

In this paper, some of the initial steps necessary for the application of Kalman filter will be discussed in general. The application of filtering for the tracking system will then be illustrated by simple examples. Design method of Kalman filter are suggested for simplifying the problem in order that it can be more easily handled on a practical computer. Practical implementation problems as well as hardware synthesis difficulties, are discussed.

In the problem of engineering design, it is often necessary to choose an appropriate model on the moving target. Simple target motion analysis on linear filtering and tracking is presented. Kalman filtering system has been developed and is being evaluated by using TMS 32010 digital signal processor(DSP). We use this DSP for the real time tracking analysis to estimation of states in moving target.

2. Basic concepts for the consideration of target model on the Kalman filter

The Kalman filter itself is defined as very precise mathematical model, but its application to actual physical systems is rarely precise science. Considerable engineering experience is needed to properly identify the system to which the filter is to be applied,to adequately model the system, and then to develop a practical program that mechanizes the filter in the on-board computer. The optimization of filter must include many factors which are difficult to describe mathematically, such as the trade-off between performance and computer size. The statistic parameters are rarely based on the actual statistics of the physical system, because these statistics are either too complicated or are not well-known. The parameters are more likely to be chosen by less formal methods which attempt to maximize the performance in spite of the imperfectly known real world.

One of the first steps in applying Kalman filter is to identify the system on which the filter is to be based. The system on which the filter is to be based is specified by a set of state variables that are defined formally by a set of mathematical relations on the behavior of target dynamic systems. In practice, for actual physical systems, there is never enough information to satisfy these mathematical relations perfectly. One of the basic design problems is the choice of state variables for the tracking filter and associated trade-off of performance versus computer requirements. The mathematical relations which give the formal definition of the state variables are : 1) the desired outputs have to be some function of the state variables, 2) the measurements have to be a function of the state variables and uncorrelated errors, 3) the state variables at one time have to be a function of the state variables at previous time, of the controls to the system between the two times, and of the uncorrelated noise inputs between the two times.

The most effective over all design of the system can usually be obtained by minimizing the amount of pre-processing of the input data before it is used in the filter. One advantage of Kalman filter is that measurements can be processed in raw form ; this can greatly simplify the sensor subsystems. The final step in the forming the state vector is the addition of any variables necessary to describe the dynamic behavior of the state variables. The total set of state variables is the minimum number of dependent variables in the differential equations that describe the system. The Kalman filter designer tipically do not have the luxury of implementing the filter based upon the best descriptive and most complete and complex model, often termed the truth model. The final filter algorithm must meet the constraints, such as on-line computer time, memory, word length, etc., and these considerations dictate using as simple a filter as possible that also meets performance specifications. Consequently, the designer must be able to exploit basic modelling alternatives to achieve a simple but adequate filter, adding to deleting compexity from the model according to performance needs and the requirements of practical constraints. Evaluation of the true performance capabilities of simplified reduced order filter is thus of critical importance in the design procedure.

The design of an effective operational Kalman filter entails an iterative process of proposed alternative designs through physical insights, tuning each, and trading off performance capabilities and computer loading.

3. Kalman filter on the moving target

The Kalman filter is a recursive scheme for estimating the state x(t) of a dynamical system represented by the following stochastic equation

$$\dot{x}(t) = f[\ x(t),\ c(t),\ n(t),\ t\ ] \quad (3\text{-}1)$$

where c(t) are known control inputs and n(t) are white Gaussian noises. For the assumption to be made here, it can be shown that the control does not affect the form of the optimum filter. It is assumed that measurements are made at discrete times according to the relation

$$m(t_m) = h[\ x(t_m),\ v(t_m)\ ]\ , \quad (3\text{-}2)$$

where $v(t_m)$ are errors in the measurements that are uncorrelated between measurements. Assuming the optimal estimates are close enough to the true values for higher-order terms to be neglected, the optimum measurement process is given by Kalman's optimum linear filter.

At a measurement time the filter equation is given by

$$\begin{aligned} \hat{x} &= \hat{x}' + E'H^t(HE'H^t+V)^{-1}(m-h(\hat{x}',t)) \\ E &= E' - E'H^t(HE'H^t+V)^{-1}HE' \\ \dot{\hat{x}} &= f(\hat{x}, t) \\ \dot{E} &= FE + EF^t + N \end{aligned} \qquad (3\text{-}3)$$

where the " ' " indicates conditions that exist just before the measurement. The error covariance matrix is defined by

$$E = \mathcal{E}[(x-\hat{x})(x-\hat{x})^t] , \qquad (3\text{-}4)$$

where $\mathcal{E}$ represents the expected or mean value, and

$$\begin{aligned} F &= \partial f/\partial x \;|x=\hat{x}', \quad H = \partial h/\partial x \;|x=\hat{x}' , \\ V &= (\partial h/\partial v)R(\partial h^t/\partial v) \;|x=\hat{x}' , \\ N &= (\partial f/\partial n)Q(\partial f^t/\partial n) \;|x=\hat{x}' . \end{aligned} \qquad (3\text{-}5)$$

The matrix R and Q are defined by

$$\begin{aligned} \mathcal{E}[v(t_m)\; v^t(t_m)] &= R , \\ \mathcal{E}[n(t)\; n^t(\tau)] &= Q\; \delta(t-\tau) . \end{aligned} \qquad (3\text{-}6)$$

4. An example of the tracking Kalman filter

4.1 Target modelling

The tracking system under appropriate consideration utilizes sensors that provide measurements of range and bearing. And this pair of measurements is most common;however, other output measurements such as range rate of Doppler and elevation are also often available.

A recent tracking system response to ground-based air defense has been guided by a variety of sensors-infrared, TV, laser -but all depend on visual acquisition of a target.

We assume that the target to be tracked should be modelled by the following discrete state equation for our study

$$x(t_{m+1}) = \phi\; x(t_m) + G\; n(t_m) \qquad (4\text{-}1)$$

where

$$x(t_m) = \begin{bmatrix} x_1(t_m) \\ x_2(t_m) \\ x_3(t_m) \\ x_4(t_m) \end{bmatrix} = \begin{bmatrix} \text{range at } t_m \\ \text{range rate at } t_m \\ \text{bearing at } t_m \\ \text{bearing rate at } t_m \end{bmatrix}$$

and state transition matrix ϕ is

$$\phi = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad G = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

and T is sampling period.

We also assume that the sensor should observe the target's range and bearing. The measurement equation is

$$m(t_m) = H\; x(t_m) + v(t_m) = \begin{bmatrix} \text{measured range at } t_m \\ \text{measured bearing at } t_m \end{bmatrix} \qquad (4\text{-}2)$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \qquad v(t_m) = \begin{bmatrix} v_1(t_m) \\ v_2(t_m) \end{bmatrix}$$

Assuming the noises $v_1(t_m)$ and $v_2(t_m)$ are independent, the measurement noise covariance matrix, $R(t_m)$, satisfies

$$R(t_m) = \mathcal{E}[v(t_m)v^t(t_m)] = \begin{bmatrix} \sigma_1^2(t_m) & 0 \\ 0 & \sigma_2^2(t_m) \end{bmatrix}$$

The Kalman filter is the most sophisticated , accurate, and costly to implement. The filter equations are given by (3-3) and (3-4). Hence, in this paper we suggest the method of implementation of Kalman filter and the possibility of real time tracking when we use microcomputer to process filter algorithm.

4.2 Design of hardware and software of the filter for real time tracking in using microcomputer

As we know, most of microcomputers not only have the constraint in processing a large amount of data at the same time on account of their limit of memory and word length, but also have shortage in processing data with high speed. Therefore, when we make up of real time tracking filter by means of microcomputer, we shall use TMS 32010 digital signal processor interfaced with microcomputer as an instrument to treat a large scale of data. The TMS32010 is the first member of the new TMS320 digital signal processing family, designed to support a wide range of high speed applications. This 16/32 bit single chip microprocessor combines the flexibility of a high speed controller with the numerical capability of an array processors. The TMS32010 contains the first MOS microcomputers capable of executing 5 million instructions per second. This high throughput is the result of the comprehensive, efficient, and easily programmed instruction set and of the highly pipelined architecture (Harvard Architecture).

4.2.1 Composition of the digital signal processing system

Figure 1 is the schematic diagram of an illustration of digital signal processing system for tracking Kalman filter using TMS32010 and it consists of host computer (microcomputer),TMS32010 DSP, program ROM, data RAM, A/D converters, and logic circuits ,etc.. The TMS 32010 utilizes a modified Harvard architecture in which program memory and data memory lies in two separate spaces. This permits a full overlap of instruction fetch and execution.

In this tracking system, the functions of host computer are the window to give and to take information necessary to tracking algorithm and the equipment to operate tracking system rather than control processor to accomplish a series of processes.

4.2.2 Development of software

We can compose software which is necessary to tracking filter by means of using the advantages of TMS 32010 DSP.

Figure 2 represents the first stage flowchart for the development procedure of the software.

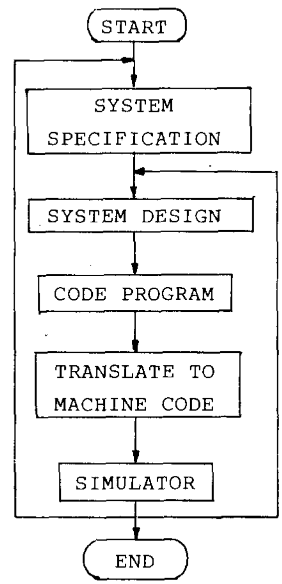

Figure 2. The flowchart of software development

At first, we must decide system constants such as noise covariances, sampling time,

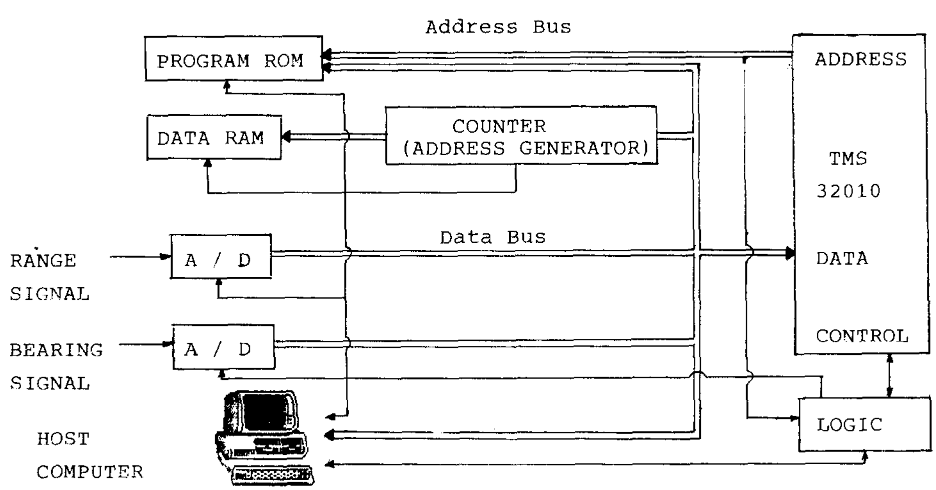

Figure 1. Digital signal processing system for tracking Kalman filter using TMS32010

etc. by using experience and outside information. Second, we must decide the method composing and processing Kalman filter algorithm. That is, it contains the choice of state variables and the design of suboptimal filter. Third, we must code filter algorithm into high-level language in order to test the correctness of the program, and then translate it to assembly language. In last stage, we can test the performance of the filter which was designed. If we are not satisfied with the result, we shall back to the first stage and repeat a series of processes.

5. Conclusion

In this paper, the order of dealing with modelling for moving targets was suggested. The filter designed for tracking was suggested as an example and was composed of microcomputer, personal computer board etc. so that it could process the information for target with high speed.
As a result, we obtained 100 microsecond in calculating one loop Kalman filter algorithm which was designed in our study. The most important result is with respect to more fast real time estimation, in other words the possibility of real time tracking on the moving target that can be used for synthesis of tracking system only by using microcomputer.

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