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IMPROVEMENT OF THE FAST KALMAN ALGORITHM'S NUMERICAL STABILITY

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I. INTRODUCTION

The design of adaptive filters with optimum learning, in the sense of minimizing the accumulated squared error between the output signal and a desired response, is of major importance in many areas of digital signal processing, estimation, and control.

Recursive least squares(LS) algorithms are ones which are belong to this class algorithm. Due to their extremely rapid convergence properties, these schemes find many applications in speech processing, noise cancellation, design of fast start-up equalizers, spectral estimation, etc. and that, the recently introduced fast Kalman algorithm (FKA) requires only O(p) opertions per recursion.[1],[2],[3],[4] This high reduction of computional complexity has raised a new growing interest in exact LS adaptive algorithms since they now require the same order of computation as the suboptimum gradient type algorithms--the least mean square (LMS) algorithm.[1],[2],[5]

Inspite of these good characteristics, the FKA has poor numerical stability when it is used in the long term operating system with forgetting factor smaller than one.[1],[5] In order to improve this numerical stability, the normalization method, resecuc variable method etc. were sugested.[4] But these methods can not make the FKA numerically stable, and CHUNG introduced the correcting constant which improved the numerical stability without loss of the signal detectability and the system adaptibility.[1]

In this paper, these role of the correcting constant is demostrated by an example and its linearized system matrix introduced by S.LIUNG.[6] The FKA is given in section II, and its stability analysis is given in section III. To demonstrate the above analysis, simulations are given in the next section.

II. THE FKA AND CORRECTING CONSTANT

The shifting poperty of the covariance matrix is the main tool to derive the FKA and standard form of the FKA is as follow.

at the t-th time step

$$\eta_t = y_t - x_t^T \theta_{t-1} \tag{2.1}$$

(form the forward residuals)

$$r_t = y_{t-p} - x_{t+1}^T \phi_{t-1} \tag{2.2}$$

(form the backward residuals)

$$\theta_t = \theta_{t-1} + K_t \eta_t \tag{2.3}$$

(update parameter vector of the forward predictor)

$$e_t = y_t - x_t^T \theta_t \tag{2.4}$$

(form the prediction error)

$$E_t = \lambda E_{t-1} + e_t \eta_t \tag{2.5}$$

(form the weighted accumulation of $e_t \eta_t$)

$$\begin{bmatrix} m_t \\ u_t \end{bmatrix} = \begin{bmatrix} E_t^{-1} e_t \\ k_t - \theta_t E_t^{-1} e_t \end{bmatrix} \tag{2.6}$$

(form the extended Kalman gain vector)

$$K_{t+1} = (K_t^* + \phi_{t-1} K_t^{**})/(1-\gamma_t K_t^{**}) \quad (2.7)$$

(update the Kalman gain vector)

$$\phi_t = \phi_{t-1} + K_{t+1}\gamma_t \quad (2.8)$$

(update parameter vector of the backward preditor)

initial values

$$k_0 = \Theta_0 = x_0 = \phi_0 = 0, \quad E_0 = \delta \ (>0) \quad (2.9)$$

The central advantage enjoyed by this FKA over the normal Kalman recursion is that of a lower computational cost --it is required only O(p) multiplication per recursion, and more significant this computational advantage is with increasing model order p.

And an auxilliary variable γ_t (defined as $\gamma_t = K_t^T x_t$) role as a likely-hood variable.[2] This makes the FKA be used to detect statistic for non-Gaussian components in the observations, simulation result indeed demonstrated that γ_t would take high value (close to 1) at non-Gaussian components. It therefore also acts as an optimal gain control in the sense that the factor $1/(1-\gamma_t)$ can adjust the gains instantaneously when non-Gaussian components are present in the observations.

III. NUMERICAL INSTABILITY OF AND CORRECTING CONSTANT

Inspite of the above good characteristices, unexplained explossions in the FKA with forgetting factor λ(<1) have been noticed to occur under some circumstances.[4],[5] Though this explossion do not occur when λ is unity, the adaptablility and the detectability of the non-Gaussian component is decresasing with recursion time increasing. At this point CHUNG sugested correcting constant which can maintain the adaptability and the detectability of the FKA with λ<1. The correcting constant α is a constant which keeps the accumulated sum of in eq.(2.5) from being negative such as;

$$E_t = \lambda E_{t-1} + e_t \eta_t + \alpha \quad (3.1)$$

Since the general analysis of the role of this correcting constant is difficult, We shall here only consider a counter example which is used to show the FKA be numerically unstable by S.Ljung et al.[6]

Let us apply the FKA in section II with eq.(3.1) instead of eq.(2.5) -- hereafter we shall call this algorithm as modified FKA — to a simple first order AR model. Here $x_t (=y_{t-1})$ is a scalar and the modified FKA gives

$$\Theta_t = \Theta_{t-1}(1-K_t y_{t-1}) - K_t y_t \quad (3.2a)$$

$$E_t = \lambda E_{t-1} + (y_t + \Theta_{t-1} y_{t-1})[y_t + \Theta_{t-1}(1 - K_t y_{t-1}) y_{t-1} - K_t y_t y_{t-1}] + \alpha \quad (3.2b)$$

$$m_t = E_t^{-1}\{y_t + [\Theta_{t-1}(1-K_t y_{t-1}) - K_t y_t] y_{t-1}\} \quad (3.2c)$$

$$u_t = K_t + [\Theta_{t-1}(1-K_t y_{t-1}) - K_t y_t] m_t \quad (3.2d)$$

$$\phi_t = [\phi_{t-1} - m_t(y_{t-1} + \phi_{t-1} y_t)]/[1 - u_t y_{t-1} + \phi_{t-1} y_t] \quad (3.2e)$$

$$K_{t+1} = m_t - \phi_t u_t \quad (3.2f)$$

Introduce the state vector X

$$X(t) = [\Theta_t, \phi_t, K_{t+1}, E_t]^T \quad (3.3)$$

Then equation (3.2) can be written as

$$X(t) = f(X(t-1), y_t, y_{t-1}) \quad (3.4)$$

The system matrox of the linearized difference equation is

$$F(t) = \frac{\partial}{\partial X} f (X , y_t , y_{t-1})\Big|_{X=X(t-1)} \quad (3.5)$$

It is still difficult to determine { F(t) } and analyze its stability properties for general input sequences { y_t }. We therefore choose the particular sequence as LJUNG in [6]

$$y_t = \begin{cases} 1 & \text{if } t=4k \\ 0 & \text{if } t=4k-1 \\ -1 & \text{if } t=4k-2 \\ 0 & \text{if } t=4k-3 \end{cases} \quad (3.6)$$

where k is an integer. This gives the nominal trajectory for large k as follow,

$$X(4k) = [0, 0, s_0^{-1}, s_0]^T \quad (3.7a)$$

$$X(4k-1) = [0, 0, 0, s_1]^T \quad (3.7b)$$

$$X(4k-2) = [0, 0, -s_0^{-1}, s_0]^T \quad (3.7c)$$

$$X(4k-3) = [0, 0, 0, s_1]^T \quad (3.7d)$$

where

$$s_0 = 1 / (1-\lambda^2) + \alpha/(1 - \lambda)$$

$$s_1 = \lambda / (1-\lambda^2) + \alpha/(1 - \lambda)$$

The calculation of eq.(3.5) for the case of eq.(3.6) is very tedious and the results are as follow,

$$F(4k) = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1-s_0^{-1} & 0 & 0 \\ 0 & 0 & 0 & -s_0^{-2} \\ 0 & 0 & 0 & \lambda \end{bmatrix} \quad (3.8a)$$

$$F(4k-1) = \begin{bmatrix} 1-s_0^{-1} & 0 & 0 & 0 \\ s_1^{-1} & (1-s_0)^{-1} & 0 & 0 \\ -s_1^{-1} & (1-s_0)^{-1} & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \quad (3.8b)$$

$$F(4k-2) = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & (1-s_0)^{-1} & 0 & 0 \\ 0 & 0 & 0 & -s_0^{-2} \\ 0 & 0 & 0 & \lambda \end{bmatrix} \quad (3.8c)$$

$$F(4k-3) = \begin{bmatrix} (1-s_0^{-1}) & 0 & 0 & 0 \\ -s_1^{-1} & (1-s_0)^{-1} & 0 & 0 \\ s_1^{-1} & (1-s_0)^{-1} & 0 & 0 \\ 0 & 0 & 0 & \lambda \end{bmatrix} \quad (3.8d)$$

It is shown that we always have

$$F(t)F(t-1) = \begin{bmatrix} F_{11} & F_{12} & 0 & 0 \\ F_{21} & 1 & 0 & 0 \\ 0 & 0 & 0 & F_{34} \\ 0 & 0 & 0 & \lambda \end{bmatrix} \quad (3.9)$$

where $F_{11} = 1+(1-\lambda)(1-\lambda^2)/m.n$

$F_{12} = -(1-\lambda^2)/[\lambda^2+(1+\lambda)\alpha]$

$F_{21} = -(1-\lambda^2)[\lambda^2+(1+\lambda)\alpha]/m.n$

$F_{34} = -\lambda(1-\lambda)^2/n^2$

and $m = \lambda+(1+\lambda)\alpha$

$n = 1+(1+\lambda)\alpha$

The matrix F(t)is time-varying, but the product F(t)F(t-1) is time invariant when t is so large that the transient of the form λ^t can be neglected. The matrix F(t)F(t-1) has one eigenvalue at the origin and one λ^2. The sum of the two others is

$$2+(1-\lambda)(1-\lambda^2)/m.n \quad (3.10)$$

Equation (3.10) show that an eigenvalue of the matrix F(t)F(t-1) is larger than unity but we can make it close arbitrarily with incerasing the correcting constant α.

IV.SIMULATIONS AND DISCUSSION

To demonstate the effect of the correcting constant we have done computer simulations. In these simulation, we use data signals generated as follow,

$$y_t = 10\sqrt{20}\sin 0.2\pi t + 100\sqrt{2}\sin 0.5\pi t + 10\sqrt{20}\sin 0.7\pi t + n_t \quad (4.1)$$

where n_t is normal distributed white noise with unity variance. The signal to noise ratio of the data signal is about 41[db], and the order of AR model used simulations is chosen as 18. The modified FKA is used in these simulations with constant forgetting factor λ 0.99 and various correcting constant α as 0, $\sqrt{2}$,1,2,5 respectively. Fig.1 shows these simulation results. In this Fig.,J(t) is the performance measure of the AR model defined short-time average of

$$20\log(|y_t - x_t^T\theta_t| \,/\, |y_t|) \quad (4.2)$$

and is plotted one sample per 20 recursion.

The results show that the FKA with α=0, α= 2 explose about t=2200, t=4600 respectively and show that the FKA with α>1 do not explose until t=5000 as our expectation. But the performance measure J(t) is very simillar except those after explosion whenever the correcting constant α increase.

V.CONCLUSIONS

The analysis of a certian example and simulations given in the previous sections show that the modified FKA is more stable than the standard FKA without loss of the performance of it. The general analysis of modified FKA's numerical stability is the open problem with more simulations in order to prove the stability of it.

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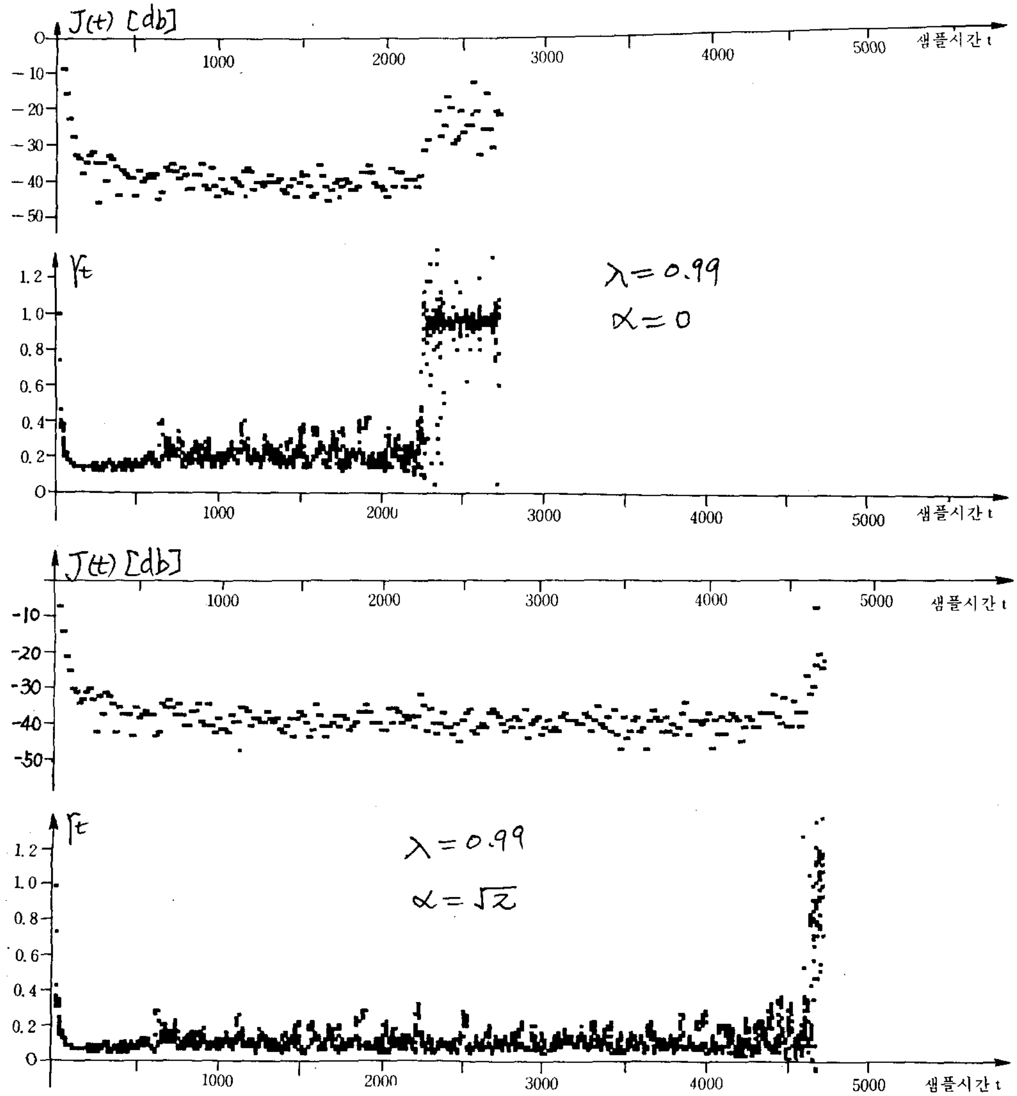

Fig.1 trajectories J(t) and γ with various α

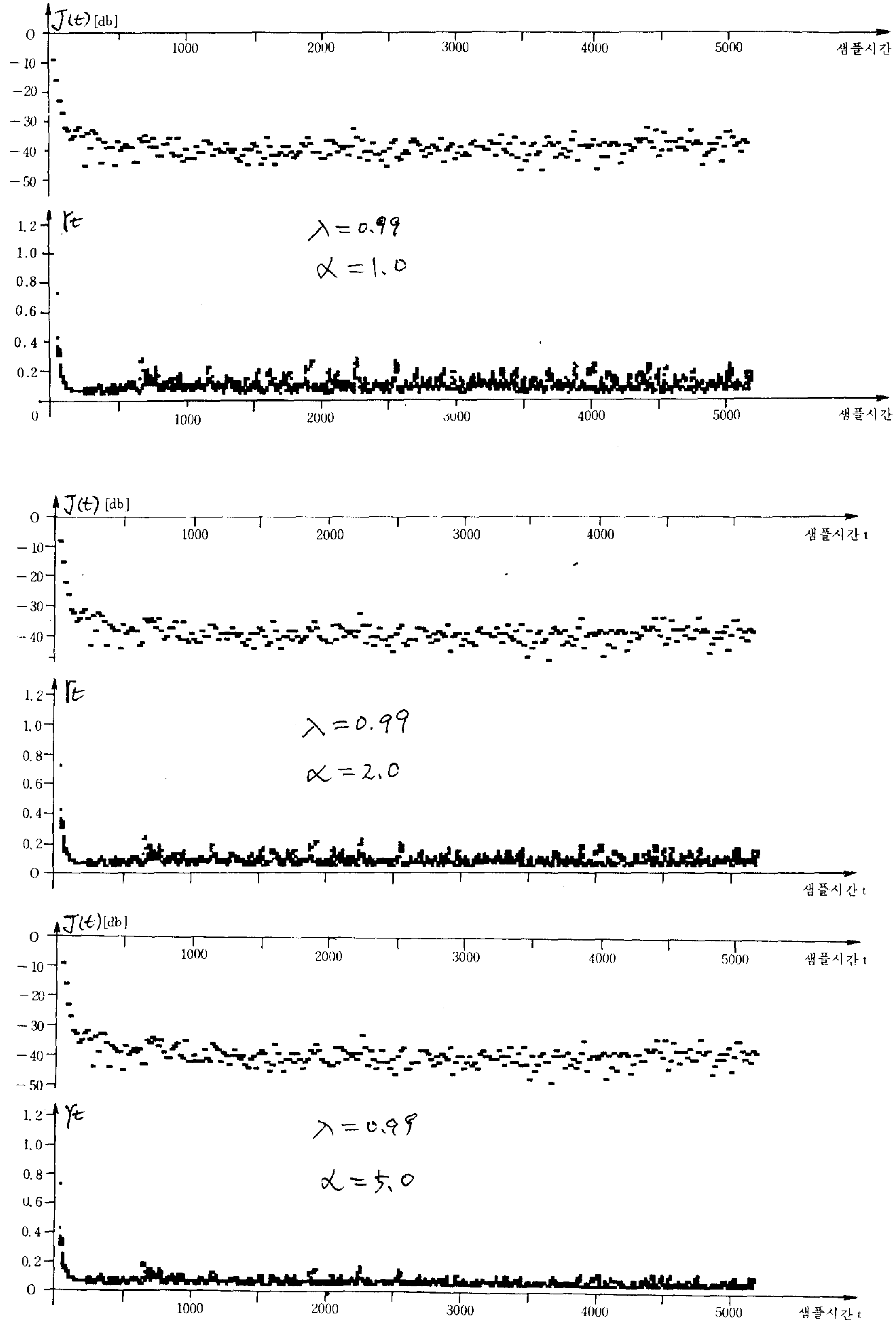

Fig.1 trajectories J(t) and γ with various α (to be continued)