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Direct Adaptive Control of Nonlinear Robot Dynamics

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Abstract:

The payload variation and modeling error can be parameterized in such a way that known nonlinear functions are multiplied linearly by parameter errors. An adaptive control algorithm is derived for a perturbed linear system with such parameterization. Hence, in this approach no linear approximation of robot system is needed for the application of an adaptive law. The stability of the adaptive control algorithm is established and also supported by a computer simulation result.

1. Introduction

Most existing robots move in such a way that they are instructed to traverse a sequence of points according to the data which were generated from kinematic equations or teach-and-replay. In such a manipulation, robots are considered as a connected linkage of servo mechanisms. This concept makes the control problem easy, but leads to the design of heavy links to satisfy the assumption accompanied by simplification. A problem with heavy links is that they necessitate the use of large motors, which, in turn, drives up the cost of the robots. For this reason, currently working robots are over-designed compared with performance. Specifically, they have unacceptably large weight to payload ratio which is, typically, in the range of 30-50. However, if robots are built with lightweight links or if they are desired to move fast, then naive control schemes do not work since the assumptions for the simple control are no longer valid. Hence, in this respect, more attention needs to be paid to the dynamics and control of robots under mild assumptions.

If the links are made with lightweight material, then the effect of payload on the robot dynamics becomes significant, i.e., the dynamics of a robot with a payload will differ significantly from those without it. The reason is that the dynamic parameters vary (in a nonlinear way) as payloads vary. The effect of payloads, as well as some modeling errors can be treated as a parameter perturbation. Hence, for dynamic control of robots, especially lightweight robots, a compensation algorithm for such a parameter perturbation should be provided. A PD-type control scheme, namely, computed torque method was developed as an attempt to control robot systems compensating such an error (Bejczy 1974). However, with computed torque method, it is not possible to eliminate output error (for example, trajectory error), although output errors may be attenuated by stable dynamics.

It is very natural to apply the idea of adaptive control to this problem setting, and the application of adaptive control law to the robot systems is nothing new (Lee and Chung 1984, Seraji 1987). However, almost all the papers concerning the adaptive control of robots are application-oriented, and up to the knowledge of author, rigorous analysis of adaptive control algorithm for robots has not appeared. In this paper, we parametrize the variation of payloads and modeling error without linearizing the system (along trajectory) and develop an adaptive control algorithm following the idea in (Nam and Arapostathis 1987). We also establish the stability adaptive control scheme in the perspective of control theory.

This paper consists of as follows: In section 2, a perturbed linear model is derived so that an adaptive control algorithm is readily applicable. We develop an adaptive control algorithm in section 3 and prove its stability in section 4. Finally, we illustrate the adaptive control scheme with computer a simulation result.

Notations: We denote by $C^r(R^+, R^m)$ the space of $r-$times continuously differentiable functions from a positive real line R^+ to R^m and by $B(0, \rho)$ an open ball of radius $\rho > 0$ centered at 0. We define $f \in O(x)$ if $\|f(x)/x\| \to 0$ as $\|x\| \to 0$. We let I_n be the $n \times n$ identity matrix.

2. Derivation of a perturbed linear system model

Throughout this paper, we consider the following dynamics of a robot having the n-degree of freedom:

$$D(q)\ddot{q} + C(q,\dot{q}) + g(q) = \tau, \tag{2.1}$$

where $q \in R^n$ is a vector of joint angles and τ is a vector of torques applied to each joint. Here, the matrix $D(q)$ is called inertia matrix, $C(q,\dot{q})$ represents Coriolis force plus Coulomb friction and g is a gravity-dependent vector. From the specifications of links, $D(q)$, $C(q,\dot{q})$, $g(q)$ can be obtained as functions of q and $\dot{q}$.

Together with (2.1), we also consider the forward kinematic equation:

$$z = G(q), \tag{2.2}$$

where $z \in R^n$ denotes the position in a fixed task-related Cartesian coordinates and the map $G: R^n \to R^n$ relates the joint coordinates with the Cartesian coordinates. For general six degree of freedom manipulator, z contains 3 position components and 3 orientation components. Here, we consider the system (2.1) and (2.2) in an open subset O of R^n in which $D(q)$ is nonsingular and the map $G: O \to G(O)$ is invertible.* In other words, we consider the robot system in a subset where no singular configuration occurs. We assume that the maps $D: R^{n\times n} \to R^{n\times n}$, $C: R^{2n} \to R$ and $g: R^n \to R^n$ are smooth and that the map $G: O \to G(O)$ is a diffeomorphism.

The functions $D(q)$, $C(q,\dot{q})$, $g(q)$ are dependent on the changes of payloads and parameter error and we regard them as causes of parameter perturbation in $D(q)$, $C(q,\dot{q})$, $g(q)$. For this reason, we assume that the functions $D(q)$, $C(q,\dot{q})$, $g(q)$ are perturbed and as a result, the exact quantities of them are not known. We denote the estimate of perturbed terms $D(q)$, $C(q,\dot{q})$, $g(q)$ by $\hat{D}(q)$, $\hat{C}(q,\dot{q})$, $\hat{g}(q)$ and the errors by $\Delta D(q) = \hat{D}(q) - D(q)$, $\Delta C(q,\dot{q}) = \hat{C}(q,\dot{q}) - C(q,\dot{q})$, $\Delta g(q) = \hat{g}(q) - g(q)$.

For the state-space representation, we need to invert the matrix $D(q)$. Consider the following matrix inversion lemma:

$$D^{-1} = (\hat{D} - \Delta D)^{-1} = \hat{D}^{-1} + \beta\hat{D}^{-1}$$

where $\beta = \hat{D}^{-1}\Delta D(I - \hat{D}^{-1}\Delta D)^{-1}$. Thus, we obtain

$$\begin{aligned}\ddot{q} &= D^{-1}(\tau - C(q,\dot{q}) - g(q)) \\ &= (\hat{D}^{-1} + \beta\hat{D}^{-1})(\tau - (\hat{C} + \hat{g}) + (\Delta C + \Delta g)).\end{aligned} \tag{2.3}$$

Hence, if we choose $\tau = \hat{D}u + \hat{C} + \hat{g}$, then (2.3) yields

$$\ddot{q} = u + \beta u + (\hat{D}^{-1} + \beta\hat{D}^{-1})(\Delta C + \Delta g). \tag{2.4}$$

Note that if $\hat{D}^{-1}\Delta D$ is small compared with I, β is approximated by $\hat{D}^{-1}\Delta D$. Hence, for a sufficiently small error ΔD, the dynamics can be described by

$$\begin{aligned}\ddot{q} &= u + \hat{D}(q)^{-1}\{\Delta D(q)u + \Delta C(q,\dot{q}) + \Delta g(q)\} \\ &\quad + \hat{D}(q)^{-1}\Delta D(q)D(q)^{-1}(\Delta C(q,\dot{q}) + \Delta g(q)).\end{aligned} \tag{2.5}$$

Note that one can choose a parameter vector $\lambda \in R^m$ for some $m > 0$ from $D(q)$, $C(q,\dot{q})$ and $g(p)$ such that all the possible modeling errors and payloads variation are reflected as changes of λ. In the following, we separate the functions of q or $\dot{q}$ from the parameter vector λ. For a matrix $\tilde{D}(q) \in R^{n\times n}$ which does not contain elements of λ, the matrix $D(q)$ may be represented as;†

$$D(q) = \Psi_D(q)\Lambda + \tilde{D}(q), \tag{2.6}$$

where $\Psi_D(q) \in R^{n\times nm}$ is a matrix of known functions of q, and $\Lambda \in R^{nm\times n}$ is a matrix consisting of λ in the following manner:

$$\Lambda = \begin{bmatrix} \lambda & 0 & \cdots & 0 \\ 0 & \lambda & \cdots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & \lambda \end{bmatrix}. \tag{2.7}$$

We denote by $\hat{\lambda} \in R^m$ the estimate of unknown vector λ. We can let

$$\hat{D}(q) = \Psi_D(q)\hat{\Lambda} + \tilde{D}(q) \quad \text{and} \quad \Delta D(q) = \Psi_D(q)\Delta\Lambda,$$

where $\hat{\Lambda}$ consists of $\hat{\lambda}$ like (2.7) and $\Delta\Lambda = \hat{\Lambda} - \Lambda$. Then,

$$\Delta D(q)u = \Psi_D(q)\begin{bmatrix} u_1\Delta\lambda \\ u_2\Delta\lambda \\ \vdots \\ u_n\Delta\lambda \end{bmatrix} = \bar{\Psi}_D(q,u)\Delta\lambda, \tag{2.8}$$

where $\bar{\Psi}_D(q,u) = \sum_{i=1}^{n}\Psi_{Di}(q)u_i$, $\Psi_{Di} \in R^{n\times m}$ denotes the i-th block sub-matrix of Ψ_D, and $\Delta\lambda = \hat{\lambda} - \lambda$. Similarly, we let

$$C(q,\dot{q}) = \Psi_C(q)\lambda + \tilde{C}(q,\dot{q}), \quad \hat{C}(q,\dot{q}) = \Psi_C(q)\hat{\lambda}_C + \tilde{C}(q,\dot{q}),$$

$$g(q) = \Psi_g(q)\lambda + \tilde{g}(q), \quad \hat{g}(q) = \Psi_g(q)\hat{\lambda}_g + \tilde{g}(q),$$

where $\Psi_C, \Psi_g \in R^{n\times m}$, $\tilde{C}, \tilde{g} \in R^n$. Then, note that for fixed q

$$\Delta D(q)\hat{D}(q)^{-1}(\Delta C(q,\dot{q}) + \Delta g(q)) \in O(\Delta\lambda). \tag{2.9}$$

Hence, neglecting the terms in (2.9) for small parameter error $\Delta\lambda$, the system (2.5) reduces to

$$\ddot{q} = u + \Psi(q,\dot{q},u)\Delta\lambda \tag{2.10}$$

where

* Noticing that $G(O)$ is usually defined to be a work space, this assumption is, in general, not a restriction.

† For the specific illustration, look at the example in section 5. There might be a case that $D(q)$ may contain a term in which a unknown parameter is multiplied not linearly. For example, the ij-th element of $D(q)$ may be represented as $d_{ij}(q) = f_1(q)\alpha + f_2(q)\alpha^2$. In this case, we can obtain the form (2.6) by treating α and α^2 as distinct parameters.

$$\Psi(q,\dot{q},u) = \hat{D}^{-1}(q)\{\tilde{\Psi}_D(q,u) + \Psi_C(q,\dot{q}) + \Psi_g(q)\}.$$

If we let $x = [q_1, \ldots, q_n, \dot{q}_1, \ldots, \dot{q}_n]^T \in R^{2n}$, then (2.10) yields

$$\dot{x} = Ax + B(u + \Psi(x,u)\Delta\lambda), \qquad (2.11)$$

where

$$A = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \in R^{2n \times 2n} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ I_n \end{bmatrix} \in R^{2n \times n}.$$

We can choose $K \in R^{n \times 2n}$ so that the eigenvalues of $A + BK$ have negative real part. For an external input $v : R^+ \to R^n$, we let

$$u(t) = Kx + v(t) \qquad (2.12)$$

so that the autonomous part is asymptotically stable. Then, we obtain from (2.11)

$$\dot{x} = (A + BK)x + B(v + \overline{\Psi}(x,v)\Delta\lambda), \qquad (2.13)$$

where $\overline{\Psi}(x,v) = \Psi(x, Kx+v)$. Summarizing this section, one can say that if a torque

$$\tau = \hat{D}(q)^{-1}(K\begin{bmatrix} q \\ \dot{q} \end{bmatrix} + v) + \hat{C}(q,\dot{q}) + \hat{g}(q) \qquad (2.14)$$

is applied to the joints, then the system (2.1) can be described by the linear perturbed model (2.13).

Remark: Note from (2.13) that the feedback (2.14) realizes a kind of computed torque method. However, unless $\overline{\Psi}(x,v)\Delta\lambda$ vanishes, output error $z = G(q)$ will not vanish, either. Hence, although computed torque method may help attenuating the effect of error $\overline{\Psi}(x,v)\Delta\lambda$ by a stable system, it cannot eliminate the output error completely. In the next section, we will show how to eliminate the effect of the error with a parameter adaptive algorithm.

3. A Direct Nonlinear Adaptive Control Scheme

In the previous section, we obtain the following equations:

$$\dot{x} = (A + BK)x + B(v + \overline{\Psi}(x,v)\Delta\lambda), \qquad (3.1)$$

$$z = G(Cx), \qquad (3.2)$$

where $C = [I_n \vdots 0] \in R^{n \times 2n}$. We will develop a direct nonlinear adaptive control algorithm for the system (3.1) and (3.2). We let a linear operator H_t be defined by

$$H_t(\gamma) = C\int_0^t e^{(A+BK)(t-\tau)}B\gamma(\tau)\,d\tau \qquad (3.3)$$

for a piecewise continuous function $\gamma : R^+ \to R^n$. Let $H(s) \equiv C(sI - (A+BK))^{-1}B$. From the specific structure of A, B and C, the following matrices are equivalent in the sense that they have the same Smith form

$$\begin{bmatrix} sI-(A+BK) & B \\ -C & 0 \end{bmatrix} \sim \begin{bmatrix} sI-A & B \\ -C & 0 \end{bmatrix} \sim \begin{bmatrix} I_{2n} & 0 \\ 0 & I_n \end{bmatrix}.$$

Hence, the inverse of $H(s)$, $H(s)^{-1}$ has poles at infinity, i.e., $H(s)^{-1}$ does not have poles in the right half plane (Kailath 1980). Thus, for a uniformly bounded function $\gamma(t) \in C^{2n}(R^+, R^n)$, $\|H_t^{-1}(\gamma)\|$ is bounded, where H_t^{-1} denotes the inverse operator of H_t. Choose

$$v = H_t^{-1}(G^{-1}(r(t)), \qquad (3.4)$$

where $r(t) \in C^{2n}(R^+, R^n)$ is a desired trajectory for the tip of the end effector to follow. Then,

$$Cx(t) = G^{-1}(r(t)) + H_t(\overline{\Psi}(x,v)\Delta\lambda) + Ce^{(A+BK)t}x(0). \qquad (3.5)$$

Introducing an augmented error,

$$\varepsilon(t) = H_t(\overline{\Psi}(x,v))\hat{\lambda} - H_t(\overline{\Psi}(x,v)\hat{\lambda}) \qquad (3.6)$$

we express equivalently as

$$H_t(\overline{\Psi}(x))\Delta\lambda(t) = \xi(t), \qquad (3.7)$$

where

$$\xi(t) = Cx(t) - G^{-1}(r(t)) + \varepsilon(t) - Ce^{(A+BK)t}x(0). \qquad (3.8)$$

Note that $\xi(t)$ is available for all $t \in R^+$.

We propose the following parameter adaptive law, which is based on a pseudogradient algorithm,

$$\Delta\dot{\lambda} = \dot{\hat{\lambda}} = -\kappa H_t(\overline{\Psi})^T(H_t(\overline{\Psi})H_t(\overline{\Psi})^T + \delta_0 I)^{-1}H_t(\overline{\Psi})\Delta\lambda(t)$$

$$= -\kappa H_t(\overline{\Psi})^T(H_t(\overline{\Psi})H_t(\overline{\Psi})^T + \delta_0 I)^{-1}\xi(t) \qquad (3.10)$$

where κ is a positive scalar gain and δ_0 is a small positive number. Let $R(t) = (H_t(\overline{\Psi})H_t(\overline{\Psi})^T + \delta_0 I)^{-1}$. Then, utilizing the identity $\dot{R} = -R\frac{d}{dt}(R^{-1})R$, we observe that (3.10) is equivalent to the following equations:

$$\dot{\hat{\lambda}} = -\kappa H_t(\overline{\Psi})^T R(t)\xi(t) \qquad (3.11)$$

$$\dot{R} = -R(\Phi(\overline{\Psi}) + \Phi(\overline{\Psi})^T)R, \quad R(0) = \frac{1}{\delta_0^n}I \qquad (3.12)$$

$$\Phi(\overline{\Psi}) = C(A+BK)\chi(t)H_t(\overline{\Psi})^T \qquad (3.13)$$

$$\dot{\chi}(t) = (A+BK)\chi(t) + B\overline{\Psi}(x), \quad \chi(0) = 0. \qquad (3.14)$$

Summarizing section 2 & 3, we obtain the following block diagram:

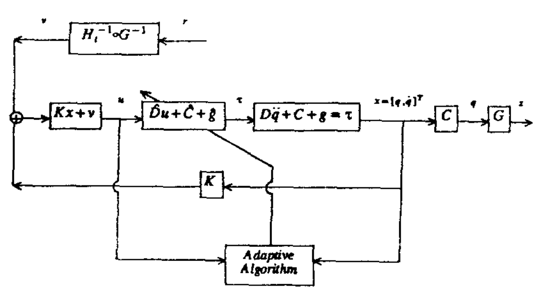

Figure 1. Block diagram of an adaptive control scheme for a robot

4. Proof of Convergence

We summarize the equations of the adaptive algorithm derived in the previous section:

$$\dot{x} = (A + BK)x + B(v + \overline{\Psi}(x,v)\Delta\lambda) \tag{4.1}$$

$$z = G(Cx) \tag{4.2}$$

$$\dot{\hat{\lambda}} = -\kappa H_t(\overline{\Psi})^T R(t)\xi(t) \tag{4.3}$$

$$\dot{R} = -R(\Phi(\overline{\Psi}) + \Phi(\overline{\Psi})^T)R, \quad R(0) = \frac{1}{\delta_0^n} I \tag{4.4}$$

$$\Phi(\overline{\Psi}) = C(A+BK)\chi(t)H_t(\overline{\Psi})^T \tag{4.5}$$

$$\dot{\chi}(t) = (A + BK)\chi(t) + B\overline{\Psi}(x,v), \quad \chi(0) = 0 \tag{4.6}$$

$$\xi(t) = Cx(t) - G^{-1}(r(t)) + \varepsilon(t) - Ce^{(A+BK)t}x(0) \tag{4.7}$$

$$\varepsilon(t) = H_t(\overline{\Psi}(x,v))\hat{\lambda} - H_t(\overline{\Psi}(x,v)\hat{\lambda}) \tag{4.8}$$

$$v = H_t^{-1}(G^{-1}(r(t)) \tag{4.9}$$

Theorem 4.1: Consider the plant (4.1-2) with the controller (4.3-4.9). Suppose that the reference input $r(t) \in C^{2n}(R^+, R^n)$ is uniformly bounded. Then, there exists an open neighborhood $U_\lambda \subset R^m$ of λ, depending on the bound of $\|r(t)\|$ such that if $\hat{\lambda}(0) \in U_\lambda$, $\|z(t) - r(t)\| \to 0$ as $t \to \infty$.

We need the following technical Lemma for the proof of theorem 4.1.

Lemma: For $M \in R^{n\times m}$ and $\delta > 0$,

$$\|M^T(MM^T + \delta I_n)^{-1/2}\| < 1.$$

Proof of Lemma: Utilizing the identity

$$P(I_m + FP)^{-1} = (I_n + PF)^{-1}P \in R^{n\times m} \tag{4.10}$$

for $P \in R^{n\times m}$ and $F \in R^{m\times n}$, we obtain

$$\|M^T(MM^T + \delta I_n)^{-1/2}\|^2 = \|M^T(MM^T + \delta I_n)^{-1}M\|$$
$$= \|(M^TM + \delta I_m)^{-1}M^TM\| < 1. \quad \blacksquare$$

Proof: Let $V(t) = \frac{1}{2\kappa}\Delta\lambda(t)^T\Delta\lambda(t)$. Recall the following relationship

$$\xi(t) = H_t(\overline{\Psi})\Delta\lambda(t) \tag{4.11}$$

From (4.3) and (4.11),

$$\dot{V}(t) = -\Delta\lambda^T H_t(\overline{\Psi})^T R H_t(\overline{\Psi})\Delta\lambda \le 0 \tag{4.12}$$

for all $t \ge 0$. Also, since $V(t) \ge 0$ and $\dot{V}(t) \le 0$, then

$$V(0) - V(\infty) = -\int_0^\infty \dot{V}\, d\tau = \int_0^\infty \Delta\lambda^T H_t(\overline{\Psi})^T R H_t(\overline{\Psi})\Delta\lambda\, d\tau < \infty$$

and hence,

$$\|R^{1/2}H_t(\overline{\Psi})\Delta\lambda\| \in L^2(R^+). \tag{4.13}$$

Since from (4.13)

$$\|\dot{\hat{\lambda}}\| \le \kappa\|H_t(\overline{\Psi})^T R^{1/2}\|\|R^{1/2}H_t(\overline{\Psi})\Delta\lambda\| \tag{4.14}$$

and since, by the previous Lemma, $\|H_t(\overline{\Psi})^T R^{1/2}\| < 1$, $\|\dot{\hat{\lambda}}\| \in L^2(R^+)$.

In the following, we prove the uniform boundedness of $\|x\|$ to show that $\|H_t(\overline{\Psi})\Delta\lambda\|$ is bounded. With P and W positive definite matrices satisfying

$$(A+BK)^T P + P(A+BK) = -W, \tag{4.15}$$

we choose $\overline{V} = e^T Pe$. We let $e(t) = x(t) - \overline{x}(t)$, where $\overline{x}(t)$ is the solution of

$$\dot{\overline{x}} = (A + BK)\overline{x} + Bv(t), \quad \overline{x}(0) = x(0). \tag{4.16}$$

Then, we obtain from (4.1), (4.9), (4.15) and (4.16) that

$$\dot{\overline{V}} = -e^T We + 2e^T P\overline{\Psi}(e(t)+\overline{x}, v)\Delta\lambda. \tag{4.17}$$

Therefore,

$$\dot{\overline{V}} \le -\alpha_w\|e\|^2 + 2\alpha_p\|e\|\|\overline{\Psi}(e+\overline{x}, v)\|\|\Delta\lambda\|, \tag{4.18}$$

where α_w is the smallest eigenvalue of W and α_p is the largest eigenvalue of P.

Since $r(t)$ is assumed to be bounded, it follows from (4.9) and (4.16) that $v(t)$ and $\overline{x}(t)$ are also bounded. Since $\overline{\Psi}$ is a smooth function of its arguments, there exist positive constants k_1 and k_2, such that for $e \in B_e(0,\rho_e) \subset R^{2n}$, some $\rho_e > 0$,

$$\|\overline{\Psi}(e+\overline{x}, v)\| \le k_1\|e\| + k_2.$$

Thus, for $e(t) \in B_e(0,\rho_e)$,

$$\dot{\overline{V}} \le (2k_1\alpha_p\|\Delta\lambda\| - \alpha_w)\|e\|^2 + 2k_2\alpha_p\|\Delta\lambda\|\|e\|. \tag{4.19}$$

Hence, if $\|\Delta\lambda\| \le \frac{\alpha_w}{2\alpha_p k_1}$, $\dot{\overline{V}} \le 0$ for $e \in B_e(0,\rho_e)$. Choose $U_\lambda = B_\lambda(\lambda, \rho_\lambda) \subset R^m$, where $\rho_\lambda = \frac{\alpha_w}{2\alpha_p k_1}$. Note from (4.12) that $\hat{\lambda}(t) \in U_\lambda$ for all $t \in R^+$, if $\hat{\lambda}(0) \in U_\lambda$. Since $e(0) = 0 \in B_e(0,\rho_e)$, if $\hat{\lambda}(0) \in U_\lambda$ then $e(t)$ thus, $x(t)$ is uniformly bounded. Hence, the boundedness of $\|\frac{d}{dt}\{H_t(\overline{\Psi})\Delta\lambda\}\|$ also follows from (4.3-8). Therefore, along with (4.13), it follows that as $t \to \infty$

$$H_t(\overline{\Psi})\Delta\lambda \to 0. \tag{4.20}$$

Note that

$$\varepsilon(t) = C(\chi(t)\hat{\lambda}(t) - \tilde{\chi}(t)), \tag{4.21}$$

where $\tilde{\chi}(t) \in R^{n\times m}$ satisfies

$$\dot{\tilde{\chi}} = (A + BK)\tilde{\chi} + B\overline{\Psi}(x,v)\hat{\lambda}, \quad \tilde{\chi}(0) = 0.$$

Since

$$\frac{d}{dt}(\chi\hat{\lambda} - \tilde{\chi}) = (A+BK)(\chi\hat{\lambda} - \tilde{\chi}) + \chi\dot{\hat{\lambda}}, \quad (\chi\hat{\lambda} - \tilde{\chi})(0) = 0,$$

it follows that

$$\varepsilon(t) = C\int_0^t e^{(A+BK)(t-\tau)}\chi\dot{\hat{\lambda}}\, d\tau. \tag{4.22}$$

Since $\dot{\hat{\lambda}} \in L^2(R^+)$ and $\chi(t)$ is uniformly bounded, $\varepsilon(t) \to 0$ as $t \to \infty$. Therefore, it follows from the (4.7) and (4.11) that $Cx(t) \to G^{-1}(r(t))$, i.e., $z(t) \to r(t)$ since G is one-to-one. ■

5. Simulation example

As a simulation example, we consider the following planar 2 degree of freedom manipulator:

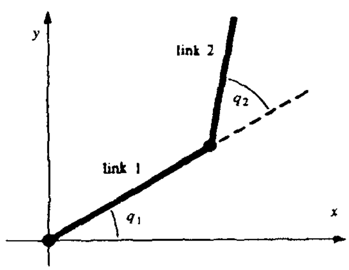

Figure 2. A planar 2 degree of freedom manipulator

$l_1 = l_2 = 0.5\,m$, $m_1 = 50\,Kg$, $m_2 = 30\,Kg$,

$I_1 = 5\,Kg.m^2$, $I_2 = 3\,Kg.m^2$,

We let q_i, l_i, m_i, and I_i the joint angle, length, mass, and the moment of inertia, respectively, of link i; we let $s_i(c_i)$ the sine (cosine) of the joint i variable and c_{1+2} $\cos(q_1+q_2)$. The data in this example was taken from (Asada 1984).

Neglecting frictions, the dynamic equation is given by

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = \begin{bmatrix}D_{11} & D_{12}\\ D_{21} & D_{22}\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + \begin{bmatrix}-\tfrac{1}{2}m_2 l_1 l_2 s_2 \dot{q}_2^2 - m_2 l_1 l_2 s_2 \dot{q}_1 \dot{q}_2\\ \tfrac{1}{2}m_2 l_1 l_2 s_2 \dot{q}_1^2\end{bmatrix} + \begin{bmatrix}\{\tfrac{1}{2}m_2 l_2 c_{1+2} + l_1(\tfrac{1}{2}m_1 + m_2)c_1\}g\\ \tfrac{1}{2}m_2 l_2 c_{1+2}\, g\end{bmatrix}, \quad (5.1)$$

where

$$D_{11} = I_1 + I_2 + \tfrac{1}{4}(m_1 l_1^2 + m_2 l_2^2) + m_2 l_1^2 + m_2 l_1 l_2 c_2,$$

$$D_{12} = D_{21} = I_2 + \tfrac{1}{4}m_2 l_2^2 + \tfrac{1}{2}m_2 l_1 l_2 c_2,$$

$$D_{22} = I_2 + \tfrac{1}{4}m_2 l_2^2$$

The kinematic equation is given by

$$\begin{bmatrix}x\\ y\end{bmatrix} = G(q) = \begin{bmatrix}l_1\cos(q_1) + l_2\cos(q_1+q_2)\\ l_1\sin(q_1) + l_2\sin(q_1+q_2)\end{bmatrix} \quad (5.2)$$

In this example, we assume that a point mass of $2Kg$ — whose actual mass may be unknown — is attached to the open tip of the link 2. Then, the parameters m_2 and I_2 will change into $\hat{m}_2 = 32\,Kg$ and $\hat{I}_2 = 3.5\,Kg.m^2$. Let $\lambda = [I_2, m_2]^T$. Then, we obtain

$$\tilde{D}(q) = \begin{bmatrix}I_1 + \tfrac{1}{4}m_1 l_1^2 & 0\\ 0 & 0\end{bmatrix}, \quad \tilde{C}(q,\dot{q}) = 0, \quad \tilde{g}(q) = 0,$$

$$\tilde{\Psi}_D(q,u) = \begin{bmatrix}u_1 + u_2 & \tfrac{1}{4}l_2^2(u_1+u_2) + l_1 l_2 c_2(u_1 + \tfrac{1}{2}u_2) + u_1 l_1^2\\ u_1 + u_2 & \tfrac{1}{4}l_2^2(u_1+u_2) + \tfrac{1}{2}l_1 l_2 c_2 u_1\end{bmatrix}$$

$$\Psi_C(q,\dot{q}) = \begin{bmatrix}0 & -\tfrac{1}{2}l_1 l_2 s_2 \dot{q}_2^2 - l_1 l_2 s_2 \dot{q}_1 \dot{q}_2\\ 0 & \tfrac{1}{2}l_1 l_2 s_2 \dot{q}_1^2\end{bmatrix}$$

$$\Psi_g(q) = \begin{bmatrix}0 & (\tfrac{1}{2}l_2 c_{1+2} + l_1 c_1)g\\ 0 & \tfrac{1}{2}l_2 c_{1+2}\, g\end{bmatrix}$$

Hence,

$$\Psi(q,\dot{q},u) = \hat{D}(q)^{-1}\begin{bmatrix}u_1 + u_2 & \tfrac{1}{4}l_2^2(u_1+u_2) + l_1 l_2 c_2(u_1 + \tfrac{1}{2}u_2) + u_1 l_1^2 - l_1 l_2 s_2(\tfrac{1}{2}\dot{q}_2^2 + \dot{q}_1\dot{q}_2) + (\tfrac{1}{2}l_2 c_{1+2} + l_1 c_1)g\\ u_1 + u_2 & \tfrac{1}{4}l_2^2(u_1+u_2) + \tfrac{1}{2}l_1 l_2(c_2 u_1 + \dot{q}_1^2) + \tfrac{1}{2}l_2 c_{1+2}\, g\end{bmatrix}$$

We compare the performance of the adaptive control with that of non-adaptive one, i.e., fixed gain feedback. We use the following data for the simulation:

$$K = \begin{bmatrix}-2 & 0 & -2 & 0\\ 0 & -2 & 0 & -2\end{bmatrix}, \quad \kappa = 1000, \quad x(0) = [\tfrac{\pi}{4}, \tfrac{\pi}{3}]^T,$$

$$r(t) = G \circ H_t([2\sin t, 4\cos t]^T).$$

Figure.3 & 4 show the the trajectories of the model and the adaptive control system, while Figure.5 & 6 show the trajectories the model and the non-adaptive system. In this simulation, non-adaptive system uses the same feedback with the adaptive one except the parameter adaptation algorithm.

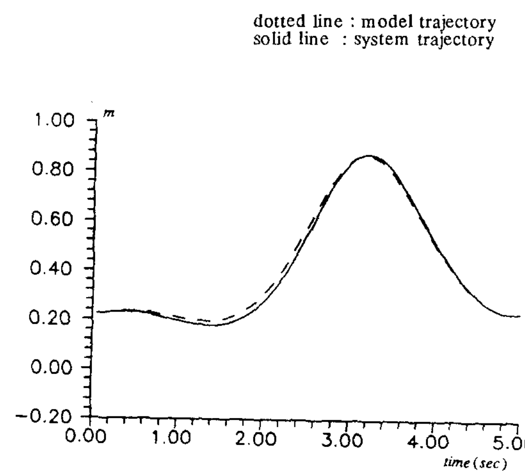

Figure 3. The trajectories (x-component) of the adaptive system and the model.

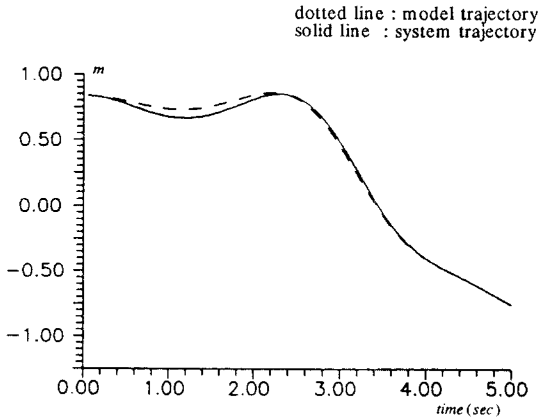

Figure 4. The trajectories (y-component) of the adaptive system and the model.

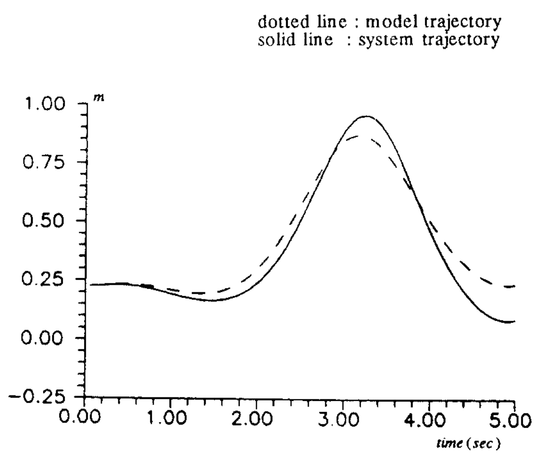

Figure 5. The trajectories (x-component) of the non-adaptive system and the model.

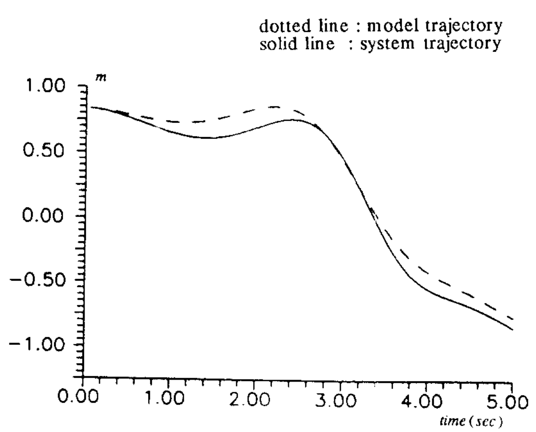

Figure 6. The trajectories (y-component) of the non-adaptive system and the model.

6. Conclusion

We parametrize the variation of payloads and modeling error in robot dynamic equation and develop an adaptive control algorithm. In this approach, we do not linearize the system along trajectory for the application of adaptive algorithm so that it becomes possible to solve both the trajectory planning and the parameter adaptation in an integrated way. Hence, the methodology used here for deriving adaptive control algorithm looks somewhat unified. If the adaptation algorithm is switched off, then the whole control scheme turns out to be a sort of computed-torque methods. This idea may be implemented practically with the advent of cheap but high performance computer. For applications, the study of robustness with respect to the sampling needs to be extended.

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