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TWO-STAGE ROBUST MODEL FOLLOWING CONTROL OF ROBOT MANIPULATORS

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Abstract: This paper is concerned with a robust model following control scheme for manipulators which contains uncertain terms. Our method consists of nonlinear compensation and linear compensation. The former ensures the robustness of the plant, the later achieves both the desired model following response and the desired initial error convergence.

1.Introduction

In recent years, a number of new robust control methods have been developed for a class of nonlinear systems. These methods can be classified into the following three types;

(A) Control methods based on the VSS theory[1][2],

(B) Control methods based on the Liapunov's method[3][4], and

(C) Control methods based on a method which is combination of (A) and (B)[5].

However, the methods of (A) and (B) have some problems; that is,

(1) the resulting control function becomes discontinuous, or the degree in continuity of the input function highly depends on the specified control accuracy.

The main drawback of the method (C), on the other hands, is that,

(2) the control accuracy can not be assigned quantitatively.

All these three methods have also the same problem, that is;

(3) the convergence characteristic of initial error response can not be specified explicitly.

Furthermore, the class of systems that can be treated by the method (B) or (C) does not contain manipulator systems.

For nonlinear mechanical systems such as manipulators, we have already proposed a robust control scheme[6] by introducing the effective expression for the dynamical model of nonlinear mechanical systems, and by using the basic idea of the control scheme proposed in the paper [5]. It is shown that the resulting control system has been able to solve the problems (1) and (2).

We have also proposed, in the papers [8] and [9], a new method named Two-Stage Robust Model Following Control System (Two-Stage-RMFCS) for nonlinear mechanical systems. The main idea of the method is a combination of the nonlinear controller treated in [6] and the linear controller treated in [8]. Consequently, we have succeeded in solving the problems (1) to (3) simultaneously.

In this paper, we discuss the application problem of Two-Stage-RMFCS to a trajectory tracking problem of a manipulator.

2.Problem Statement

We describe the problem which will be solved in this paper. Let us consider the following system;

Manipulator<S>:

$$\begin{cases} J(q)\ddot{q}+D(q)f(\dot{q})+H\dot{q}+g(q)=u & \text{(1a)} \\ y=q \quad (q(t)=q_0 \text{ for } t\leq 0) & \text{(1b)} \end{cases}$$

Reference model<S_R>:

$$\begin{cases} \dot{q}_R=Ir_R & \text{(2a)} \\ y_R=q_R \quad (q_R(t)=q_{R0} \text{ for } t\leq 0). & \text{(2b)} \end{cases}$$

The error signal is defined by

$$\text{Error}: e=y-y_R \quad (e(t)=e_0 \text{ for } t\leq 0) \quad (3)$$

where

- q:nx1 vector of the joint angle.
- $J(q)$:nxn inertia matrix.
- $D(q)$:nxn^2 matrix defining Coriolis and centrifugal terms.
- H :nxn diagonal matrix of viscous friction torque.
- $g(q)$:nx1 vector defining gravity terms.
- I :nxn unit matrix.
- 0 :nxn zero matrix.
- u,r_R:nx1 vectors of input force.
- y :nx1 controlled variable.
- y_R :nx1 controlled variable of reference model.

and

$$f(\dot{q})=[\dot{q}_1\dot{q}_1,\ldots\dot{q}_1\dot{q}_n,\ldots,\dot{q}_n\dot{q}_1,\ldots,\dot{q}_n\dot{q}_n]^T.$$

In general, we can assume the following assumptions pertaining to the manipulator whose joints are all rotational.

[A1] $J(q)=J(q)^T>0$ for any x. (4)

[A2] Matrices $J(q)$,$D(q)$,H and $G(q)$ are all bounded.

[A3] We can measure the joint angle q and the joint angular velocity $\dot{q}$, and $f(\dot{q})$.

[A4] r_R is at least two times differentiable.

Now we can state the problem considered here as follows.

[PROBLEM] Consider a manipulator with the assumptions [A1] to [A4]. And specify the reference model $\langle S_R \rangle$; namely,

$$(a)G_R(s)=I, \qquad (5a)$$

and the convergence characteristic of e_0 as,

$$(b)G_e(S)=\frac{s+2M}{(s+M)^2}I \ (M>0). \qquad (5b)$$

Then, design the control input u so that the controlled variable of system $\langle S \rangle$ may follow the trajectory,

$$y_d=y_R+L^{-1}(G_e(s)e_0) \qquad (6a)$$

with the arbitrary accuracy of ε ; namely,

$$(c)\|y-y_d\|<\varepsilon \quad \text{for } t\geq 0. \qquad (6b)$$

Where, $L^{-1}(\cdot)$ denotes the inverse Laplace transformation. And for simplicity, we set $\dot{e}_0=0$ ($\dot{q}(t)=\dot{q}_d(t)=0$ for $t\leq 0$). □

3.Two-stage-RMFCS for Manipulator

For the purpose of solving the above problem, we consider the following two-stage construction of a controller.
(Stage-1) Construct an intermediate linear model $\langle S_M \rangle$ whose initial condition coincides with the initial condition of manipulator $\langle S \rangle$. Then, design a robust tracking compensator $\langle C_N \rangle$ so that y may follow y_M.
(Stage-2) Regard the intermediate linear model $\langle S_M \rangle$ as a controlled object, and construct a two-degree-of-freedom controller for $\langle S_M \rangle$ so that the resulting system may have the desired properties.

3.1) Stage1:Design of a robust tracking compensator$\langle C_N \rangle$

Construct the following n input n output stable intermediate linear model $\langle S_M \rangle$ for the manipulator$\langle S \rangle$.

$$\langle S_M \rangle : \begin{cases} \ddot{q}_M+k_2\dot{q}_M+k_1q_M=u_M & (7a) \\ y_M=q_M & (7b) \end{cases}$$

where, k_1, k_2 denote scalar constants, and y_M denotes the controlled variable of intermediate linear model.

Now, Consider the following input candidate for the solution of the problem stated in chapter 2.

$$u=\hat{J}(q)(-k_1q-k_2\dot{q}+u_M) +\hat{D}(q)f(\dot{q})+\hat{H}\dot{q}+\hat{g}(q)+u_R \qquad (8)$$

where $\hat{J}(q)\in R^{nxn}$, $\hat{D}(q)\in R^{nxn2}$, $\hat{H}\in R^{nxn}$ and $\hat{g}(q)\in R^n$ denote the nominal bounded matrices of $J(q)$, $D(q)$, H and $g(q)$, respectively, $u_R\in R^n$ denotes the new input from the robust compensator to be designed.

Let us substitute Eq.(8) into $\langle S \rangle$,and define the state error as follows:

$$z=\begin{bmatrix} q-q_M \\ \dot{q}-\dot{q}_M \end{bmatrix}. \qquad (9)$$

By using this notation of state error and $\langle S_M \rangle$, we can derive the following error system:

$$\begin{cases} \dot{z}=A_Mz+B_Mw & (10a) \\ w=J(q)^{-1}u_R+\sum_{k=1}^{6} C_K(q)a_k & (10b) \\ z(t)=0 \quad (t\leq 0) & (10c) \end{cases}$$

where

$$A_M=\begin{bmatrix} 0 & I \\ -k_1I & -k_2I \end{bmatrix}, B_M=\begin{bmatrix} 0 \\ I \end{bmatrix} \qquad (11a)$$

$$C_1(q)=J(q)^{-1}(\hat{D}(q)-D(q)), a_1=f(\dot{q}) \qquad (11b)$$

$$C_2(q)=J(q)^{-1}(\hat{H}-H) \quad , a_2=\dot{q} \qquad (11c)$$

$$C_3(q)=J(q)^{-1}(\hat{g}(q)-g(q)), a_3=1 \qquad (11d)$$

$$C_4(q)=(I-J(q)^{-1}\hat{J}(q))k_1 \quad , a_4=q \qquad (11e)$$

$$C_5(q)=(I-J(q)^{-1}\hat{J}(q))k_2 \quad , a_5=\dot{q} \qquad (11f)$$

$$C_6(q)=J(q)^{-1}\hat{J}(q)-I \quad , a_6=u_M \qquad (11g)$$

Remark1:From [A1],[A2],Eqs.(8) and (11), we can obtain

$$J(q)^{-1}=(J(q)^{-1})^T>0, \qquad (12a)$$

and we can estimate the bounded constant values which satisfy the inequalities,

$$G_m(J(q)^{-1})>J \qquad (12b)$$

$$\|C_k(q)\|<C_k \quad (k=1,2..,6). \qquad (12c)$$

where $G_m(W)$ denotes the minimum eigenvalue of nxn square matrix W.

Then , from [6] we can obtain the following lemma.

[Lemma1] Let $z=[z_1^T,z_2^T]^T$ and consider the linear system

$$\begin{cases} \dot{z}=A_Mz+B_Mw \ (z(t)=0 \text{ for } t\leq 0) & (13a) \\ v=B_M^TPz. & (13b) \end{cases}$$

where $P\in R^{2nx2n}$ is the solution of Lyapunov equation

$$A_M^TP+PA_M=-Q \ (\text{Any } Q=\text{diag}[Q_1I \mid Q_2I]>0). \qquad (14)$$

Then, for any p>0, if there exists an input w which satisfies

$$\|v\|<p \quad \text{for} \quad t\geq 0, \tag{15}$$

we can obtain the following inequality:

$$\|z_1\|<mp \quad \text{for } t\geq 0 \tag{16}$$

where m is the scalar constant calculated from A_M, B_M and Q.

Proof) Solving Eq.(14) and substituting the solution into Eq.(13b), we obtain

$$\dot{z}_1=-az_1+bv. \tag{17}$$

where

$$a=\frac{Q_1k_2}{Q_1+k_1Q_2}>0, \quad b=\frac{2k_1k_2}{Q_1+k_1Q_2}>0 \ . \tag{18}$$

Next, consider the positive definite function

$$L=z_1^Tz_1/2. \tag{19}$$

After differentiation of L along the solution of Eq.(17), substitution of Eq.(15) yields

$$\dot{L}<-\|z_1\|(a\|z_1\|-bp). \tag{20}$$

From Eq.(20) and the condition z(0)=0 , if m is set as

$$m=b/a=2k_1/Q_1, \tag{21}$$

we can obtain Eq.(16). ■

Remark2:From Eqs.(15),(16) and (17), the boundedness of z_2 can be guaranteed.

[Lemma2] Suppose that the arbitrary positive constants d, p and the following input for system(9) are given:

$$u_R=-\frac{v}{\|v\|+d}\sum_{k=0}^{6}h_k\|a_k\|. \tag{22a}$$

Moreover, assume that the following conditions are satisfied:

$$h_k>\frac{1}{J}(C_k+C_{max}\frac{d}{p}) \ (k=0,1,..,6), \tag{23a}$$

where

$$C_0=\|(B_M^TPB_M)^{-1}B_M^TPA_M\| \tag{23b}$$

$$C_{max}=\max\{C_0,C_1,...,C_6\}. \tag{23c}$$

Then, we obtain Eq.(15). Here d can be regarded as the parameter which indicates the degree in continuity of input function.

Proof) Consider a positive definite function

$$L=v^T(B_M^TPB_M)^{-1}v.$$

After differentiation of L along the solution of Eqs.(8) and (22), and consideration of Remark 2 derives the conclusion of the Lemma. ■

From the above two Lemmas, we can reach the following Proposition.

[Proposition 1] Consider a manipulator $\langle S\rangle$ and the input function of Eqs.(8) and (22). Suppose that a suitable intermediate linear model $\langle S_M\rangle$ and Q are selected, and arbitrary ε and d are given. Moreover, suppose that a scalar m which appears in Lemma1 can be obtained , and the condition

$$h_k>\frac{1}{J}(C_k+C_{max}md\frac{1}{\varepsilon}) \ (0\leq k\leq 6) \tag{24}$$

are satisfied.

Then we obtain

$$\|y-y_M\|<\varepsilon \quad \text{for } t\geq 0 \tag{25}$$

Proof)It is clear from Lemmas. ■

The structure of the proposed controller is shown in Fig.1.

3.2)Stage2:Design of two degree of freedom controller $\langle C_L\rangle$

In the previous section, we have obtained the conclusion that the controlled variable y of the manipulator $\langle S\rangle$ follows the controlled variable y_M of the intermediate linear model $\langle S_M\rangle$ with the accuracy ε. Next, regarding $\langle S_M\rangle$ as a controlled object, let us construct a linear controller for $\langle S_M\rangle$ such that the resulting system may satisfy properties of Eqs.(5a) and (6b). To do this, we consider a two degree of freedom controller for $\langle C_L\rangle$. Considering the specifications that the desired transfer function is I and that the convergence characteristic of initial error is specified by pole assignment, the structure of such $\langle C_L\rangle$ can be depicted as Fig.2.

Consequently, the following proposition can be obtained.

[Proposition 2] Consider the intermediate linear model $\langle S_M\rangle$ and the input :

$$u_M=\ddot{r}_R+k_2\dot{r}_R+k_1r_R+b_2(\dot{r}_R-\dot{q}_M)+b_1(r_R-q_M). \tag{26}$$

Then, if

$$k_1+b_1=M^2 \tag{27a}$$

$$k_2+b_2=2M, \tag{27b}$$

we can obtain

$$y_M=y_R+L^{-1}(G_e(s)e_0). \tag{28}$$

Proof) Fig.2 indicates that the transfer function from r_R to y_M is I. If we set r_R=0, then the differential equation for the system can be written as

$$\ddot{e}+k_2\dot{e}+k_1e=-b_2\dot{e}-b_1e. \tag{29}$$

From Eq.(29) and the condition $\dot{e}_0$=0, we can get the transfer matrix from e_0 to $e(q_M)$ as follows:

$$G_e(s)=\frac{s+(k_2+b_2)}{s^2+(k_2+b_2)s+(k_1+b_1)}I \quad (30)$$

This completes the proof.

3.3) Design procedure

From 3.1 and 3.2, the following theorem can be obtained.

[Theorem] Consider the manipulator <S> , the following specification:
(a)Reference model: $G_R(s)$......Eq.(5a)
(b)Convergence characteristic of e_0 : $G_e(s)$......Eq.(5b)
(c)Accuracy: ε.................Eq.(6),
and the input

$$u=\hat{J}(q)(-k_1q-k_2\dot{q}+u_M) +\hat{D}(q)f(\dot{q})+\hat{H}\dot{q}+\hat{g}(q)+u_R \quad (31a)$$

$$u_R=-\frac{v}{\|v\|+d}\sum_{k=0}^{6} h_k\|a_k\|. \quad (31b)$$

$$v=B_M^T Pz \quad (31c)$$

$$u_M=\ddot{r}_R+k_2\dot{r}_R+k_1r_R +b_2(\dot{r}_R-\dot{q}_M)+b_1(r_R-q_M). \quad (31d)$$

Then, if the conditions (24) and (27) hold , we get the equation

$$\|y-y_d\|<\varepsilon \quad \text{for } t\geq 0. \quad (32)$$

Based on the theorem we propose the following design procedure:

(0) Specification
  (1) Selection of $G_R(s)$, $G_e(S)$, ε, d and Q.
(I) Design of robust compensator <C_N>
  (2) Selection of $\hat{J}(q)$, $\hat{D}(q)$, $\hat{H}$ and $\hat{g}(q)$.
  (3) Selection of k_1 and k_2 (intermediate linear model)
  (4) Calculation of m:Eq.(21)
  (5) Calculation of J and C_k: Eqs. (12) and (23)
  (6) Calculation of h_k:Eq.(24)
  (7) Implementation of u
(II)Design of two degree of freedom controller<C_L>
  (8) Selection of b_1 and b_2:Eq.(27)
  (9) Implementation of u_M

In Fig.3, we show the structure of the proposed Two-Stage-RMFCS.

4.Simulation

To illustrate the effectiveness of the controller proposed in this paper, a simple simulation study has been made. We have considered a trajectory tracking problem of the two-link manipulator illustrated in Fig.4. Here, let I_i, r_i, m_i and H_i be the moment of inertia of link i about the center of mass, the distance from joint i to the center of mass of link i, the mass of link i and the viscous friction coefficients of joint i, respectively. The real and nominal values of the above parameters are tabulated in Table 1.

Now, let us consider the specifications as follows:

(a) $G_R(s)=I$

(b) $G_e(s)=\frac{s+2M}{(s+M)^2}I \quad (M>0)$

(c) $\varepsilon=0.01$

and,

(d) $r_R=-\frac{1}{4}\sin(4\pi t)+t \quad (0\leq t\leq 0.5)$.

Next, set d=0.1, $k_1=1$, $k_2=2$, $q_1=50$, $q_2=1$, and using these informations, the Two-Stage-RMFCS has been constructed. The simulated responses of the resulting system are plotted. In Fig.5(a), the responses of y_{R1} (reference model) and y_1 (real system) are shown in the case of M=20,40. This figure shows that in spite of the existence of difference between real and nominal values of physical parameters, y_1 follows y_{R1} with the specified convergence characteristic of initial error. In Fig.5(b), the control input u_1 in the case of M=20 is compared with the ideal input. Where the ideal input implies that the input which makes y_1 follows y_{R1} exactly for any time. This figure indicates that the proposed control system can supply a reasonable input. We have also obtained the similar results with respect to 2nd-joint. From these results, we can conclude that the proposed control system works effectively.

5.Conclusion

This paper has proposed a design method of a robust model following control system for a manipulator. This method has the following properties,
(1) We can specify a reference model ($G_R(s)$), a convergence characteristic of initial error ($G_e(s)$) and the accuracy (ε) explicitly and independently.
(2) The control input becomes continuous.
(3) From the property (2), we convince that this control system can be used in the situation that there exists a small parasitical dynamics.
(4) Only the rough information on the controlled object is needed for design of the control system.

The further problem to be done is to investigate the robustness of this controller to the parasitical dynamics.

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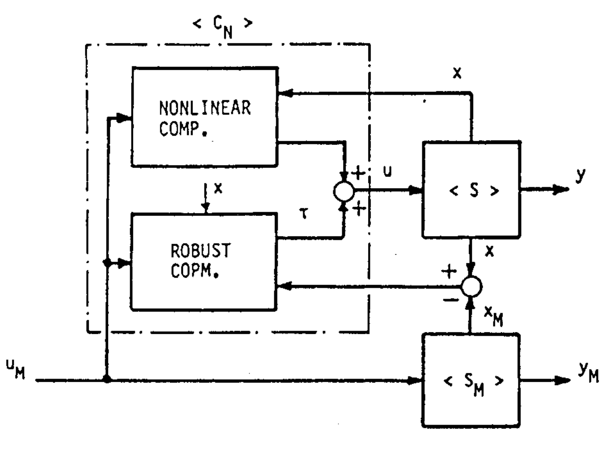

Fig.1 Stage1:Robust tracking controller

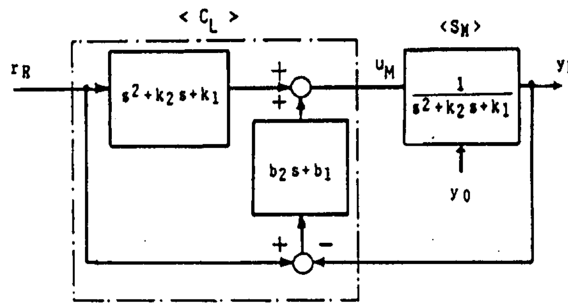

Fig.2 Stage2:Two-degree-of-freedom controller

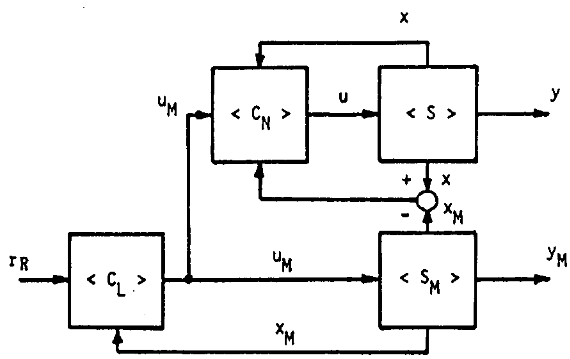

Fig.3 Two-Stage RMFCS

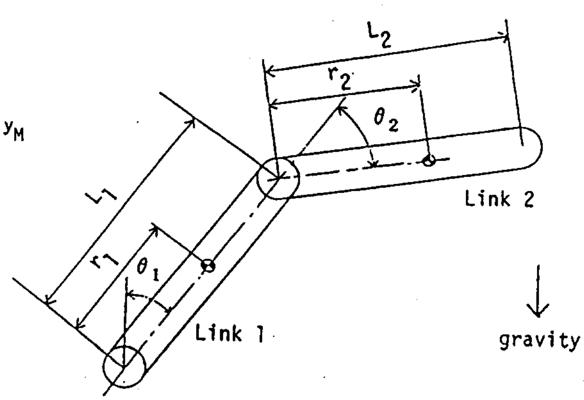

Fig.4 2-link manipulator

parameter	(a)real	(b)nominal
I_1	0.208(kgm^2)	0.250(kgm^2)
I_2	0.208(kgm^2)	0.250(kgm^2)
m_1	10(kg)	12(kg)
m_2	10(kg)	12(kg)
H_1	0.5(Nms)	0.6(Nms)
H_2	0.5(Nms)	0.6(Nms)
r_1	0.25(m)	0.25(m)
r_2	0.25(m)	0.25(m)
l_1	0.5(m)	0.5(m)
l_2	0.5(m)	0.5(m)

Table 1 Parameters

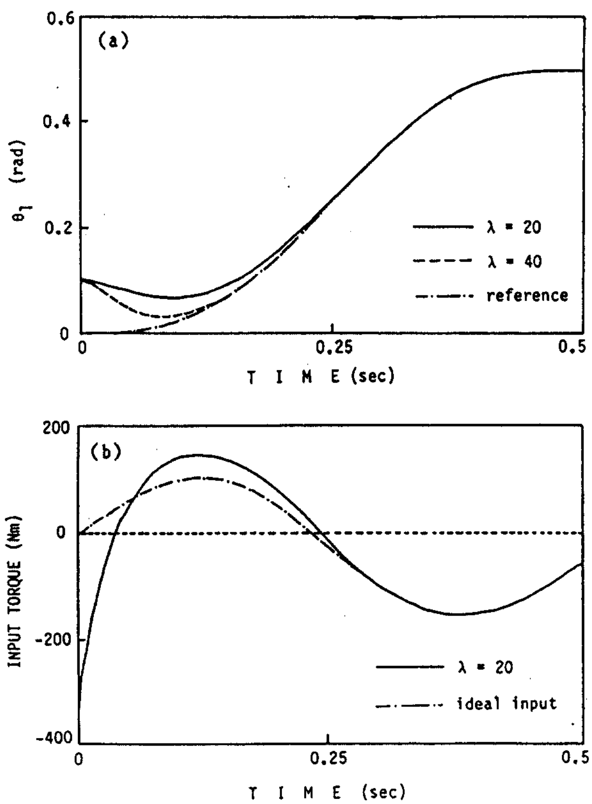

Fig.5 Simulation results