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Analysis of Dynamic Performance of Redundant Manipulators Using the Concept of Aspects

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Abstract

For kinematically redundant manipulators, conventional dynamic control methods of local torque optimization showed the instability which resulted in physically unachievable torque requirements. In order to guarantee stability of the null space vector method which resolves redundancy at the acceleration level, Maciejewski [1] analyzed the kinetic behavior of homogeneous solution component and proposed the condition to identify regions of stability and instability for this method. In this paper, a modified null space vector method is first presented based on the Maciejewski's condition which is a function of a manipulator's configuration. Secondly, a new control method which is based on the concept of *aspects* is proposed.

It was shown by computer simulations that the modified null space vector method and the proposed method have a common property that a preferred aspect is preserved during the execution of a task. It was also illustrated that both methods demonstrate a drastic reduction of torque loadings at the joints in the tracking motion of a long trajectory when compared with the null space vector method, and thus guarantee the stability of joint torque.

1. Introduction

Kinematically redundant manipulators have more joint degrees-of-freedoms (DOF) than are required to complete a specified task. The majority of researches on utilizing redundancy have been focused on the resolution of redundancy at the kinematic level. The kinematics of manipulators is presented in the differential form by

$$\dot{x} = J(\theta)\dot{\theta} \tag{1}$$

where $\dot{x}$ is an m-dimensional vector specifying the end-effector velocity, $\dot{\theta}$ is an n-dimensional vector denoting the joint velocity, and J is the m by n manipulator Jacobian matrix.

For redundant manipulators $m < n$, the general solution to Eq. (1) is typically given by the resolved motion method [2] and represented in the form

$$\dot{\theta} = J^{+}(\theta)\dot{x} \pm \kappa \, \{I - J^{+}(\theta)J(\theta)\} \nabla H(\theta) \tag{2}$$

where J^{+} denotes the pseudoinverse or Moore-Penrose inverse of J [3], $(I - J^{+}J)$ is a projection operator onto the null space of J, and $\nabla H(\theta)$ is the gradient vector of a performance function $H(\theta)$. Especially, the signs $+$ and $-$ in the above equation are related to the maximization and minimization of $H(\theta)$, respectively. The positive scalar constant κ in Eq. (2) is based on the hardware limits of joint velocities. The second term in this equation is the homogeneous solution to Eq. (1) since it results in no end-effector velocity. The homogeneous solution is frequently used to optimize a performance function $H(\theta)$ under the constraint of specified end-effector velocity by choosing $H(\theta)$ to be some function of θ. Some of the performance functions that have been applied for secondary criteria such as singularity avoidance, higher flexibility, and obstacle avoidance, include JRA (Joint Range Availability) [2,4], manipulability measure [5], condition number [6], compatibility index [7], minor measure [8], and artificial potential function [9]. The homogeneous solution can also be used to optimize secondary criteria defined in Cartesian space either to impose a priority to the manipulator variables [10] or to avoid obstacles [16]. Alternative formulations for instantaneously optimizing a secondary criterion have also been presented by Baillieul [12] and Chang [13], which seek exact locally optimal joint configurations at the joint position level rather than at the joint velocity level.

In all of the above techniques, the specified end-effector trajectory is the primary criterion. Unfortunately, the addition of a secondary criterion can significantly affect the primary criterion of trajectory tracking even though the secondary criterion is mapped mathematically into the null space of J. This effect is a result of the physical limitations of the actuators which prevent instantaneous achievement of the secondary criterion. Klein and Chirco [14] illustrated that the dynamic performance of a redundant manipulator showed significant end-effector tracking errors when the scalar constant κ in a homogeneous solution, i.e., the second term of Eq. (2), which determines how closely the secondary criterion will be tracked, becomes large.

A more dramatic difficulty with using homogeneous solutions is the instability in the null space vector method [15] where redundancy is resolved at the acceleration level to instantaneously minimize joint torque. In this case, the joint acceleration is related to the end-effector acceleration by differentiating Eq. (1) to obtain

$$\ddot{x} = J\ddot{\theta} + \dot{J}\dot{\theta} \tag{3}$$

where once again the general solution for $\ddot{\theta}$ is expressed in the form

$$\ddot{\theta} = J^{+}(\ddot{x} - \dot{J}\dot{\theta}) + (I - J^{+}J)\ddot{\phi} \tag{4}$$

where $\ddot{\phi} \in \Re^n$ is an arbitrary vector. In actual manipulator control, given a desired trajectory, $x_d(\cdot)$., the command joint acceleration $\ddot{\theta}_d$ is represented by a feedback control scheme as follows:

$$\ddot{\theta}_d = J^{+}(\ddot{x}_d + K_v\dot{e} + K_p e - \dot{J}\dot{\theta}) + (I - J^{+}J)\ddot{\phi} \tag{5}$$

where $e \triangleq x_d - x$ is the tracking error, K_v and K_p are constant velocity and position feedback gain matrices. The command torque τ of model-based control is in general generated by the measured values of joint angle and velocity vectors, namely $\boldsymbol{\theta}$ and $\dot{\boldsymbol{\theta}}$. That is,

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\theta})\,\ddot{\boldsymbol{\theta}}_d + \boldsymbol{N}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \tag{6}$$

where $\boldsymbol{M}(\boldsymbol{\theta}) \in \Re^{n\times n}$ is a symmetric, positive definite inertia matrix, and $\boldsymbol{N}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}) \in \Re^{n}$ is a vector containing terms such as Coriolis, centripetal, and gravitational torques. If $\tilde{\boldsymbol{\tau}}$ is used to denote the torque due to the first term of Eq. (5), i.e., the pseudoinverse solution, then $\tilde{\boldsymbol{\tau}}$ can be obtained as:

$$\tilde{\boldsymbol{\tau}} = \boldsymbol{M}\,\boldsymbol{J}^{+}(\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_v\,\dot{\boldsymbol{e}} + \boldsymbol{K}_p\,\boldsymbol{e} - \dot{\boldsymbol{J}}\,\dot{\boldsymbol{\theta}}) + \boldsymbol{N}. \tag{7}$$

The optimization problem of locally minimizing joint torque is recast as finding the vector $\ddot{\boldsymbol{\phi}}$ to minimize

$$\left\|(\boldsymbol{M} - \boldsymbol{J}^{+}\boldsymbol{J})\ddot{\boldsymbol{\phi}} + \tilde{\boldsymbol{\tau}}\right\|. \tag{8}$$

This is a straight forward least squares problem which can be solved by the pseudoinverse [3] with the solution given by

$$\ddot{\boldsymbol{\phi}} = -\left[\boldsymbol{M}\,(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\right]^{+}\tilde{\boldsymbol{\tau}}. \tag{9}$$

Thus the command joint acceleration $\ddot{\boldsymbol{\theta}}_d$ can be obtained in the form

$$\begin{aligned}\ddot{\boldsymbol{\theta}}_d &= \boldsymbol{J}^{+}(\ddot{\boldsymbol{x}}_d + \boldsymbol{K}_v\,\dot{\boldsymbol{e}} + \boldsymbol{K}_p\,\boldsymbol{e} - \dot{\boldsymbol{J}}\,\dot{\boldsymbol{\theta}}) \\ &\quad - \left[\boldsymbol{M}\,(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\right]^{+}\tilde{\boldsymbol{\tau}}. \end{aligned} \tag{10}$$

The above equation was simplified with use of the identity $\boldsymbol{B}[\boldsymbol{C}\boldsymbol{B}]^{+} = [\boldsymbol{C}\boldsymbol{B}]^{+}$ because $\boldsymbol{B} = \boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J}$ is hermitian and idempotent [16]. The value of the minimum torque can be obtained by substituting Eq. (10) into Eq. (6) and then using Eq. (7) to obtain

$$\boldsymbol{\tau} = \tilde{\boldsymbol{\tau}} - \boldsymbol{M}\left[\boldsymbol{M}\,(\boldsymbol{I} - \boldsymbol{J}^{+}\boldsymbol{J})\right]^{+}\tilde{\boldsymbol{\tau}}. \tag{11}$$

Unfortunately, because this is only a local minimization technique, it has been shown in [15] that the command joint acceleration given by Eq. (10) can induce large joint torques that may require physically unrealizable joint torques in order to maintain the desired end-effector trajectory.

In order to place realistic limitations on the use of redundancy, Maciejewski [1] analyzed the kinetic effects of a homogeneous solution and presented the condition for identifying the instability of the null space vector method. He also showed that this condition is only a function of a manipulator's configuration and thus can be used to determine desirable regions of operation. However, he did not present a real-time dynamic control law which may be guided by his condition.

This paper is organized as follows. In Section 2, the modified null space vector method using the Maciejewski's condition, which is a new real-time dynamic control law, is presented. In Section 3, our own method, i.e., the $\boldsymbol{J}$-Minor based Dynamic Control, which is hereinafter abbreviated "JMDC", is proposed. Comparison of JMDC with the null space vector method and the modified one is performed through computer simulation in Section 4. Especially this computer simulation illustrates an important point that the goal pursued by Maciejewski's condition is in common with the basic idea of JMDC in order to guarantee stability of joint torque. Concluding remarks are made in Section 5.

2. Modified Null Space Vector Method

The null space vector method proposed by Hollerbach and Suh [15] leads to stability problem, even though locally this method reduces actuator torques. Maciejewski [1] presented that the dependency of the direction of the homogeneous (or null space) joint acceleration on the homogeneous joint velocity (and vice versa) can be used to identify the regions of stability and instability for the null space vector method. Mathematically, this condition which we will call "Maciejewski's condition" in this paper can be expressed by

$$\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H \geq 0, \tag{12}$$

where $\dot{\boldsymbol{\theta}}_H$ and $\ddot{\boldsymbol{\theta}}_H$ are homogeneous joint velocity and acceleration vectors, respectively. When Eq. (12) is true, the homogeneous acceleration term, i.e., the second term in Eq. (10), will increase the magnitude of the homogeneous joint velocity and subsequently increase the torque requirements. This, in effect, amounts to a positive feedback system and results in the instability of local torque optimization noted in [15]. In order to guarantee global stability when using the null space vector method, the homogeneous joint acceleration must not be applied when Eq. (12) holds.

As another contribution of Maciejewski, he proved that the condition given by Eq. (12) is solely a function of a manipulator's configuration. It is thus possible to determine regions of operation for which the null space vector method is inherently stable or unstable. However, Maciejewski did not present a real-time control law to overcome instability of the null space vector method using this condition, while he focused on proving that the condition is only a function of $\boldsymbol{\theta}$.

In this paper, we propose the modified null space vector method guided by Maciejewski's condition. In this method. the command torque is generated according to this condition as follows:

$$\boldsymbol{\tau} = \tilde{\boldsymbol{\tau}} \qquad \text{if } \dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H \geq 0 \tag{13}$$

$$\boldsymbol{\tau} = \tilde{\boldsymbol{\tau}} - \boldsymbol{M}\left[\boldsymbol{M}\,(\boldsymbol{I} - \boldsymbol{J}^{+}\,\boldsymbol{J})\right]^{+}\tilde{\boldsymbol{\tau}} \text{ if } \dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H < 0 \tag{14}$$

where

$$\dot{\boldsymbol{\theta}}_H = (\boldsymbol{I} - \boldsymbol{J}^{+}\,\boldsymbol{J})\,\dot{\boldsymbol{\theta}}, \tag{15}$$

$$\ddot{\boldsymbol{\theta}}_H = -\left[\boldsymbol{M}\,(\boldsymbol{I} - \boldsymbol{J}^{+}\,\boldsymbol{J})\right]^{+}\tilde{\boldsymbol{\tau}}, \tag{16}$$

and $\tilde{\boldsymbol{\tau}}$ is given by Eq. (7). In Eq. (15), the vector $\dot{\boldsymbol{\theta}}$ is a value of joint velocity vector which can be measured from a redundant robot system. The algorithm of the modified null space vector method is illustrated in detail through the flow chart shown in Fig. 1. Consequently, Maciejewski's condition can be incorporated in actively avoiding the instability region of operation by using the modified null space vector method rather than passively identifying the regions of stability and instability for the null space vector method. Besides, the original formulation of Maciejewski's condition includes the radius of curvature of a homogeneous solution space, which hinders this condition from being implemented in on-line. On the contrary, Maciejewski's condition used in the modified null space vector method does not include this radius of curvature as shown in Eqs. (15) and (16), which enables the on-line application of Maciejewski's condition to the null space vector method.

3. $\boldsymbol{J}$-Minor based Dynamic Control

The $\boldsymbol{J}$-Minor based Dynamic Control (JMDC) is based on the concept of *aspects* which are functions of the manipulator

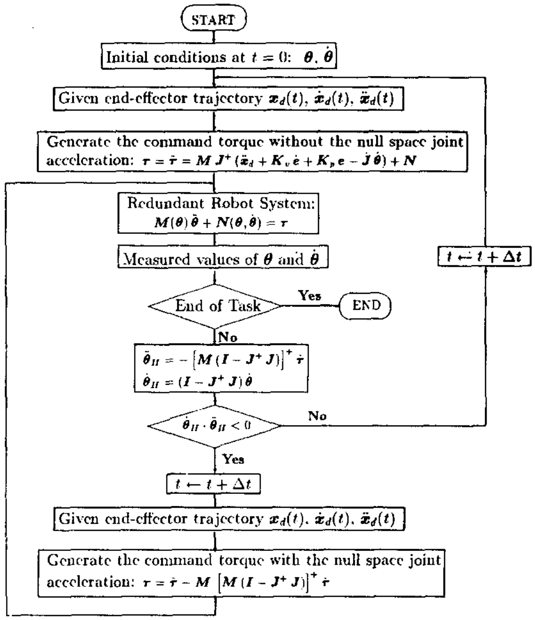


Fig. 1 Flow chart of the modified null space vector method.

configuration. The aspects have close relation to the *full row rank minors* of the manipulator Jacobian matrix. Borrel and Liégeois [17] found that there exist various classes of configurations called "aspects". The admissible domain in the joint space is divided into ${}_nC_m$ aspects for m task variables and n joint variables. One of the separating surfaces between aspects is the locus of joint coordinates corresponding to one of the m-th order minors of the manipulator Jacobian matrix $\boldsymbol{J} \in \Re^{m\times n}$ equal to zero. Mathematically, an aspect is defined in [17].

Based on the definition of the aspect, we can suggest the hypothesis that *the necessary condition to guarantee the stability of joint torque is to preserve the aspect to which an initial configuration at the beginning of a task belongs.* Thus the JMDC method for the dynamic control of redundant manipulators aims at preserving this preferred aspect by using the performance function which has a direct control over each full row rank minor, and generating the command joint velocity $\dot{\boldsymbol{\theta}}_d$ from the resolved motion method given by Eq. (2) as follows:

$$\dot{\boldsymbol{\theta}}_d^{i+1} = \boldsymbol{J}^+(\boldsymbol{\theta}^i)\dot{\boldsymbol{x}}_d^i + \kappa\left\{\boldsymbol{I} - \boldsymbol{J}^+(\boldsymbol{\theta}^i)\boldsymbol{J}(\boldsymbol{\theta}^i)\right\}\nabla H(\boldsymbol{\theta}^i) \quad (17)$$

where i denotes the current state at time $t = i \cdot \Delta t$ (Δt: sampling time); $\boldsymbol{\theta}^i$ and $\dot{\boldsymbol{\theta}}^i$ are the measured values of joint angle and velocity at time t, respectively. The positive scalar constant κ in the above equation is based on the hardware limits of joint velocities. Especially, the performance function $H(\boldsymbol{\theta})$ in the second term of Eq. (17) plays an important role in the JMDC method because the aspect can be preserved for some specific function.

For the kinematic control of redundant manipulators, Chang [8] proposed the minor measure as the product of full row rank minors of the Jacobian matrix, which is defined as:

$$H = \left|\prod_{i=1}^{p}\delta_i\right|^{1/p} \quad (18)$$

where δ_i's for $i = 1, 2, \ldots, p$ are the minors of rank m of the Jacobian matrix $\boldsymbol{J} \in \Re^{m\times n}$; with $p = {}_nC_m$, the combination of m taken out of n. It is noted here that each minor is maximized and is kept at a nonzero value by maximizing H. Thus the aspect can be preserved by using this minor measure as the performance function in Eq. (17).

The command joint angle (or displacement for a prismatic joint) $\boldsymbol{\theta}_d$ is calculated by numerical integration:

$$\boldsymbol{\theta}_d^{i+1} = \boldsymbol{\theta}_d^i + \dot{\boldsymbol{\theta}}_d^i \cdot \Delta t \quad (19)$$

where $\dot{\boldsymbol{\theta}}_d^0$ is assumed to be equal to $\dot{\boldsymbol{\theta}}^0$ for $i = 0$. The command joint acceleration $\ddot{\boldsymbol{\theta}}_d$ is generated by numerical differentiation:

$$\ddot{\boldsymbol{\theta}}_d^{i+1} = \frac{\dot{\boldsymbol{\theta}}_d^{i+1} - \dot{\boldsymbol{\theta}}_d^i}{\Delta t} \cong \frac{\dot{\boldsymbol{\theta}}_d^i - \dot{\boldsymbol{\theta}}_d^{i-1}}{\Delta t} \quad (20)$$

where $(\dot{\boldsymbol{\theta}}_d^{i+1} - \dot{\boldsymbol{\theta}}_d^i)$ is assumed to be approximately equal to $(\dot{\boldsymbol{\theta}}_d^i - \dot{\boldsymbol{\theta}}_d^{i-1})$, and $\ddot{\boldsymbol{\theta}}_d^0$ is assumed to be zero. The above numerical differentiation can generate relatively large values of the command joint accelerations, which may in turn induce large joint torques, especially at the beginning stage of motion when $\dot{\boldsymbol{\theta}}_d^0 = 0$ and a large value of $\dot{\boldsymbol{\theta}}_d^1$ due to the improper choice of κ in Eq. (17) are assigned to Eq. (20). This difficulty can be fixed by selecting a small value of κ which is enough to induce smooth self-motion. It is noted that the command joint acceleration is not influenced by any noise signal because the measured joint velocities are not involved in Eq. (20).

By using the computed-torque control law [18] which is well known for non-redundant manipulators, the command torque can be easily generated in the form

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{\theta}^i)\left\{\ddot{\boldsymbol{\theta}}_d^i + \boldsymbol{K}_v'(\dot{\boldsymbol{\theta}}_d^i - \dot{\boldsymbol{\theta}}^i) + \boldsymbol{K}_p'(\boldsymbol{\theta}_d^i - \boldsymbol{\theta}^i)\right\} + \boldsymbol{N}(\boldsymbol{\theta}^i, \dot{\boldsymbol{\theta}}^i), \quad (21)$$

where $\boldsymbol{K}_v' \in \Re^{n\times n}$ and $\boldsymbol{K}_p' \in \Re^{n\times n}$ are position and velocity feedback gain matrices, respectively. It is worthwhile noticing that the JMDC method does not have any torque optimization scheme explicitly, but focuses on suppressing the switching of aspects at the kinematic level.

4. Computer Simulation

In this section, the characteristics of the modified null space vector method and the hypothesis presented in the previous section will be verified by the computer simulations which are performed for the planar 3-DOF manipulator. The links are all identical and are modeled as thin uniform rods with lengths of 1 m and masses of 10 kg. The joints are labeled 1,2,3 from the base. The simulated movement of the end-effector is a straight Cartesian trajectory starting and ending with zero velocity, with constant acceleration over the first and the last half of the movement, respectively. To generate a long trajectory of the end-effector, the command end-effector acceleration is given by $\ddot{x}_d = [1.5 \quad 1.0]^T$ m/s^2 and $\ddot{\boldsymbol{x}}_d = [-1.5 \quad -1.0]^T$ m/s^2 for the first and the last half of the trajectory, respectively, and the total duration time T is chosen as 2 s. For the convenience, this trajectory is termed as trajectory A.

In order to examine the relationship between the stability of joint torque and the aspects, the simulation for the null space vector method was first performed in which the planar 3-DOF manipulator was commanded to track trajectory A, given an initial configuration $\boldsymbol{\theta} = [-50° \quad 140° \quad -140°]^T$. The simulation results of trajectory A for the null space vector method are shown in Fig. 2(a)-(d). As can be seen in Fig. 2(a), this method induces uniform configurations for some short excursion of the end-effector from the beginning of trajectory A. However, the configurations of the manipulator become non-uniform after the mid-course of the movement although the end-effector itself follows the command end-effector trajectory well. Fig. 2(b) illustrates the profile of joint torques computed by the method. In this figure, physically unachievable joint torques which may exceed hardware torque limits have occurred at about $t = 1.25$ and $t = 1.8$ s. Thus the null space vector method has unrealistic characteristics of joint torques.

To quantitatively examine the instability of joint torques mentioned above, we observed how the full row rank minors behave especially when extremely large joint torques are required. For the planar 3-DOF manipulators with revolute joints, the minors are given by

$$\delta_1 = \det[J^1 \quad J^2] = \ell_1\ell_2 S_2 + \ell_1\ell_3 S_{23} \tag{22}$$

$$\delta_2 = \det[J^2 \quad J^3] = \ell_2\ell_3 S_3 \tag{23}$$

$$\delta_3 = \det[J^3 \quad J^1] = -\ell_2\ell_3 S_3 - \ell_1\ell_3 S_{23} \tag{24}$$

where J^i is the i-th column vector of the matrix $\boldsymbol{J}$: $S_2 = \sin\theta_2$, $S_3 = \sin\theta_3$, and $S_{23} = \sin(\theta_2+\theta_3)$. Figs. 2(c) illustrate the profiles of these minors for the null space vector method. Observing Figs. 2(b), and 2(c), we surprisingly found that that the peak joint torques, specifically torques of joint 1 and 2, occur almost exactly when the minor δ_2 becomes zero.

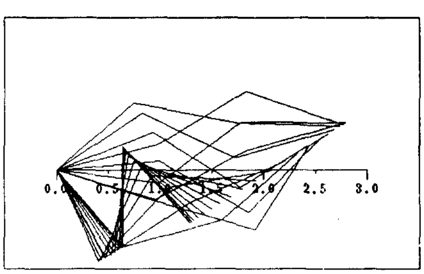
The relation between minors and joint torques can be explained in more detail as follows. Chang [8] proved that the manipulability measure [5] can be rewritten in terms of full row rank minors:

$$w = \sqrt{\det(\boldsymbol{J}\,\boldsymbol{J}^T)} = \left(\sum_{i=1}^{p} \delta_i^2\right)^{1/2} \tag{25}$$

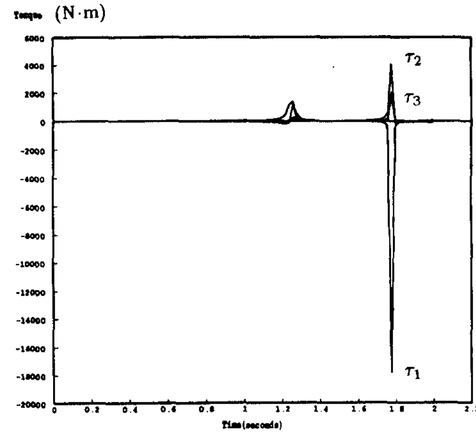
where δ_i's, $i = 1, 2, \ldots, p$, with $p = {}_nC_m$, are the minors of rank m of the matrix $\boldsymbol{J} \in \Re^{m\times n}$ with $m < n$. The pseudoinverse of $\boldsymbol{J}$ with full rank is given by

$$\boldsymbol{J}^+ = \boldsymbol{J}^T(\boldsymbol{J}\,\boldsymbol{J}^T)^{-1}. \tag{26}$$

Reviewing Eqs. (25) and (26), we can state that the smaller



(a)



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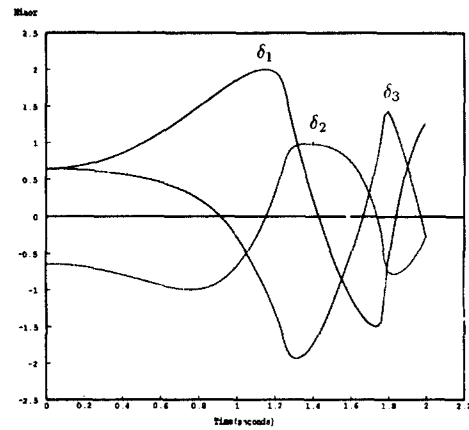

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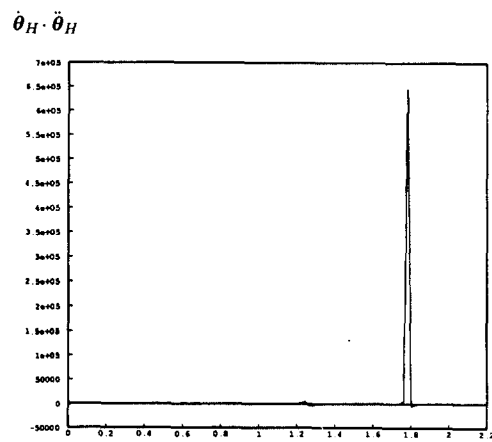


(d)

Fig. 2 Simulation results of trajectory A for the null space vector method: (a) arm motion; (b) torque profiles; (c) minor profiles; (d) plot of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$.

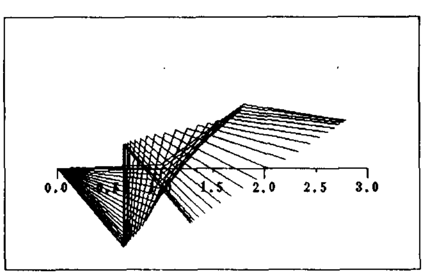
values of the minor δ_i induce the larger values of the elements of $\boldsymbol{J}^+$. The minimum norm acceleration, i.e., the first term of Eq. (10), also becomes large due to the pseudoinverse $\boldsymbol{J}^+$, and then results in the large values of $\tilde{\tau}$. Thus the command torque for the null space vector method becomes large owing to $\tilde{\tau}$. Therefore it can be concluded that the small values of full row rank minors result in a remarkable increase of the torque required to track a desired end-effector trajectory.

To illustrate whether the modified null space vector method can guarantee the global stability of joint torque or not, the second computer simulation was performed for trajectory A. As shown in Fig. 3 (a), the arm motion for this method is uniform and stays within one kind of aspects. This is verified in Fig. 3 (c) where the aspect to which the initial configuration belongs is preserved all the time because any minor among δ_1, δ_2, and δ_3 does not become zero. In fact, the method does not have any direct control over the full row rank minors. But, Maciejewski's condition results in preserving an aspect when used in the modified null space vector method. As anticipated for this arm motion, the joint torques are drastically held down at low values in Fig. 3 (b). The profile of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$ is presented in Fig. 3 (d) where the torque due to the homogeneous joint acceleration $\ddot{\boldsymbol{\theta}}_H$, i.e., the second term of Eq. (10), was applied only when $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H < 0$, i.e., during the time interval from $t = 1.0$ to $t = 1.65$ s and from $t = 1.8$ to $t = 2.0$ s. In the meanwhile, the profile of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$ for the null space vector method is shown in Fig. 2 (d) where the torque due to the homogeneous joint acceleration was always applied during the execution of the task. Especially, the peak value of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$ in Fig. 2 (d), i.e., about 6.5×10^5, is observed at the same instant as the peak joint torques occur in Fig. 2 (b). Maciejewski [1] stated that a configuration is unstable when Eq. (12) is true. As shown in Figs. 3 (b) and 3 (d), this is not true for the modified null space vector method because it requires sufficiently low values of torques and thus guarantees stability even when Eq. (12) holds. Therefore, it should be pointed out that unstable configurations depend on not whether the sign of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$ is positive or not but how large the positive peak value of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$ is.

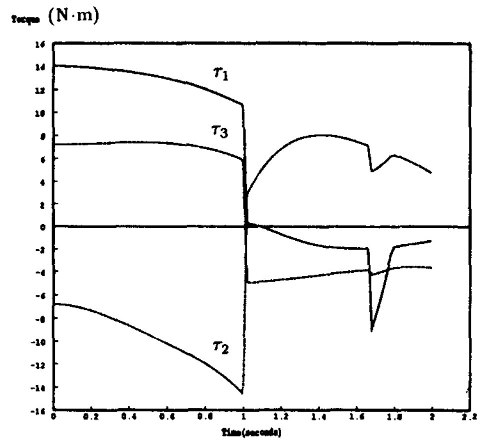
In the simulation of the JMDC method, the minor measure [8] was used as a performance function. This measure is given for the planar 3-DOF manipulator by

$$
\begin{aligned}
H &= \left\{\det[J^1 \;\; J^2] \cdot \det[J^2 \;\; J^3] \cdot \det[J^3 \;\; J^1]\right\}^{1/3} \\
&= \{(\ell_1\ell_2 S_2 + \ell_1\ell_3 S_{23})\,\ell_2\ell_3 S_3\,(-\ell_2\ell_3 S_3 - \ell_1\ell_3 S_{23})\}^{1/3},
\end{aligned}
$$

where J^i is the i-th column vector of the matrix $\boldsymbol{J}$. The constant κ in Eq. (17) was chosen as 0.001 by considering smooth self-motion at the beginning stage of motion. The simulation



(a)



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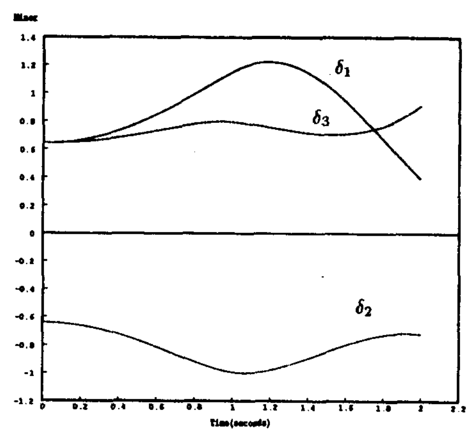

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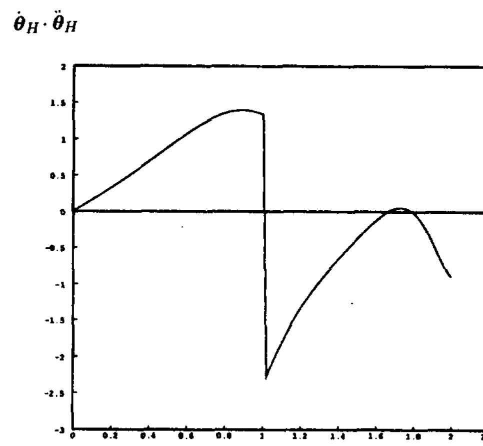
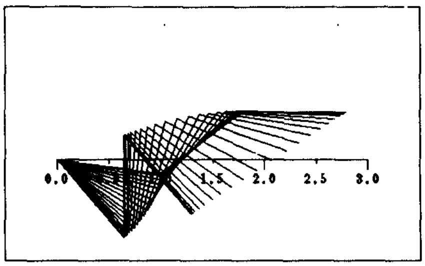


(d)

Fig. 3 Simulation results of trajectory A for the modified null space vector method: (a) arm motion; (b) torque profiles; (c) minor profiles; (d) plot of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$.

results for trajectory A are shown in Fig. 4(a)-(d). The uniform arm motion illustrated in Fig. 4(a) implies that the configurations of the manipulator belong to only one aspect. This is verified in Fig. 4(c) which demonstrates no switching of aspects. As expected, the joint torques shown in Fig. 4(b) are drastically reduced when compared with the null space vector method. Fig. 4(d) illustrates that the sign of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$ is almost negative except the transition time from acceleration to deceleration, i.e., $t = 1$ s. Moreover, the positive peak value of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$ is small. Thus the configurations shown in Fig. 4(a) are inherently stable. Especially, it is worth while noticing that the arm motion, torque profiles, and minor pro-



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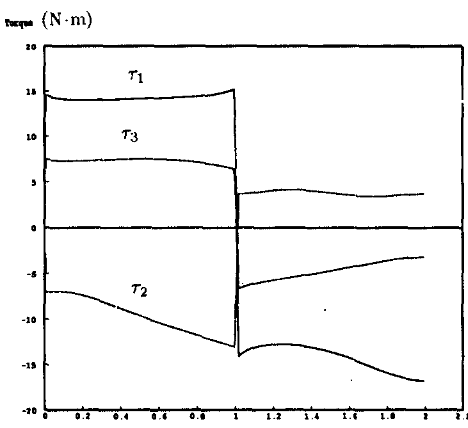

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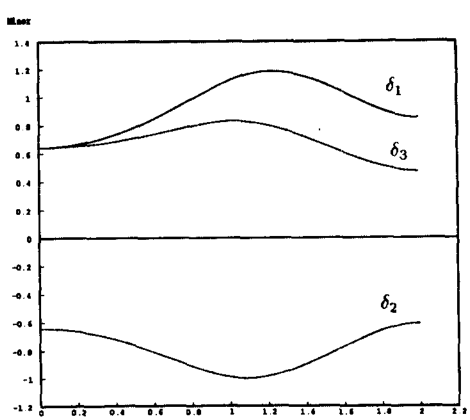

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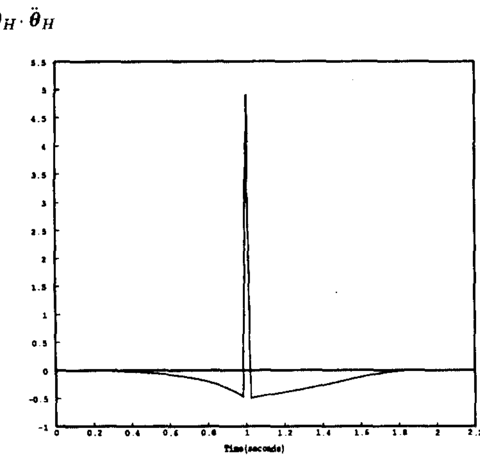

(d)

Fig. 4 Simulation results of trajectory A for the JMDC method: (a) arm motion; (b) torque profiles; (c) minor profiles; (d) plot of $\dot{\boldsymbol{\theta}}_H \cdot \ddot{\boldsymbol{\theta}}_H$.

files for the JMDC method are similar to those for the modified null space vector method when one compares Fig. 3(a)-(c) with Fig. 4(a)-(c). This implies that the modified null space vector method in which the torque due to the homogeneous joint acceleration is applied according to Maciejewski's condition is related to preserving an aspect and thus has the similar effect on joint torques as the JMDC method. However, a heavy computational load is demanded for this method since several pseudoinverse operations are required in computing the terms $\boldsymbol{J}^+$ and $[\boldsymbol{M}(\boldsymbol{I} - \boldsymbol{J}^+\boldsymbol{J})]^+$. On the contrary, the JMDC method requires only one pseudoinverse operation, namely $\boldsymbol{J}^+$, so it is more suitable for real-time control than the modified null space vector method.

5. Conclusion

This paper presented two real-time dynamic control laws to guarantee the global stability of joint torque for kinematically redundant manipulators: 1) modified null space vector method and 2) $\boldsymbol{J}$-minor based dynamic control method. The first method is in a modified form of the local torque optimization algorithm proposed by Hollerbach and Suh, and is guided by the Maciejewski's condition which is a function of a manipulator's configuration. In this method, Maciejewski's condition is incorporated in actively avoiding the instability region of operation rather than passively identifying the regions of stability and instability. The $\boldsymbol{J}$-Minor based Dynamic Control (JMDC) method is based on the concept of aspects which are also functions of a manipulator's configuration. The JMDC method starts with the basic understanding of full row rank minors of the manipulator Jacobian matrix. In our investigation, we found that these minors which cause the switching of aspects have close relation with the dynamics of redundant manipulators, which has not been treated carefully. The key idea of JMDC method is at the kinematic level to preserve the aspect to which an initial configuration belongs, and thereby at the dynamic level to guarantee the stability of joint torque.

It was shown through computer simulations that the modified null space vector method in which the torque due to a homogeneous joint acceleration is applied according to Maciejewski's condition preserves an aspect, while it does not include any direct control over full row rank minors. Thus it can be concluded that Maciejewski's condition ultimately aims at preserving an aspect. It was also pointed out that unstable configurations of a manipulator depend on not whether the sign of $\dot{\theta}_H \cdot \ddot{\theta}_H$ is positive or negative but how large the positive peak value of $\dot{\theta}_H \cdot \ddot{\theta}_H$ is, while Maciejewski stated that a configuration is always unstable when $\dot{\theta}_H \cdot \ddot{\theta}_H > 0$.

It was proved by computer simulations that preserving an aspect is a necessary condition to guarantee the stability of joint torque in the global sense. Since the JMDC method directly preserves an aspect by using the minor measure, it is simpler in algorithm and more efficient in real-time control than the modified null space vector method which has the similar effect on joint torques as the JMDC method. It was demonstrated that both JMDC and modified null space vector methods generate reasonably low values of joint torques when compared with the null space vector method.

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