'91 KACC 1991. 10. 22~24

Position Control for Multifingered Robot Hand

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ABSTRACT

The problem of fine manipulation is considered in this paper. By fine manipulation, we mean the positioning of the object relative to the palm as opposed to gross manipulation by the arm. The compliance in the fingers and the object is modeled by linear springs. It is shown that the motion of the fingers and object can be predicted by minimizing a quadratic objective function. A method for simulating position control algorithms is developed.

1. Introduction

With the current state of the art in robotics, position control algorithms are simpler and more robust than force control algorithms [4-7].

The first studies of grasp stability were reported by [1]. They defined a grasp to be stable, if the object returns to the original position after a perturbation of its position with respect to the hand and proposed to grasp an object in such a way that the potential energy stored in the elastic fingers should be minimized.

[3] have considered stability to mean a situation in which the object does not slip from the hand. They applied numerical optimization techniques to select a three-point grasp for a hand with three single-link fingers.

The redundancy or indeterminacy in the finger-object system can be resolved if fingers are modeled as compliant members. [8] developed a simple geometric relation between the stiffness of the grasp and the spatial configuration of the virtual springs at the contacts. He formulated the potential function of the grasp as the sum of the potential functions from all linear and angular springs. A grasp is in equilibrium if and only if the gradient of its potential function is zero.

The variable compliance method was proposed by [6] to resolve the static indeterminacy by using the principle of geometric compatibility, which is extensively used for passive structures encountered in solid mechanics problems.

The fingers and the object were modeled as elastic bodies and the region of contact as a deformable surface patch [10]. The nature of the constraints arising out of conditions for compatibility and static equilibrium motivated the formulation of the model as a nonlinear constrained minimization problem. He minimized the total potential energy of the system subject to the nonlinear, equality and inequality constraints on the system, using a successive quadratic programming method.

In this paper, the problem then consists of analyzing the effect of finger displacements on (a) the contact forces at the interface (b) the position of the object. The objective is to explore if controlled finger displacements will result in finger forces that will maintain the object in equilibrium. we model the contact compliance through linear elastic springs. we explicitly incorporate unilateral constraints and frictional constraints in this model. In Sections 2 and 3, the details of the model and the formulation are presented. Section 4 deals with a minimum principle which can be used to simulate quasi-static systems in which friction work is zero. The application of this principle and examples are included in Section 5.

2. Modeling of the Finger-Object Contact

The compliance in the finger and the object is very difficult to model. Even if principles of linear elasticity are invoked, it is quite difficult to obtain closed form solutions for the equivalent stiffness or the contact stresses. We will assume that the compliance on each surface can be modeled by a normal and a tangential spring. Further, we assume that the displacements at the contact as well as the gross motion of the object are small. Our analysis is quasi-static. The contacts

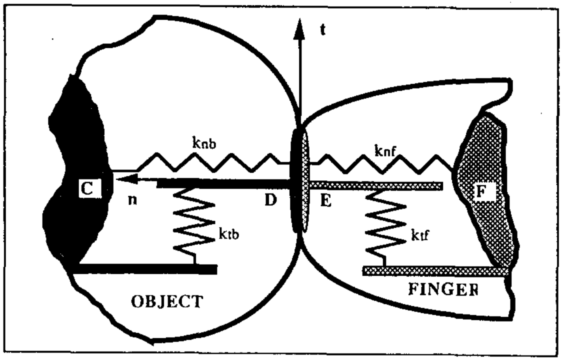


Figure 2.1: Elastic Finger and Object in Two Dimensions

between the fingers and the object are modeled as point contact with friction.

In Figure 2.1 C is assumed to be a point on a solid rigid core which is sufficiently far away from the contact. In other words, it is unaffected by the contact stresses at point D. Similarly F lies on the solid part of the finger. D and E are contact points on the body and finger respectively. Thus the displacements at D and E represent the displacements at the surfaces of the body and the finger respectively. The displacements at C and F represent rigid body displacement of the body and finger respectively. **n** and **t** are unit normal and tangential vectors at the contact point as shown in the finger. k_{nb} and k_{tb} are the normal and tangential stiffness of the body and k_{nf} and k_{tf} are the normal and tangential stiffness of the finger. In principle, the finger stiffness can not only model the stiffness in the structure, and transmission and actuation system, but also the compliance that can be generated electronically. Therefore, this analysis is realistic for compliance control algorithms too. $\mathbf{u}_c$, $\mathbf{u}_d$, $\mathbf{u}_e$ and $\mathbf{u}_f$ are displacements at points C, D, E and E respectively. We restrict ourselves to planar geometries with the observation that the nature of the problem does not change in three dimensions.

The assumptions made in the analysis are reiterated:

1. The contact occurs at a point.
2. The friction is modeled as Coulomb friction.
3. A quasi-static analysis is adequate.
4. The displacement of the contact point is small.
5. A lumped stiffness model is valid.
6. The object and the fingers are planar.

The finger force **F**, which is the force at the interface exerted *on* the body, is proportional to the spring deformations:

$$\mathbf{F} = F_n\,\mathbf{n} + F_t\,\mathbf{t}$$

where

$$F_n = k_{nb}(\mathbf{u}_d-\mathbf{u}_c)\bullet\mathbf{n} = k_{nf}(\mathbf{u}_f-\mathbf{u}_e)\bullet\mathbf{n}$$

$$F_t = k_{tb}(\mathbf{u}_d-\mathbf{u}_c)\bullet\mathbf{t} = k_{tf}(\mathbf{u}_f-\mathbf{u}_e)\bullet\mathbf{t}$$

3. Constraint Condition

The Coulomb friction constraints require that the finger force vector lies within the friction cone for rolling contact and that the finger force vector lies on the boundary of the friction cone for sliding contact. Therefore

$$\mu^2 F_n^2 \geq F_t^2 \quad \text{(for rolling contact)}$$

$$-\frac{(\mathbf{u}_e-\mathbf{u}_d)\bullet\mathbf{t}}{\left\|(\mathbf{u}_e-\mathbf{u}_d)\bullet\mathbf{t}\right\|}\mu F_n = F_t \quad \text{(for sliding contact)}$$

where μ is the coefficient of friction at the point of contact.

The relative displacement at the contact, which is $\mathbf{u}_e$-$\mathbf{u}_d$, can be divided into normal and tangential components. That is, the relative normal displacement ($(\mathbf{u}_e-\mathbf{u}_d)\bullet\mathbf{n}$) and the relative tangential displacement ($(\mathbf{u}_e-\mathbf{u}_d)\bullet\mathbf{t}$). The former is constrained because the fingertip (E) cannot penetrate the boundary of the object (D):

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet\mathbf{n} \geq 0$$

If contact is sustained, the equality condition holds.

The tangential displacement is constrained depending on the nature of the contact:

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet\mathbf{t} = 0 \quad \text{(for rolling contact)}$$

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet\mathbf{t} \neq 0 \quad \text{(for sliding contact)}$$

All the above contact constraints can be summarized in the following manner:

A. Rolling Contact Case

$$F_n = k_{nb}(\mathbf{u}_d-\mathbf{u}_c)\bullet\mathbf{n} = k_{nf}(\mathbf{u}_f-\mathbf{u}_e)\bullet\mathbf{n} \quad (3.A.1)$$

$$F_t = k_{tb}(\mathbf{u}_d-\mathbf{u}_c)\bullet\mathbf{t} = k_{tf}(\mathbf{u}_f-\mathbf{u}_e)\bullet\mathbf{t} \quad (3.A.2)$$

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet\mathbf{t} = 0 \quad (3.A.3)$$

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet\mathbf{n} = 0 \quad (3.A.4)$$

$$\mu F_n \geq F_t \geq -\mu F_n \quad (3.A.5)$$

$$F_n \geq 0 \quad (3.A.6)$$

B. Sliding Contact Case

$$F_n = k_{nb}(\mathbf{u}_d-\mathbf{u}_c)\bullet\mathbf{n} = k_{nf}(\mathbf{u}_f-\mathbf{u}_e)\bullet\mathbf{n} \quad (3.B.1)$$

$$F_t = k_{tb}(\mathbf{u}_d-\mathbf{u}_c)\bullet \mathbf{t} = k_{tf}(\mathbf{u}_f-\mathbf{u}_e)\bullet \mathbf{t} \quad (3.B.2)$$

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet \mathbf{n} = 0 \quad (3.B.3)$$

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet \mathbf{t} \neq 0 \quad (3.B.4)$$

$$-\frac{(\mathbf{u}_e-\mathbf{u}_d)\bullet \mathbf{t}}{\|(\mathbf{u}_e-\mathbf{u}_d)\bullet \mathbf{t}\|}\mu F_n = F_t \quad (3.B.5)$$

$$F_n \geq 0 \quad (3.B.6)$$

A unified approach to the two rolling and sliding problems yield:

$$F_n = k_{nb}(\mathbf{u}_d-\mathbf{u}_c)\bullet \mathbf{n} = k_{nf}(\mathbf{u}_f-\mathbf{u}_e)\bullet \mathbf{n} \quad \text{(from 3.A.1 \& 3.B.1)}$$

$$F_t = k_{tb}(\mathbf{u}_d-\mathbf{u}_c)\bullet \mathbf{t} = k_{tf}(\mathbf{u}_f-\mathbf{u}_e)\bullet \mathbf{t} \quad \text{(from 3.A.2 \& 3.B.2)}$$

$$\left[-\frac{(\mathbf{u}_e-\mathbf{u}_d)\bullet \mathbf{t}}{\|(\mathbf{u}_e-\mathbf{u}_d)\bullet \mathbf{t}\|}\mu F_n - F_t\right]\left[(\mathbf{u}_d-\mathbf{u}_e)\bullet \mathbf{t}\right] = 0$$

(from 3.A.3 & 3.B.5)

$$(\mathbf{u}_d-\mathbf{u}_e)\bullet \mathbf{n} = 0 \quad \text{(from 3.A.4 \& 3.B.3)}$$

$$\mu F_n \geq F_t \geq -\mu F_n \quad \text{(from 3.A.5)}$$

$$F_n \geq 0 \quad \text{(from 3.A.6 \& 3.B.6)}$$

4. The Minimum Potential Energy Principle

If a given force depends on the position alone, $\mathbf{F} = \mathbf{F}(\mathbf{r})$, and the quantity $\mathbf{F}\bullet d\mathbf{r}$ can be expressed in the form of a perfect differential

$$\mathbf{F}\bullet d\mathbf{r} = -d\Phi(\mathbf{r})$$

where the function $\Phi(\mathbf{r})$ depends only on the position vector $\mathbf{r}$ and does not depend explicitly on the velocity $\dot{\mathbf{r}}$ or the time t, the force field is said to be *conservative*, and the function $\Phi(\mathbf{r})$ is known as the potential energy.

The principle of minimum potential energy states that the displacement which satisfies the differential equations of equilibrium, as well as the conditions at the bounding surface, yields a smaller value for the potential energy of deformation than any other displacement, which satisfies the same conditions at the bounding surface.

The potential energy stored in a finger-object system is equal to the strain energy, which, in our model, is the potential energy stored in the springs:

$$\Psi = \frac{1}{2}\mathbf{U}^T\mathbf{K}\mathbf{U}$$

where

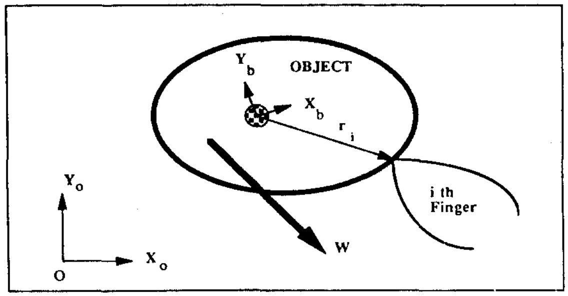


Figure 4.1: Relationship between Finger Displacements and Body Twist.

$$\mathbf{U} = \begin{bmatrix} (\mathbf{u}_d-\mathbf{u}_c)\bullet \mathbf{n} \\ (\mathbf{u}_f-\mathbf{u}_e)\bullet \mathbf{n} \\ (\mathbf{u}_d-\mathbf{u}_c)\bullet \mathbf{t} \\ (\mathbf{u}_f-\mathbf{u}_e)\bullet \mathbf{t} \end{bmatrix} \quad \mathbf{K} = \begin{bmatrix} k_{nb} & 0 & 0 & 0 \\ 0 & k_{nf} & 0 & 0 \\ 0 & 0 & k_{tb} & 0 \\ 0 & 0 & 0 & k_{tf} \end{bmatrix}$$

In Figure 4.1 let X_o-Y_o be a fixed frame and X_b-Y_b be a reference frame fixed to the body. $\mathbf{W}$ is the 3x1 generalized vector of external forces and moments on the object and $\mathbf{r}_i$ is the position vector of the i[th] contact point in the body fixed frame. As $\mathbf{r}_i$ is moved to its new location, $\mathbf{r}_i'$, the grasped object is displaced by an infinitesimal twist $\hat{t} = (\Delta_b^T, \delta z_b)^T$, where $\Delta_b = (dx_b, dy_b)^T$ is the infinitesimal linear displacement of the center of mass of the object and δz_b is infinitesimal angular displacement of the object.

Using a second order Taylor series approximation, we can get the difference between $\mathbf{r}_i$ and $\mathbf{r}_i'$:

$$\begin{bmatrix} dr_{ix} \\ dr_{iy} \end{bmatrix} = \begin{bmatrix} dx_b - \delta z_b r_{iy} - \frac{1}{2}(\delta z_b)^2 r_{ix} \\ dy_b + \delta z_b r_{ix} - \frac{1}{2}(\delta z_b)^2 r_{iy} \end{bmatrix} \quad (4.1)$$

If $\mathbf{W}$ is a conservative force, let the potential energy associated with the vector $\mathbf{W}$ be defined by Θ. Therefore the total potential energy of this system, Φ, is the combination of the sum of the potential energies associated with each contact (Ψ_i) and Θ such that

$$\Phi = \sum_{i=1}^{n} \Psi_i + \Theta \quad (4.2)$$

where Ψ_i is the potential energy stored in the springs at the i[th] contact point, Θ is the potential energy associated with the external forces and moments, and n is the number of contact points.

By minimizing Φ subject to the constraint conditions, we can solve for $\mathbf{u}_c$, $\mathbf{u}_d$, $\mathbf{u}_e$ (displacements), $\mathbf{F}$ (finger force) of each contact and $\hat{t}$ (the object twist) provided all finger motor displacements ($\mathbf{u}_f$) are known. This is the basis for simulating position control algorithms.

However, in the event that sliding occurs, the system is no longer conservative since the frictional force does nonzero work. Hence this method is no longer applicable. In [9], it is shown that a similar minimum principle, called the "minimum power principle" can be used to solve such problems. This is not pursed in this paper, but is suggested as a direction for furture research.

5 Simulation

5.1 Programming

The statement of the problem is as follows:

Given:	1. $\mathbf{n}_i$, $\mathbf{t}_i$ and $\mathbf{r}_i$ 2. Finger displacements, $\mathbf{u}_{if}$ 3. External forces acting on the object, $\mathbf{W}$

Find:	1. The object displacement, $\hat{\mathbf{t}} = (dx_b, dy_b, \delta z_b)^T$ 2. Displacements, $\mathbf{u}_{ic}$, $\mathbf{u}_{id}$ and $\mathbf{u}_{ie}$ 3. Finger forces, $\mathbf{F}_i = F_{in}\,\mathbf{n}_i + F_{it}\,\mathbf{t}_i$

The normal and tangential directions and the coordinate of i^{th} contact point are given by

$$\mathbf{n}_i = \begin{bmatrix} n_{ix} \\ n_{iy} \end{bmatrix}, \mathbf{t}_i = \begin{bmatrix} t_{ix} \\ t_{iy} \end{bmatrix}, \mathbf{r}_i = \begin{bmatrix} r_{ix} \\ r_{iy} \end{bmatrix}$$

The external force, $\mathbf{W}$, has the form

$$\mathbf{W} = \begin{bmatrix} W_x \\ W_y \\ \varpi_z \end{bmatrix}$$

The displacement of the object, $\hat{\mathbf{t}}$, is

$$\mathbf{Q}_b = \begin{bmatrix} dx_b \\ dy_b \\ \delta z_b \end{bmatrix} = \begin{bmatrix} q_{b1} \\ q_{b2} \\ q_{b3} \end{bmatrix}$$

and the displacements at the i^{th} finger-object interface are denoted by $\mathbf{Q}_i$:

$$\mathbf{Q}_i = \begin{bmatrix} \mathbf{u}_{ic}\cdot\mathbf{n}_i \\ \mathbf{u}_{ic}\cdot\mathbf{t}_i \\ \mathbf{u}_{id}\cdot\mathbf{n}_i \\ \mathbf{u}_{id}\cdot\mathbf{t}_i \\ \mathbf{u}_{ie}\cdot\mathbf{n}_i \\ \mathbf{u}_{ie}\cdot\mathbf{t}_i \end{bmatrix} = \begin{bmatrix} q_{i1} \\ q_{i2} \\ q_{i3} \\ q_{i4} \\ q_{i5} \\ q_{i6} \end{bmatrix}$$

The finger displacements (at the rigid substrates) are given by

$$\mathbf{Q}_{if} = \begin{bmatrix} \mathbf{u}_{if}\cdot\mathbf{n}_i \\ \mathbf{u}_{if}\cdot\mathbf{t}_i \end{bmatrix} = \begin{bmatrix} q_{i7} \\ q_{i8} \end{bmatrix}$$

We consider here example of two fingered grasps with rolling contacts. For more than two fingered grasps, we can expand this method easily. A first order approximation for rotation is used. Consider an equilibrium configuration and define all displacement variables so that $\mathbf{Q}_b = \mathbf{0}$, $\mathbf{Q}_i = \mathbf{0}$ and $\mathbf{Q}_{if} = \mathbf{0}$ (i = 1, ..., n) at equilibrium. Now let the motors of the fingers command nonzero displacements $\mathbf{Q}_{if}$. The objective is determine the resulting displacements $\mathbf{Q}_b$, $\mathbf{Q}_i$ (i = 1, ..., n) and the resulting contact forces.

The basic problem then is:

Minimize	$\Phi = G^T Q + \frac{1}{2} Q^T H Q$
Subject to	$AQ = C$ $BQ \geq D$

where

$$\mathbf{Q}^T = \left(\mathbf{Q}_1^T, \mathbf{Q}_2^T, \mathbf{Q}_b^T\right)^T$$

$$\mathbf{Q}_1^T = (q_{11}, q_{12}, q_{13}, q_{14}, q_{15}, q_{16})$$

$$\mathbf{Q}_2^T = (q_{21}, q_{22}, q_{23}, q_{24}, q_{25}, q_{26})$$

$$\mathbf{Q}_b^T = (q_{b1}, q_{b2}, q_{b3})$$

$$\mathbf{G}^T = \left(\mathbf{G}_1^T, \mathbf{G}_2^T, \mathbf{G}_b^T\right)^T$$

$$\mathbf{G}_1^T = (0, 0, 0, 0, -k_{nf}q_{17}, -k_{tf}q_{18})$$

$$\mathbf{G}_2^T = (0, 0, 0, 0, -k_{nf}q_{27}, -k_{tf}q_{28})$$

$$\mathbf{G}_b^T = (-W_x, -W_y, -\varpi_z)$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}$$

$$\mathbf{H}_1 = \mathbf{H}_2 = \begin{bmatrix} k_{nb} & 0 & -k_{nb} & 0 & 0 & 0 \\ 0 & k_{tb} & 0 & -k_{tb} & 0 & 0 \\ -k_{nb} & 0 & k_{nb} & 0 & 0 & 0 \\ 0 & -k_{tb} & 0 & k_{tb} & 0 & 0 \\ 0 & 0 & 0 & 0 & k_{nf} & 0 \\ 0 & 0 & 0 & 0 & 0 & k_{tf} \end{bmatrix}$$

$$
\mathbf{A} = \begin{bmatrix}
-1 & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & 0 & \beta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 & \alpha & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 & \beta & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
n_{1x} & t_{1x} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & r_{1y} \\
n_{1y} & t_{1y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -r_{1x} \\
0 & 0 & 0 & 0 & 0 & 0 & n_{2x} & t_{2x} & 0 & 0 & 0 & 0 & -1 & 0 & r_{2y} \\
0 & 0 & 0 & 0 & 0 & 0 & n_{2y} & t_{2y} & 0 & 0 & 0 & 0 & 0 & -1 & -r_{2x}
\end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix} \mathbf{B}_1 & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}
$$

$$
\mathbf{B}_1 = \mathbf{B}_2 = \begin{bmatrix}
-1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 \\
-\mu & \gamma & \mu & -\gamma & 0 & 0 \\
-\mu & -\gamma & \mu & \gamma & 0 & 0
\end{bmatrix}
$$

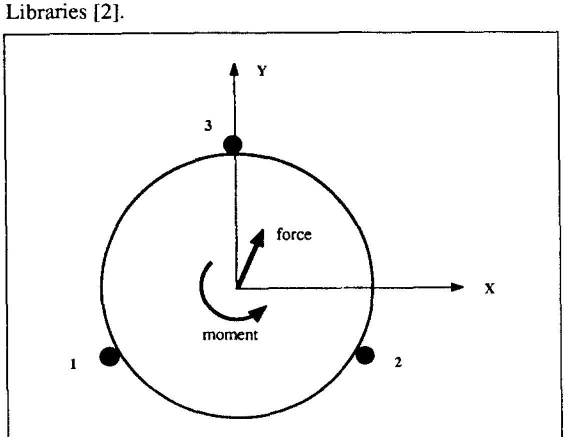
$$
\mathbf{C}^{\mathbf{T}} = \left(\alpha q_{17}, \beta q_{18}, 0, \alpha q_{27}, \beta q_{28}, 0, 0, 0, 0, 0\right)
$$

$$
\mathbf{D}^{\mathbf{T}} = \left(0, 0, -q_{17}, 0, 0, 0, 0, -q_{27}, 0, 0\right)
$$

$$
\alpha = \frac{k_{nf}}{k_{nb}}, \ \beta = \frac{k_{tf}}{k_{tb}}, \ \gamma = \frac{k_{tb}}{k_{nb}}
$$

Here we have assumed that the coefficient of friction is large so that conditions of pure rolling exist at each contact. Therefore the objective function is quadratic and positive definite and the constraints are convex. It is convenient to use the quadratic programming technique.

Next we present examples which we solved using the QPROG routine in International Mathematical and Statistical Libraries [2].



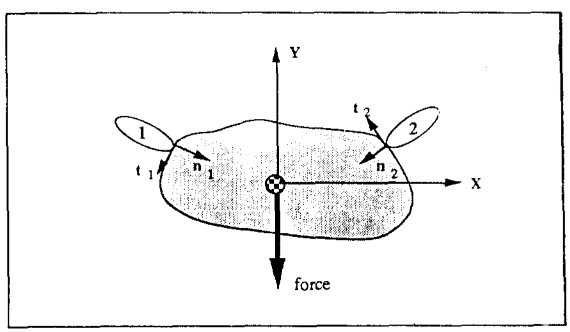
5.2 Examples

Example 1) A Planar Circular Disk with a Three-Fingered Grasp
(see Figure 5.2.a)

Number of Fingers		3
Coordinates of Contact Points (m)	1	(-0.8666, -0.5)
	2	(0.8666, -0.5)
	3	(0., 1.)
Load Force (N)		0.1i + 0.2j applied at (0, 0)
Load Couple (N-m)		0.3k
Stiffness (N/m) $k_{nb}, k_{tb}, k_{nf}, k_{tf}$		100
Contact Forces (N) $\mathbf{F}_i$	1	(4.2496, 2.5216)
	2	(-4.4164, 2.3483)
	3	(0.0666, -5.0651)
Equilibrating Forces (N) F_{iE}	1	(-0.0804, 0.0199)
	2	(-0.0864, -0.1534)
	3	(0.0666, -0.0668)
Interaction Forces (N) F_{iI}	1	(4.333, 2.5017)
	2	(-4.333, 2.5017)
	3	(0., -4.9983)
Coefficient of Friction		$\mu = 0.25$
Ratio of F_{it}/F_{in}	1	-0.0122
	2	-0.0346
	3	-0.0131

Example 2) A General Object with a Two-Fingered Grasp
(see Figure 5.2.b)

	1ST FINGER FORCE		2ND FINGER FORCE	
Load	Normal	Tangential	Normal	Tangential
-0.00	0.5000	-0.0625	0.5000	0.0625
-0.01	0.4994	-0.0674	0.4994	0.0674
-0.02	0.4988	-0.0724	0.4998	0.0724
-0.03	0.4981	-0.0774	0.4981	0.0774
-0.04	0.4975	-0.0823	0.4975	0.0823
-0.05	0.4969	-0.0873	0.4969	0.0873
-0.06	0.4963	-0.0923	0.4963	0.0923
-0.07	0.4957	-0.0972	0.4957	0.0972
-0.08	0.4950	-0.1022	0.4950	0.1022
-0.09	0.4944	-0.1071	0.4944	0.1071
-0.10	0.4938	-0.1121	0.4938	0.1121
-0.11	0.4932	-0.1171	0.4932	0.1171
-0.12	0.4926	-0.1220	0.4926	0.1220
-0.13	0.4924	-0.1231	0.4924	0.1231
-0.14	0.4924	-0.1231	0.4924	0.1231



	Coordinates		Unit Normals
i	x	y	n_x n_y
1	(-4.,	1.8)	(0.9806, -0.1961)
2	(4.,	1.8)	(-0.9806, -0.1961)

Figure 5.2 (a) A Planar Circular Disk with a Three Fingered Grasp (b) A General Object with a Two-Fingered Grasp

The example in Figure 5.2.b involves an object of arbitrary shape and a two fingered grasp. Here the normal and tangential stiffnesses of the body and fingers are all 10 units. Each finger is given a 0.1 unit displacement in the normal direction, while the tangential displacements are a - 0.0125 unit displacement for the first finger and a 0.0125 unit displacement for second finger respectively. The friction coefficient is 0.25. The load is a force **W** which is increased from 0 to 0.14 units. The results show that sliding occurs from **W** $\geq$ 0.14 units.

6. Conclusion

The modeling and problem formulation for computing finger forces and the object displacement in a multifingered grasp with compliant contacts are presented. We considered two types of contact, rolling and sliding. A unified approach to rolling and sliding contacts is described. When the friction forces do zero work (rolling and frictionless sliding), the system can be simulated by extremizing an objective function derived from the potential energy of the system. Simulations of systems include sliding contacts with friction is beyond the scope of this paper, although the method described here is able to predict sliding.

We simulated examples of rolling contact using the QPROG routine in International Mathematical and Statistical Libraries. Finally, we presented two examples to illustrate the methodology for simulation.

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