# The Robustness of Continuous Self Tuning Controller for Retarded System

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#### Abstract

In this paper, the robustness of self turning controller on the continuous time-delay system is investigated. The polynomial identification method using continuous time exponentially wighted least square algorithm is used for estimating the time-delay system parameters. The pole-zero and pole palcement method are adopted for the control algorithm. On considering the control weighting factor and relizability filter the effect of unmodeled dynamics of the plant are examined by the simulation.

#### 1. Introduction

The industrial process has the characteristics of the nonlinearity and the delay caused by the change of the operating point and environments. The design of the adaptive control for the dynamics has been reported by many researches. The self tuning controller for the process with time delay has been basically treated in discrete time approach. [1] But, The discrete time approach has some problems, such as, the choice of sampling interval, zeros of sampled data systems, time delays, and sensivity to time variation. But these problems can be avoided by using the continous time approach has been adopted.

The main algorithm for self tuning control is the GMV(generalized minimum variance) using an emulator.[2] The polynomial identification algorithm is the modified continuous time exponentially-weighted least square algorithm for estimating the time-delay system parameters. We can show that the robustness of the control system has ralation with the control weighting factor.

# 2. The Continuous Time Self Tuning Control System

The plant transfer function is described as followes.

$$Y(s) = e^{-sT} \frac{B(s)}{A(s)} U(s) + \frac{C(s)}{A(s)} V(s)$$
 (1)

where,

$$U(s)$$
,  $Y(s)$ : input, output  
 $A(s) = \alpha_0 S^n + \alpha_1^{n-1} S^{n-1} + \cdots + \alpha_n$   
 $B(s) = b_0 S^m + b_1 S^{m-1} + \cdots + b_m$ 

T : System time delay

V(s) : disturbance

The emulator is employed to realize the system that compensate for the high relative degree, time delay, and non-minimum phase zero points of the system. The approximation to the compensation of the time delay is the pade polynomial expansion method.

$$e^{-sT} \ge \frac{T(-s)}{T(s)}$$

where.

$$T(s) = \sum_{i=0}^{n} \frac{(2n-i)!n!}{(2n)!i!(n-i)!} (sT)^{i}$$
 (2)

The emulator using the above approximation is given by

$$\phi(s) = \frac{P(s)}{Z(s)} e^{sT}Y(s) 
= \frac{P(s)}{Z(s)} \frac{T(s)}{T(-s)} Y(s) 
= \phi^*(s) + \phi^*(s) 
= \frac{F(s)}{C(s)Z^*(s)} Y(s) 
+ \frac{E(s)B(s)}{T(s)C(s)Z^-(s)} U(s) + e^*(s)$$
(3)   
=  $\frac{F(s)}{F \text{ filter}} Y(s) 
+ \frac{G(s)}{G \text{ filter}} U(s) + e^*(s)$ (4)

The rules for the polynomials Z(s), E(s), F(s) are as follows

$$Z(s) = Z^{+}(s) + Z^{-}(s)$$
 (5)

where, represent

Z+(s): Realizable Part Z-(s): Nonrealizable Part

Diaphantine Equation

$$T(s)P(s)C(s) = A(s)Z^{+}(s)E(s) + F(s)T(-s)Z^{-}(s)$$
 (6)

The equation(3) can be transformed as the linear in the parameter form

$$\varphi^*(s) = X^{\mathsf{T}}(s)\Theta = \left[\begin{array}{c} X_{\mathsf{u}}(s) \\ X_{\mathsf{y}}(s) \end{array}\right] \left[\begin{array}{c} \Theta_{\mathsf{n}} \ \Theta_{\mathsf{y}} \end{array}\right]$$

where the data vector  $\mathbf{X}^{T}(s)$  and parameter  $\boldsymbol{\theta}$  are given as

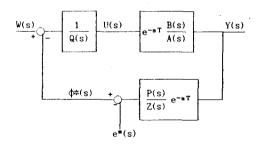
$$X_{\mathbf{u}}(s) = \frac{1}{G \text{ filter}(s)} \begin{bmatrix} s_{\mathbf{m}-1}^{\mathbf{m}} \\ \vdots \\ t \end{bmatrix} U(s)$$

$$X_{y}(s) = \frac{1}{\text{F filter(s)}} \begin{bmatrix} s_{n-1}^{n} \\ \vdots \\ \vdots \end{bmatrix} Y(s)$$

and  $\Theta$  is given as

$$\Theta_{\mathbf{u}} = \begin{bmatrix} g_0 \\ \vdots \\ g_n \end{bmatrix} \qquad \Theta_{\mathbf{y}} = \begin{bmatrix} f_0 \\ \vdots \\ f_n \end{bmatrix} \tag{9}$$

Now, the feedback loop using the above emulator is considered



## figure1. The nominal feedback loop

The Controller is described as,

$$Q(s)U(s) = W(s) - \Phi^*(s) \tag{10}$$

where.

U(s) : the control output,

p\*(s) : the emulator output,

W(s) : the set point,

Q(s) : the weighting transfer function

$$Q(s) = \frac{Q_n(s)}{Q_d(s)}$$

From the equation (3),(10) the Control ouput is

$$U(s) = \frac{Q_d(s)C(s)Z(s)T(s)}{D(s)} W(s)$$

+ 
$$\frac{Q_d(s)F(s)Z(s)T(s)}{D(s)}$$
 Y(s)

where,

$$D(s) = Q_n(s)C(s)Z(s)T(s) + Q_d(s)E(s)B(s)$$
 (11)

The Loop gain is

$$L(s) = \frac{1}{Q(s)} \frac{P(s)}{Z(s)} \frac{B(s)}{A(s)}$$
 (12)

The Closed loop system is

$$Y(s) = e^{-sT} \frac{L(s)}{1 + L(s)} \frac{Z(s)}{P(s)} [w(s) + e^{*}(s)]$$
(13)
$$+ \frac{1}{1 + L(s)} \frac{C(s)}{A(s)} V(s)$$

We can give the pole-placement, pole-zero, and detuned mrac control from the above generalized control system

We can obtain the adaptive system using the above nonadaptive control and estimation method. The implicit self and estimation control scheme that estimates the parameter of the emulator directly without using Diophantine Equation is adopted. The emulator signal \$\psi(t)\$ can be realized employing the Realizability Filter.

$$\phi_{-}(s) = \Lambda(s)\phi(s) \tag{14}$$

Let A(s) be.

$$\Lambda(s) = e^{-sT} \frac{Z(s)}{P(s)} = \frac{T(-s)}{T(s)} \frac{Z(s)}{P(s)}$$
(15)

+ 
$$\frac{G(s)}{T(s)C(s)} \frac{T(-s)}{T(s)P(s)} U(s)$$

The corresponding linear in the parameter form is as follows

$$\uparrow \land (s) = X \land T(s) + e \land (s)$$
 (17)

$$\chi_{\uparrow}(s) = \Lambda(s)\chi(s)$$
  
 $e_{\uparrow}(s) = \Lambda(s)e(s)$ 

To identify the parameters of time delay system, the estimated model can be described as the polynomial form in the parameters to be estimated.

The realizability-filtered emmulaaator output can be put estimated system output.

Then, the estimation error is

$$e(t) = y(t) - y(t) = y(t) - \phi(t)$$
 (18)

where, the linear in the parameter form of the estimated output is

$$\hat{\Phi}_{-}(t) = X(t)\hat{\Theta}(t)$$

$$= [X_{u}(s)][\hat{\Theta}_{u}(s)|\hat{\Theta}_{y}(s)] \qquad (19)$$

$$[X_{y}(s)]$$

where.

$$\chi_{y}(s) = \frac{1}{C(s)} \begin{bmatrix} s_{n-1}^{n} \\ \vdots \\ i \end{bmatrix} pfy(s)$$

$$X_{u}(s) = \frac{1}{T(s)C(s)Z(s)} \begin{bmatrix} s \\ i \end{bmatrix} PFU(s)$$
 (20)

$$\hat{\Theta}_{u}(s) = \begin{bmatrix} \hat{g}_{0} \\ \hat{g}_{max} \end{bmatrix} \qquad \hat{\Theta}_{y}(s) = \begin{bmatrix} \hat{f}_{0} \\ \hat{f}_{max} \end{bmatrix}$$

To find a parameter estimate e(t), the least-square criterion cannot only be used in relation to parameter estimation in linear parameter system but also be extended to polynomial in the parameter systems.

In the continuous time domain, the cost function is

$$J(\hat{\theta}(t), t) = \frac{1}{2} e[\hat{\theta}(t) - \hat{\theta}_0]^T So[\hat{\theta}(t) - \hat{\theta}_0]$$
(21)  
 
$$+ \frac{1}{2} \int_0^t e^{-\beta(t-\tau)} \hat{\theta}(t, \tau)^2 d\tau$$

where,  $\beta$  is the non-negative scalar forgetting factor  $(\beta \ge 0)$  and  $S_0$  is the positive definite mastrix initial cost weighting is  $S_0 > 0$ .

In the minimizing the cost function the condition ensures the exitence and uniqueness of the solution, but the condition is not good enough in practice because the second derivative  $J_2(e(t), t)$  become nearly singular.

The recursive solution of the linear parameter system for the cost function above is

$$\frac{d}{dt} \hat{\Theta}(t) = S^{-1}(t)X(t)\hat{e}(t)$$

$$\frac{d}{dt} S^{-1}(t) = \beta S^{-1}(t) - S^{-1}(t) \mathcal{F}^{T}(t) S^{-1}(t)$$
 (22)

It is better to update the square root of S(t) rather than S(t) itself because of the above numerical reasons.

### 3. Simulation

In the simulation, the effects of the two kinds of unmodeled dynamics are investigated.

The first case is the unmodeled time delay, and the second is the unmodeled pole-zero placement.

The control scheme is the detuned model reference control.

The control weighting factor is arranged for the robustness of the control system.

The assumed designed parameters are as followes

$$\frac{B(s)}{A(s)} = \frac{2}{s+3}$$
, T = 1

$$C(s) = s + 2$$
 ,  $Q(s) = \frac{qs}{s + 1}$ 

$$P(s) = 1 + 0.3s$$
.  $Z(s) = 1 + 0.03s$ 

CASO	Unnodeled Delay	(Wandeled pole-placement	9	Approximation Pade
-	0	1	-	12
2	0	100 s <sup>1</sup> + 8x + 100	1	12
3	2	1	ı	12
1	1	100 s <sup>2 0</sup> + 8s + 100	1	12
5	ı	ı	0,5	2
٤	1	ı	0.4	£

## 4. Conclusions

The simulation for continuous time implicit adaptive controller for the time-delay system show the following results.

- (1) The proper selecting of the control weighting factor and the order of Pade approximation improves the robustness of unmodeled dynamics.
- (2) The study for the efficient choice of weighting factor is required.

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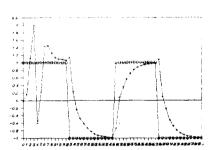


figure 2. case 1

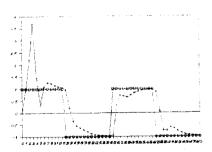


figure 3. case 2

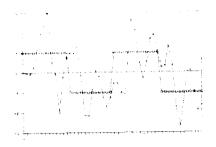


figure 4. case 3

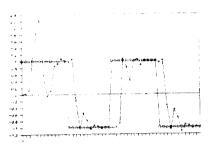


figure 5. case 4

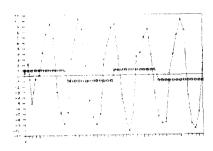


figure 6. case 5

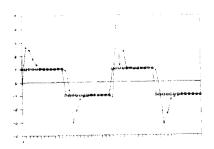


figure 7, case 6