'91 KACC 1991. 10. 22~24

A STUDY ON THE ROBUST MODEL-FOLLOWING CONTROL SYSTEMS WITH NONLINEAR PLANT

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ABSTRACT

This paper proposes a robust model following control systems with nonlinear time varying plant, which realies good properties such as asymptotic stability, disturbance rejection and model-following with reduced sensitivity for plant parameter variation. The schemes do not incorporate any parameter identification algorithms, but the adaptation is realized through signal synthesis in a fixed parameter structure.

1. INTRODUCTION

The variable structure model-following control (VSMFC) system is an adaptive model-following control (AMFC) system designed as a variable structure system (VSS) by applying the theory of VSS so that sliding mode exists [1][2].

The study of the VSMFC for the single input systems is enlivened, but for the multi-input systems is not only difficult to choose the variable structure control gain but also afraid of stability and convergence [1]. And it may be impractical to apply the discontinuous chattering input directly to the plant [3].

In this paper, a new method to the robust model following control (RMFC) systems [4][5] is considered. The control gain does not require the solution of a set of differential equations, thus the structure is simple. The advantage of designing RMFC systems is that the transient response of the model plant error can be prescribed by the design. The control system exhibits insensitivity to parameter variations and noise disturbances.

The theory is applied to the problem of model-following control for a class of nonlinear time-varying plants [6]-[8]. The design procedure and the performance of the resulting control system are illustrated by a design of the pendulum position control systems[9][10].

2. PROBLEM FORMULATION

The state equations of a nonlinear time varying multivariable plant are as follow. [9]

$$\dot{x}_p = A_p(x_p, t)\, x_p + B_p(x_p, t)\, u_p + a(x_p, t) \qquad (1)$$

where $x_p \in R^n$ is the plant state, $u_p \in R^m$ is the control input, $a \in R^n$ is a nonlinear time-varying vector including disturbances. The plant matrices A_p and B_p may be nonlinear time-varying, but the nominal value of the elements of these matrices are assumed to be known to the designer. The following assumptions specify the class of nonlinear plants considered in this paper. ① $B_p(x_p, t) = B_t F(x_p, t)$, a full rank matrix B_t is available, a norm bounded matrix $F(x_p, t)$ is positive definite ② $A_p(x_p, t) - A_t = B_t C(x_p, t)$, the matrix A_t and the upper bounds of the norms of the matrix $C(x_p, t)$ are available ③ $a(x_p, t) = B_t c(x_p, t)$, the upper bounds of the norms of the vector $c(x_p, t)$ is available ④ the pair (A_t, B_t) is controllable

The model specifying state behavior expected from the controlled plant is described by the linear, time-invariant, differential equation.

$$\dot{x}_m = A_m x_m + B_m u_m \qquad (2)$$

where $x_m \in R^n$ is the model state, $u_m \in R^l$ is a piecewise continuous bounded reference input. A_m is asymptotically stable and the pairs (A_m, B_t) is controllable. A matrix A_m satisfying the assumptions can be obtained as $A_m = A_t + B_t K_1$ with K_1 obtained, for example, by demanding A_m to have specific eigenvalues.

In model-following systems, the plant is controlled in such way that the dynamic behavior of plant approximates that of a specified model. The controller should force the error between the model and plant to zero as time tends to infinity.

The state error vectors are represented by Eq.(3) and a new transformed vector is defined by Eq.(4). The dimension of the new vector is equal to that of the control input vector.

$$e = x_m - x_p \tag{3}$$

$$s = Ge \tag{4}$$

where $e \in \mathbf{R}^n$, $s \in \mathbf{R}^m$ and $G(m \times n)$ is the transformed matrix.

$$\dot{e} = \dot{x}_m - \dot{x}_p \tag{5}$$

$$\dot{s} = G\dot{e} = G[A_m e + (A_m - A_p)x_p + B_m u_m - B_p u_p - a] \tag{6}$$

For the perfect model following case, Eq.(6) is zero and the equivalent control is obtained by solving the equation for u_p.

$$u_{peq} = (GB_p)^{-1} G[A_m e + (A_m - A_p)x_p + B_m u_m - a] \tag{7}$$

Substituting Eq.(7) into Eq.(5), Eq.(8) is obtained.

$$\dot{e} = [I - B_p(GB_p)^{-1}G][A_m e + (A_m - A_p)x_p + B_m u_m - a] \tag{8}$$

In Eq.(8), x_p, u_m and a are considered as disturbances for the error dynamics [1]. For these total disturbance rejection, Eq.(9) holds for any x_p, u_m and a.

$$\begin{aligned} &[I - B_p(GB_p)^{-1}G](A_m - A_p)x_p = 0 \\ &[I - B_p(GB_p)^{-1}G]B_m u_m = 0 \\ &[I - B_p(GB_p)^{-1}G]a = 0 \end{aligned} \tag{9}$$

So that the linear systems, Eq.(9), have a solution, the following conditions are obtained [11].

$$\begin{aligned} \mathrm{rank}[B_p] &= \mathrm{rank}[B_p ; A_m - A_p] \\ &= \mathrm{rank}[B_p ; B_m] \\ &= m \end{aligned} \tag{10}$$

Eq.(10) is the perfect model-following conditions [1] which is a general necessary conditions to the model-following control systems.

3. ROBUST MODEL-FOLLOWING CONTROL SYSTEMS

In many case, the variation in plant matrices can be expressed as the sum of a known nominal matrix and a variation matrix.

$$\begin{aligned} \dot{x}_p &= [A_p^\circ + \Delta A_p(x_p, t)]x_p + [B_p^\circ + \Delta B_p(x_p, t)]u_p + a(x_p, t) \\ &= [A_p^\circ x_p + B_p^\circ u_p] + [\Delta A_p(x_p, t)x_p + \Delta B_p(x_p, t)u_p] + a(x_p, t) \end{aligned} \tag{11}$$

where, A_p° and B_p° are the nominal matrices of $A_p(x_p, t)$ and $B_p(x_p, t)$, respectively. Eq.(5) and (6) became eq.(12) and (13).

$$\begin{aligned} \dot{e} &= A_m e + (A_m - A_p^\circ)x_p + B_m u_m - B_p^\circ u_p \\ &\quad - [\Delta A_p(x_p, t)x_p + \Delta B_p(x_p, t)u_p + a(x_p, t)] \\ &= \dot{e}^\circ - \Delta\dot{e} \end{aligned} \tag{12}$$

$$\dot{s} = GA_m e + G(A_m - A_p^\circ)x_p + GB_m u_m - GB_p^\circ u_p - G\Delta\dot{e} \tag{13}$$

3.1 Design of a robust model following control law

Because s is a scalar in the single input system, $G(1 \times n)$ is a vector, therefore, we can choose the element of G so that the transient state error response is desirable. However, in the multivariable system, $s(m \times 1)$ and $u_p(m \times 1)$ are vectors, thus $G(m \times n)$ is a matrix. In order to correspond one by one between s and u_p element, the matrix GB_p° in Eq.(13) should be a unit matrix $I(m \times m)$.

$$GB_p^\circ = I \tag{14}$$

$$\dot{s} = GA_m e + G(A_m - A_p^\circ)x_p + GB_m u_m - u_p - G\Delta\dot{e} \tag{15}$$

The linear gain matrices are defined as Eq.(16) and Eq.(15) became Eq.(17).

$$\begin{aligned} G_e &= GA_m \\ G_p &= G(A_m - A_p^\circ) \\ G_m &= GB_m \end{aligned} \tag{16}$$

$$\dot{s} = G_e e + G_p x_p + G_m u_m - u_p - G\Delta\dot{e} \tag{17}$$

Rearranging about the control law, Eq.(18) is obtained.

$$\begin{aligned} u_p &= G_e e + G_p x_p + G_m u_m - \dot{s} - G\Delta\dot{e} \\ &= G_e e + G_p x_p + G_m u_m - \dot{s}^\circ \end{aligned} \tag{18}$$

In the Eq.(4), the error vector goes to zero as time tends to infinity, that is, the $\dot{s}^\circ$ vector goes to zero. In order to satisfy that, the sign of $\dot{s}_i^\circ$ different from the sign of s_i°

$$s_i^\circ \dot{s}_i^\circ < 0 \tag{19}$$

Inequality (19) is the condition to occur the sliding motion in VSS. In the VSS theory [12][13], the system representative point could be brought from any initial position to the switching hyperplanes. Once the operating point reaches the switching hyperplanes, the control will switch between the gains to force the representative point to move along the switching hyperplanes. Every time the representative point leaves the switching hyperplanes the controller changes the feedback structure to force the point to return to the switching hyperplanes. This special motion is called the sliding motion. Sliding motion occurs if, at a point on a switching surface, the direction of motion along the error state trajectories on either side of the surface are not away from the switching surface. The state then slides and remains for some finite time on the surface $s_i(e)=0$. Then the error between model and plant goes to zero and the model-following is obtained.

Because Eq.(18) is a set of differential equations such as AMFC, we take the term of s° instead of $\dot{s}^\circ$. Then, inequality Eq.(19) became an equality Eq.(20).

$$\dot{s}_i^\circ = -\alpha s_i^\circ \qquad (20)$$

where α is a positive constant. Therefore, the control law obtained as Eq.(21).

$$u_p = G_e e + G_p x_p + G_m u_m + \alpha s^\circ \qquad (21)$$

The variation in $A_p(x_p, t)$ and $B_p(x_p, t)$ can be expressed as the sum of a known nominal matrix and a variation matrix. Since the bounds on the variations are limited, s is a very close approximation to s°. Furthermore, carrying on the model following, s° and s are approximate to zero and the final term in Eq. (21) is relatively small in comparison with other terms. Thus, taking s instead of s°, we can compose the control function as Eq.(22) and the block diagram as Fig.1.

$$u_p = G_e e + G_p x_p + G_m u_m + \alpha s \qquad (22)$$

In Fig.1, the input of the model, u_m, is a command value or a feedback input by the optimal theory or the pole placement theory.

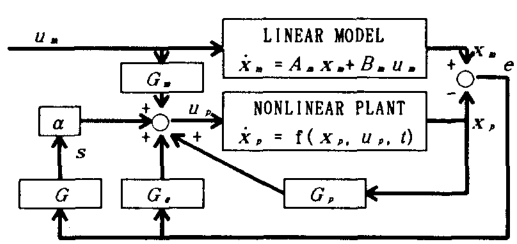


Fig.1 Robust model-following control systems

3.2 Hyperplane design by eigenvalue assignment

The perfect model following condition Eq.(9) means that the matrices A_m, A_p and B_p can be transformed to Eq.(23).

$$B_p = \begin{bmatrix} 0 \\ \hline B_2 \end{bmatrix}\begin{matrix} n-m \\ m \end{matrix} \qquad A_m, A_p = \begin{bmatrix} 0 & I & 0 \\ \hline & a_{ij} & \end{bmatrix}\begin{matrix} n-m \\ m \end{matrix} \qquad (23)$$

Where, $B_2(m \times m)$ is the nonsingular square matrix. $i=n-m+1, n$ and $j=1, n$. And, from Eq.(14), we consider the following equation.

$$G = (Q B_p^\circ)^{-1} Q \qquad (24)$$

where a matrix $Q(m \times n)$ is defined the hyperplane matrix. The selection of the matrix Q is very important.

For the perfect model following case, substituting Eq.(24) into Eq.(8), Eq.(25) is obtained and Eq.(4) became Eq.(26).

$$\dot{e} = [I - B_p (Q B_p)^{-1} Q] A_m e \qquad (25)$$
$$s = G e = 0 \qquad (26)$$

In this case, n error state variables in Eq.(25) can be expressed in terms of the remaining $n-m$ error state variables using the m algebraic Eq.(26). Since Eq.(27) always holds, Eq.(28) is formed [14].

$$[B_p(QB_p)^{-1}Q]^2 = B_p(QB_p)^{-1}QB_p(QB_p)^{-1}Q = B_p(QB_p)^{-1}Q \qquad (27)$$
$$\text{rank}[B_p(QB_p)^{-1}Q] = \text{rank}[B_p] = m \qquad (28)$$

Therefore, the most rank of $[I - B_p(QB_p)^{-1}Q]$ in Eq.(25) is $n-m$ and any $A_m(n \times n)$ matrix pre-multiplied by $[I - B_p(QB_p)^{-1}Q]$ will have at most rank, $n-m$. The remaining unforced system Eq.(25) must be asymptotically stable, which implies that all $n-m$ eigenvalues of the matrix $[I - B_p(QB_p)^{-1}Q]A_m$ have negative real parts. The eigenvalues can be placed arbitrarily in the complex plane by suitable choice of matrix $Q(m \times n)$. The design objective is to choose Q so that the error tends to zero with suitable transient motion. The polynomial from the desired eigenvalues is reduced to

$$p(\lambda) = c_1 + c_2\lambda + \cdots + c_{n-m}\lambda^{n-m-1} + \lambda^{n-m} \qquad (29)$$

We set up the matrix $Q(m \times n)$ as Eq.(30) so that the eigenvalues of Eq.(29) equal to those of Eq.(25).

$$Q = \left[\begin{array}{c|c} c_1 \cdots c_{n-m} & \\ \hline & I_m \\ 0 & \end{array}\right] \qquad (30)$$

3.3 Robustness of the RMFC systems

In AMFC, the design of controller can be failed if the inverse matrix does not exist. But, in this algorithm, the inverse matrix in Eq.(24) always exist because the multiplication $Q\,(m \times n)$ matrix by $B_p^\circ(n \times m)$ matrix became the nonsingular square matrix $B_2(m\times m)$ in Eq.(23).

$$QB_p=\left[\begin{array}{c:c} c_1\cdots c_{n-m} & \\ \hdashline 0 & \end{array} I_m\right]\left[\begin{array}{c}0\\ \hdashline B_2\end{array}\right]=B_2 \quad (31)$$

From Eq.(23), Eq.(30) and Eq.(31), the error dynamics Eq.(25) became as below.

$$[I-B_p\{QB_p\}^{-1}Q]A_m$$

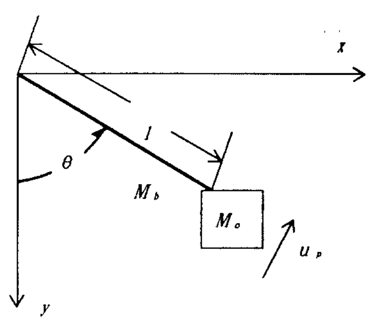
$$=\left[\begin{array}{c:c:c} 0 & \begin{array}{c} I_{n-m} \\ \hdashline -c_1\cdots\cdots\cdots -c_{n-m} \\ \hdashline 0 \end{array} & 0 \end{array}\right]\begin{array}{l} n-m \\ 1 \\ m-1 \end{array} \quad (32)$$

(column widths: 1, $n-m$, $m-1$)

Eq.(32) means that ① the rank of the error dynamics is $(n-m)$, ② $(n-m)$ eigenvalues of the error dynamics equals to the desired eigenvalues of Eq.(29), ③ the eigenvalues can be placed arbitrarily in the complex plane by suitable choice of Q, ④ the system is not influenced by the parameter variations in A_p and B_p.

4. DESIGN EXAMPLE

The design procedure and the performance of the resulting control system are illustrated by a simple example, a position control of the the pendulum shown in Fig.2 [9][10].



The equation of motion is

$$l^2(M_b/3+M_o)\ddot{\theta}+\phi\dot{\theta}+g\,l(M_b/2+M_o)\sin\theta=u_p \quad (33)$$

where u_p is the control torque, l(=0.3[m]) is the pendulum length, M_b (=1.2[kg]) is the distributed mass of the link, M_o (= 0-0.5[kg]) is a disturbance in the form of a time varying lumped mass, θ is the angle between pendulum and vertical axis, ϕ (= 0 - 0.03[N.ms/rad])is an uncertain parameter representing the viscous friction coefficient and g is the gravitational constant.

Defining the state vector as $x_p^T=(\theta,\dot{\theta})$, we have

$$\dot{x}_p=\begin{bmatrix}0 & 1\\ 0 & \dfrac{-\phi}{0.036+0.09M_o}\end{bmatrix}x_p+\begin{bmatrix}0\\ \dfrac{1}{0.036+0.09M_o}\end{bmatrix}u_p+\begin{bmatrix}0\\ -\dfrac{(19.6+32.7M_o)\sin x_{p1}}{0.4+M_o}\end{bmatrix} \quad (34)$$

The linear model to be tracked is as fellows:

$$\dot{x}_m=\begin{bmatrix}0 & 1\\ -26 & -2\end{bmatrix}x_m+\begin{bmatrix}0\\ 26\end{bmatrix}u_m \quad (35)$$

where, eigenvalues of model : $\lambda_m = -1\pm j5$

Digital simulations have been made taking ϕ=0.02, reference input u_m as a square wave with a frequency of 0.2[Hz] and an amplitude of 1[rad], and disturbance M_o(t) as a square wave with a frequency of 0.2[Hz] and with values ranging from 0 to 0.5[kg].

The simulation conditions are as fellows:
the initial conditions have been assumed zero, except x_{p1}=0.5[rad].
perfect model following condition :
rank$[B_p]$=rank$[B_p; A_m-A_p]$=rank$[B_p; B_m]$=1

computer time interval	:	dt	=	0.02 [sec]
positive constant	:	α	=	0.1t/dt
degree of polynomial	:	n-m	=	2-1 = 1
desired eigenvalue	:	λ	=	-10
desired polynomial	:	$p(\lambda)$	=	λ+10
hyperplane matrix	:	Q	=	[10 1]
control gain matrices	:	G	=	[0.585 0.0585]
		G_e	=	[-1.521 0.468]
		G_p	=	[-1.521 -0.97]
		G_m	=	[1.521]

The simulation results are illustrated in Fig.3-5. They indicate that the continuous control law allows a remarkable smoothness of the control signal to [10], together with a considerable reduction of its level to [9]. Therefore, the proposed control scheme may be very useful for industrial applications.

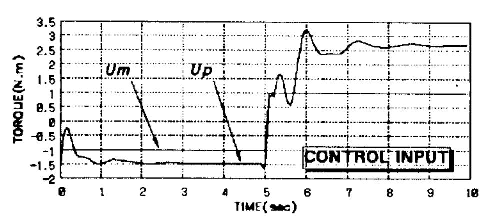


Fig.3 Control input of the pendulum position control

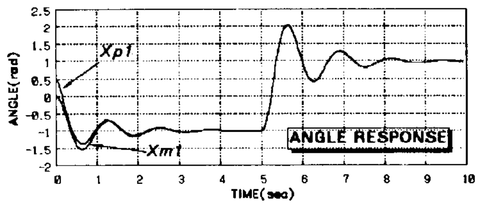


Fig.4 Angle response of the pendulum position control

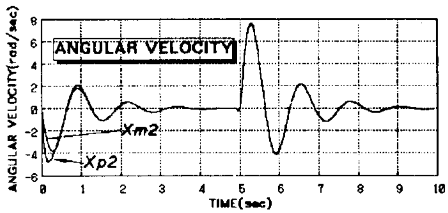


Fig.5 Angular velocity response of the pendulum position control

5. CONCLUSION

In order to eliminate the unpractical chattering of the control signal typical of discontinuous control laws, we propose a new design concept for a robust model following control systems for a class of nonlinear plants in which the control law is a continuous function of all its arguments. Such a scheme guarantees that state error remains bounded and tends exponentially to an arbitrarily small neighborhood of the zero state. This system realizes good properties such as asymptotic stability, disturbance rejection and model following with reduced sensitivity for plant parameter variation. This algorithm can be easily applied to the single-input system or the multi-input system, the linear system or the nonlinear system. And the control structure is simpler than other model following control structure. The transient response of the model plant error can be prescribed arbitrarily by the design. An example is used to demonstrate the design procedures and the excellence of the performance of the RMFC systems.

REFERENCE

[1] K.K.D.Young: "Design of Variable Structure Model-Following Control Systems", *IEEE Trans. Automatic Control*, Vol.AC-23, No.6, pp.1079-1085, (1978)

[2] R.A.Decarlo, S.H.Zak and G.P.Matthews: "Variable Structure Control of Nonlinear Multivariable Systems: A Tutorial", *Proc.IEEE*, Vol.76, No.3, pp.212-232, (1988)

[3] A.S.I.Zinober, O.M.E.El-Ghezawi and S.A.Billings: "Multivariable variable-structure adaptive model-following control systems", *IEE Proc.*, Vol.129, Pt.D, No.1, pp.6-12, (1982)

[4] T.Okada, M.Kihara and O.Asai: "Robust Model-Following Control System and its Application", *Trans. Japan Soc. Aero. Space Sci.*, Vol.27, No.76, pp.78-93, (1984)

[5] C.M.Dorling and A.S.I.Zinober "Robust hyperplane design in multivariable variable structure control systems", *Int.J.Control*, Vol.48, No.5, pp.2043-2054, (1988)

[6] M.W.Spong and M.Vidyasagar: "Robust Linear Compensator for Nonlinear Robotic Control", *IEEE J. Robotics and Automation*, Vol.RA-3, No.4, pp.345-351, (1987)

[7] K.Osuka and T.Ono: "On Robust Stabilization of an Uncertain System via Output Feedback-An H^{∞} Theoretical Approach-", *Trans. of the Society of Instrument and Control Engineers*, Vol.25, No.12, pp.1379-1381, (1989)

[8] K.Osuka, T.Sugie and T.Ono: "PD-Type Two-Stage Robust Tracking Control for Robot Manipulators", *Trans. of the Society of Instrument and Control Engineers*, Vol.25, No.1, pp.46-53, (1989)

[9] G.Ambrosino, G.Celentano and F.Garofalo: "Robust Model Tracking Control for a Class of Nonlinear Plants", *IEEE Trans. Automatic Control*, Vol.AC-30, No.3, pp.275-279, (1985)

[10] A.Balestrino, G.D.Maria and A.S.I.Zinober: "Nonlinear Adaptive Model-Following Control", *Automatica*, pp.559-568, (1984)

[11] C.G.Cullen: *Matrices and Linear Transformation*, Addison Wesley Publishing Co., p.90, (1972)

[12] V.I.Utkin: "Variable Structure with Sliding Mode, A Survey", *IEEE Trans. Automatic Control*, Vol.AC-22, pp.212-222, (1977)

[13] S.H.Kwon, H.Y.Chun, G.T.Park and C.H.Lee: "A Study on the Adaptive Model-Following Control Systems by Slide Mode", *Trans. of the Korean Institute of Electrical Eng.*, Vol.34, No.10, pp.407-417, (1985)

[14] N.E.Gough, Z.M.Ismail and R.E.King: "Analysis of Variable Structure Systems with Sliding Modes", *Int.J. Systems, Sci.*, Vol.15, No.4, pp.401-409, (1984)