Local Instability of Orthotropic Twin-Web Section Members

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INTRODUCTION

In recent years, the fiber reinforced polymeric composite materials are widely utilized in civil engineering field because of their attractive physical and mechanical properties. Especially they are adopted as main structural members in the structures which demand lightweight, high specific strength and/or stiffness, non-conductivity, and corrosion resistance of materials. Since, in general the fiber reinforced polymeric plastic composite structural shapes are manufactured by using the pultrusion process, they are competitive with steel and/or concrete members even in cost.

In spite of their availability and diversity of structural section configuration, most of the structural engineers are hesitant to use fiber reinforced plastic members due to the absent of reliable design criteria and guidelines. For this reason it is necessary to study the short-term and long-term behaviors based on the analytical and experimental investigation, so that the reliable design guidelines could be established.

BUCKLING OF ISOLATED PLATES

Within the classical small deflection theory of thin plates, the governing differential equation of homogeneous orthotropic plate subjected to uniform compression, as shown in Fig. 1, takes the form(Lekhnitskii, 1984)[1]:

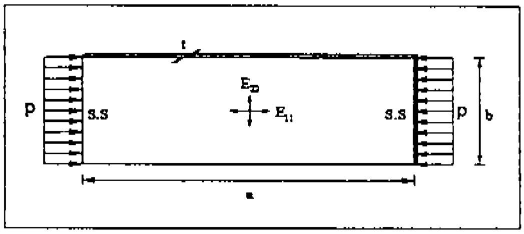

Fig. 1 Orthotropic Plate Subjected to Uniform Compression

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+p\frac{\partial^2 w}{\partial x^2}=0 \quad (1)$$

where the w is out of plane displacement, p is the uniform compression force. The D_{11}, D_{22}, D_{12}, and, D_{66} are the flexural and twisting rigidities of the plate, and they are defined as follows:

$$D_{11}=\frac{E_{11}t^3}{12(1-\nu_{12}\nu_{21})},\quad D_{22}=\frac{E_{22}t^3}{12(1-\nu_{12}\nu_{21})},\quad D_{66}=\frac{G_{12}t^3}{12},\quad D_{12}=\nu_{12}D_{22}=\nu_{21}D_{11} \quad (2\text{-a,b,c,d})$$

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E_{11}, E_{22} = Young's Moduli in fiber(1-1) and transverse(2-2) directions,

G_{12} = shear modulus in 1-2 plane,

ν_{12}, ν_{21} = major and minor Poisson's ratios, respectively,

t = thickness of plate.

Introducing the non-dimensional parameters ξ =x/a and η =y/b, Eq. (1) may be expressed in the form:

$$\frac{D_{11}}{a^4}\frac{\partial^4 w}{\partial \xi^4}+\frac{2(D_{12}+2D_{66})}{a^2b^2}\frac{\partial^4 w}{\partial \xi^2 \partial \eta^2}+\frac{D_{22}}{b^4}\frac{\partial^4 w}{\partial \eta^4}+\frac{p}{a^2}\frac{\partial^2 w}{\partial \xi^2}=0 \quad (3)$$

With the loaded edges being simply supported, the solution of Eq. (3) can be taken as follows:

$$w=f(\eta)\sin m\pi\xi \quad (4)$$

where m is an integer representing the number of half-sine waves of buckled plates in ξ - direction.

When Eq. (4) is substituted into Eq. (3), we can find the fourth-order homogeneous linear differential equation. After finding the general solution of the ensuing differential equation, Eq.(4) can be written as:

$$w=(A_1\cosh\alpha\eta+A_2\sinh\alpha\eta+A_3\cos\beta\eta+A_4\sin\beta\eta)\sin m\pi\xi \quad (5)$$

where

$$\alpha=m\pi s\lambda_2\sqrt{1+\sqrt{1-\left(\frac{\lambda_1}{\lambda_2}\right)^4\left(1-\frac{k}{m^2s^2\lambda_1^2}\right)}} \quad (6)$$

$$\beta=m\pi s\lambda_2\sqrt{-1+\sqrt{1-\left(\frac{\lambda_1}{\lambda_2}\right)^4\left(1-\frac{k}{m^2s^2\lambda_1^2}\right)}} \quad (7)$$

in which:

$$s=\frac{b}{a},\ \lambda_1=\left(\frac{D_{11}}{D_{22}}\right)^{\frac{1}{4}} \quad (8\text{-}1,2)$$

$$\lambda_2=\left(\frac{D_{12}+2D_{66}}{D_{22}}\right)^{\frac{1}{2}},\ k=\frac{pb^2}{\pi^2\sqrt{D_{11}D_{22}}} \quad (8\text{-}3,4)$$

With substitution of boundary conditions of unloaded edges into Eq. (5), we may obtain the characteristic equation of plates, and they are given in Table 1.

Table 1. Characteristic equation of plates

Boundary condition	Characteristic equations
Simple-Free	$SFR=(\alpha^2+\beta^2)(\alpha R^2\cosh\alpha\sin\beta-\beta S^2\sinh\alpha\cos\beta)$
Fixed-Free	$FFR=\{2\alpha\beta RS+\alpha\beta(R^2+S^2)\cosh\alpha\cos\beta+(\beta^2R^2-\alpha^2S^2)\sinh\alpha\sin\beta\}$
Simple-Symmetry	$SSY=\alpha\beta(\alpha^2+\beta^2)\cdot\cosh\frac{\alpha}{2}\cos\frac{\beta}{2}$
Fixed-Symmetry	$FSY=\alpha\beta(\alpha\sinh\frac{\alpha}{2}\cos\frac{\beta}{2}+\beta\cosh\frac{\alpha}{2}\sin\frac{\beta}{2})$

BUCKLING OF TWIN-WEB SECTION MEMBER

In local buckling problem of structural shape composed of several plate components, following assumptions for the junction of plate components are adopted following Bulson(1955)[2]:

(1) the junction of the plate components remains straight when buckling occurs,

(2) original angles between plate components are unchanged during buckling,

(3) there is an equilibrium of moments about junction.

For the buckled shape of the twin-web section subjected to uniform compression, as shown Fig 2, the planes of symmetry at the web and flange exist, respectively.

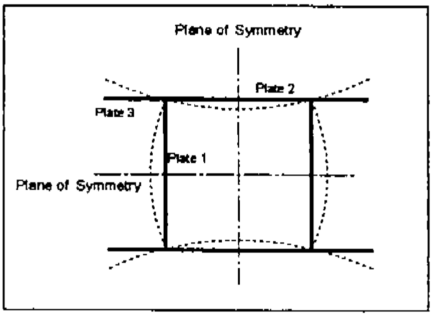

Fig. 2 Buckled Shape of Twin-Web Section

Using the plane of symmetry boundary conditions for efficiency of calculation, we can obtain the following characteristic equation:

$$\left(\frac{SSY}{FSY}\right)_1+\left(\frac{b_2}{b_1}\right)\left(\frac{t_1}{t_2}\right)^3\left(\frac{SSY}{FSY}\right)_2 + \left(\frac{b_3}{b_1}\right)\left(\frac{t_1}{t_3}\right)^3\left(\frac{SFR}{FFR}\right)_3 = 0 \qquad (9)$$

In Eq. (9) SSY, FSY, SFR, and, FFR are expressed in Table 1. From Eq. (9) we can calculate the web buckling coefficient of orthotropic twin-web section member numerically.

The local buckling stress σ_{cr} of the structural members can be expressed with a web buckling coefficient k_1, following Timoshenko(1961)[3]:

$$\sigma_{cr} = k_1 \frac{\pi^2\sqrt{E_{11}E_{22}}}{12(1-\nu_{12}\nu_{21})\left(\frac{b}{t}\right)_1^2} \qquad (10)$$

Since the buckling stress of plate components must be the same, following relations between plate components can be made.

$$k_2 = k_1\left(\frac{b_2}{b_1}\right)^2\left(\frac{t_1}{t_2}\right)^2 \qquad (11)$$

$$k_3 = k_1\left(\frac{b_3}{b_1}\right)^2\left(\frac{t_1}{t_3}\right)^2 \qquad (12)$$

Substituting Eqs. (11) and (12) into Eq. (9), the buckling equation, including only web buckling coefficient k_1, can be obtained.

The minimum web buckling coefficients for orthotropic and isotropic twin-web sections with various b_2/b_1 are shown in Fig. 3 and Fig. 4, respectively. The orthotropic material properties used in Fig. 3 are taken from the data of the structural members in MMFG(1989)[4] which is widely used in America, and isotropic material properties used in Fig. 4 are those of structural steel.

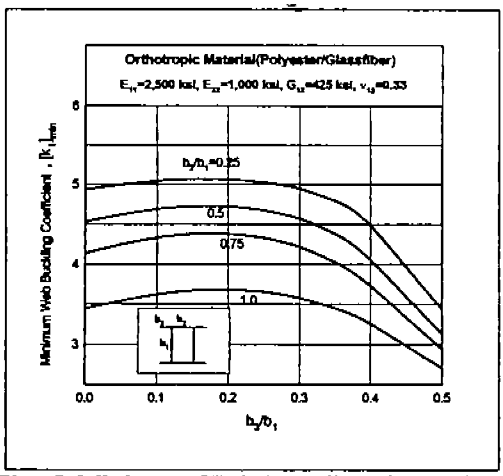

Fig. 3 Minimum Web Buckling Coefficient for Orthotropic Material

DISCUSSION AND CONCLUSION

In this study, the analytical solution of local buckling problem of fiber reinforced plastic twin-web section compression members is derived.

Using presented equation, graphical form of results was suggested so that the engineers can design fiber reinforced plastic twin-web section column more easily.

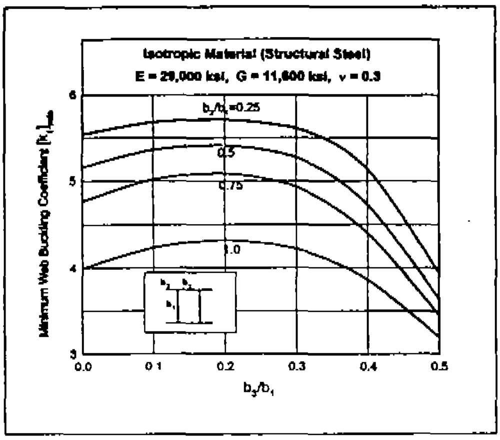

Fig. 4 Minimum Web Buckling Coefficient for Isotropic Material

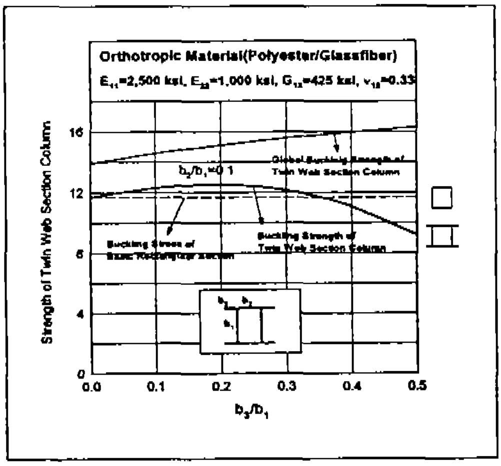

Fig. 5 Effect of Outstanding Flange

In equation derivation, all plate components assumed to have the same thickness, which is normal for practical sections of this type.

To verify the solution developed, the graphical form of minimum web buckling coefficient k_1 of centrally loaded isotropic twin-web section column, whose Poisson's ratio $\nu=0.3$, is calculated as shown in Fig. 5. Identical results given by Bulson(1969)[5] are obtained. As shown in the figure, as the ratio(b_3/b_1) is increased, the critical stress is also increased. The critical stress is maximum at $b_3/b_1 = 0.15 \sim 0.20$. When the width of outstanding portion of the flange is increased, the buckling strength falls under that of basic rectangular box section. Therefore, it is adverse if the outstanding flange width is wider than one-third of the web depth.

In order to establish the complete set of design criteria relating to the local buckling behavior of twin-web section column, linearly varying edge loading condition must also be investigated so that the behavior under eccentric edge loading could be considered.

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