Dynamic Robust Path-Following Using A Temporary Path Generator
for Mobile Robots with Nonholonomic Constraints

"Seunghy Lee", and Jongguk Yim", and Jonghun Park"

* School of Mechanical Eng., Hanyang University, Sungdong-Ku Seoul, 133-791, Korea
(Tel: 82-2-2297-3786; Fax: 82-2-2298-4634; E-mail: lsh123@hanyang.ac.kr)
** School of Mechanical Eng., Hanyang University, Sungdong-Ku Seoul, 133-791, Korea
(Tel: 82-2-2297-3786; Fax: 82-2-2298-4634; E-mail:jgyim@hanmail.com)
*** Department of Mechanical Eng., Hanyang University, Sungdong-Ku Seoul, 133-791, Korea
(Tel: 82-2-2290-0435; Fax: 82-2-2298-4634; E-mail: jongpark@email.hanyang.ac.kr)

Abstract

The performance of dynamic path following of a wheeled mobile robot with nonholonomic constraints has some drawbacks such as the influence of the initial state. The drawbacks can be overcome by the temporary path generator and modified output. But with the previous input-output linearization method using them, it is difficult to tune the gains, and if there are some modeling errors, the low gain can make the system unstable. And if a high gain is used to overcome the model uncertainties, the control inputs are apt to be large so the system can be unstable. In this paper, an $H_{\infty}$ controller is designed to guarantee robustness to model parameter uncertainties and to consider the magnitude of control inputs. And the solution to Hamilton Jacobi (HJ) inequality, which is essential to $H_{\infty}$ control design, is obtained by nonlinear matrix inequality (NLMI).

1. Introduction

In the previous research, the temporary path generator and a modified output function to overcome the drawbacks of the dynamic path following were proposed [9]. In that paper, the controller is designed with the input-output linearization method and the robustness of the controller is slightly mentioned. If the gains of the controller are large, the controller can overcome the model uncertainties, but the control inputs are apt to be large so that the system can be unstable. To overcome such problems, there have been several researches on the stabilizing problem of the nonholonomic wheeled mobile robot when there are model uncertainties [6,7].

In this paper, an $H_{\infty}$ controller is designed to guarantee robustness to model parameter uncertainties and to consider the magnitude of control inputs. The $H_{\infty}$ controller is derived by the $L_2$ -gain analysis based on the concept of the energy dissipation. And the Hamilton Jacobi inequality, which is the consequence of the derivation and is essential to $H_{\infty}$ control design, is transformed to nonlinear matrix inequality (NLMI). Although solving the NLMI yields a convex optimization problem, this is not finite dimensional as the linear case. However, if the matrices forming the NLMI are bounded, then we only need to solve a finite number of LMIs [8].

This paper is organized as follows. Section 2 presents the dynamic equation and input-output linearization of the wheeled mobile robot system. Section 3 presents the temporary path generator and the modified output for dynamic path following. Section 4 presents the robust control algorithm. In Section 5, simulations for 2-wheeled nonholonomic mobile platform are discussed. In Section 6, we present our conclusions.

2. Dynamic Path Following

![Fig. 1. Schematic of the wheeled mobile platform.]

We consider a wheeled mobile platform whose schematic top view is shown in Figure 1. If we ignore the passive wheels, the configuration of the platform can be described by four generalized coordinates [9]

$$q = [x, y, \theta, \dot{\theta}]^T$$

(1)

The dynamic equation of the system [2] is described by

$$M(q)\ddot{q} + C(q, \dot{q}) = E(q) - A^T(q)\lambda,$$

(2)

where $\lambda$ is the vectors of Lagrange multiplier for nonholonomic constraints. Eq. (2) can be represented in the state space form of

$$\dot{x} = f(x) + g(x)(\tau + w),$$

(3)

where $x = [x, y, \theta, \dot{\theta}, \eta, \eta_1]^T$,

$$f(x) = \begin{bmatrix} S\eta \\ -[S' M S\eta + S' C] \end{bmatrix}^T,$$

$$g(x) = \begin{bmatrix} 0 \\ (S'M)^{-1} \end{bmatrix}, \eta = \begin{bmatrix} \eta \\ \eta_1 \end{bmatrix}, \eta_1 = \begin{bmatrix} \dot{\theta} \\ \dot{\theta}_1 \end{bmatrix},$$

$$S = \begin{bmatrix} c\beta \cos \phi & c\beta \cos \phi & c\beta \cos \phi & c\beta \sin \phi \\ c\beta \sin \phi & c\beta \sin \phi & c\beta \sin \phi & c\beta \sin \phi \\ 1 & 0 & 0 & 1 \end{bmatrix} : \text{columns are in the null space of } A(q)$$

and $w$ is the disturbance caused by model uncertainties.

For simplification, suppose

515