Fault Tolerant Control of Magnetic Bearings

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ABSTRACT

Fault tolerant control algorithm for heteropolar magnetic bearings are presented. This fault tolerant control utilizes grouping of currents as C-cores in order to isolate magnetic fluxes. Hardware requirements to maintain fault tolerant control are reduced since decoupling chokes are not required in this control scheme. The currents supplied to each pole are redistributed, if some coils fail suddenly, such that the resultant magnetic forces should remain invariant through coil failure events. Load capacity before magnetic saturation is reduced through coil failures while maintaining the same magnetic forces before and after failure.

Nomenclature

\( a \): pole face area
\( g_0 \): nominal air gap
\( q \): number of active poles
\( n \): number of coil turns
\( \mu_0 \): permeability of air
\( \theta \): pole face angle
\( i_j \): current inputs
\( x, y \): rotor displacements

1. Introduction

Magnetic bearings are filling a greater number of applications in industry since they have many advantages over conventional fluid film or rolling element bearings, such as lower friction losses, free of lubrication, operation at temperature extremes, quiet operation, and high speeds. Magnetic suspension produces active damping and stiffness which arises from the control action, so system parameters can be designed to avoid resonance or for optimum damping through the resonances while in operation. Highly critical applications of these machinery elements may demand a fault-tolerant control strategy. Fault-tolerant control of magnetic bearing system provides continued operation of the bearing even if its power amplifiers or coils suddenly fail. The goal of the present work is to develop a fault-tolerant control algorithm such that bearing actuators can preserve the same magnetic forces even after some components such as coils or power amplifiers fail.

Fault-tolerant actuators have been investigated by several researchers. Maslen and Meeker [1, 2] showed that a magnetic bearing with multiple coil failure can produce decoupled control forces if the remaining coil currents are properly redistributed. The fault-tolerant control can be maintained for an 8-pole magnetic bearing including material path reluctances for up to 5 coils out of 8 failed [3, 4]. The fault-tolerant scheme utilizing the grouping of currents reduces the required number of controller outputs and decoupling chokes [5]. This fault tolerance may reduce load capacity of the bearing because the redistribution of currents to the failed bearing may lead to saturation in the bearing material.

The present work utilizes the bias voltage linearization to determine the redistribution of the remaining coil currents such that the same linearized magnetic forces are preserved even after the magnetic bearing actuator experiences failure.

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2. Bearing Model

Magnetic forces are determined from magnetic flux density and may be reduced by flux leakage, fringing, saturation of magnetic material, and eddy current effects. If eddy current effects are negligible, and the flux density is linear with magnetomotive force, Maxwell’s equations can be fairly accurately approximated by 1D magnetostatic relations. A 16-pole heteropolar radial magnetic bearing with 8 C-core type coil windings are shown in Figure 1.

Fig. 1 The 16-Pole Heteropolar Magnetic Bearing

The flux density vector in the air gap is described as:

$$ B = V(x, y) I $$

where the current vector is:

$$ I = [i_1, i_2, \ldots, i_8]^T $$

The magnetic forces along the $\varphi$ direction are then related to current inputs and rotor displacements as:

$$ f_\varphi = -I^T Q_\varphi(x, y) I, \varphi = x \text{ or } y $$

where

$$ Q_\varphi(x, y) = V^T \frac{\partial D}{\partial \varphi} V $$

$$ D = \text{diag}(g_j(x, y) a/(2\mu_0)) $$

and where the air gap is described as:

$$ g_j = g_0 - x \cos \theta_j - y \sin \theta_j $$

The current inputs to each pole are generally expressed with a linear combination of a bias voltage $v_b$, and control voltages $v_{cr}$ and $v_{cy}$. The current vector of an 16-pole magnetic bearing is defined as:

$$ I = Tv $$

where the distribution matrix is:

$$ T = [T_b, T_s, T_p] $$

and the voltage vector is:

$$ v = [v_b, v_{cr}, v_{cy}]^T $$

If some coils fail, the full (8x1) current vector is related to the reduced current vector by introducing a failure map matrix $H$.

$$ I = Hv $$

For example, if the 4-5-7th coils fail, $H$ matrix is:

$$ H = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 
\end{bmatrix} $$

The reduced current vector is expressed as:

$$ \hat{I} = \hat{T} v $$

where the reduced distribution matrix $\hat{T}$ is defined as:

$$ \hat{T} = [\hat{T}_b, \hat{T}_s, \hat{T}_p] $$

where

$$ \hat{T}_b = [\hat{t}_1, \hat{t}_2, \ldots, \hat{t}_8]^T, $$

$$ \hat{T}_s = [\hat{t}_{4+1}, \hat{t}_{4+2}, \ldots, \hat{t}_{4+8}]^T, $$

$$ \hat{T}_p = [\hat{t}_{8+1}, \hat{t}_{8+2}, \ldots, \hat{t}_{8+9}]^T $$

The unknown distribution matrix $\hat{T}$ is determined to generate the desired magnetic forces. The magnetic forces are controlled with a combination of 8 currents one for each pole in normal operation. If some coils fail, the remaining coils must provide the magnetomotive forces to generate the desired magnetic forces.
3. Fault Tolerant Distribution of Currents

The general magnetic forces including a distribution gain matrix are:

\[ f_\varphi = v^T M_\varphi v \]  

(7)

where

\[ M_\varphi = \hat{T}^T H^T Q_\varphi(x,y) \hat{H} \hat{T} \]  

(8)

The magnetic forces are quadratically dependent on the voltage vector \( v \). The bias voltage \( v_b \) is adjusted in a manner that maximizes the load capacity of the magnetic bearing. The magnetic forces in Eq. (7) are linearized about the bearing center position and about the bias voltage \( v_b \). The linearized magnetic forces are then:

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = - \begin{bmatrix}
k_{\varphi x} & k_{\varphi y} \\
k_{\varphi y} & k_{\varphi y}
\end{bmatrix} \begin{bmatrix}
x \\
y
\end{bmatrix} + \begin{bmatrix}
k_{\varphi x} & k_{\varphi y} \\
k_{\varphi y} & k_{\varphi y}
\end{bmatrix} \begin{bmatrix}
v_x \\
v_y
\end{bmatrix}
\]  

(9)

The position related force coefficients (position stiffnesses) are calculated as

\[ k_{\varphi x y} = - \frac{\partial f_\varphi}{\partial x} = -T_b^T Q_{\varphi x y} T_b v_y^2, \]  

(10)

and the voltage related force coefficients (voltage stiffnesses) are:

\[ k_{\varphi x y} = \frac{\partial f_\varphi}{\partial v} = -2T_b^T Q_{\varphi x y} T_b v_y, \]  

(11)

where the parameters \( \varphi \) and \( \omega \) both represent \( x \) or \( y \) direction.

Though some coils fail, employing an optimal current distribution \( T \) may decouple the linearized forces, and even maintain the same decoupled magnetic forces as those of an unfailed magnetic bearing. The distribution of currents can be designed as a part of control law so that the effects of coil failure can be very much mitigated with control action. The distribution matrix \( T \) is determined such that \( M_\varphi \) should remain invariant through coil failures.

There may exist multiple candidates of \( \hat{T} \)'s that satisfy the decoupling conditions. The criterion for choosing the best candidate is the one that will yield the maximum load capacity prior to any saturation. To accomplish this a distribution matrix \( \hat{T} \) can be determined by using the Lagrange Multiplier method to minimize the Euclidean norm of the flux density vector \( B \). The cost function is defined as:

\[ J = B(\hat{T})^T P B(\hat{T}) \]  

(13)

where the diagonal weighting matrix \( P \) is also selected to maximize the load capacity.

The Lagrange Multiplier method is then used to solve for the \( T \) that satisfies Eq. (13). Define:

\[ L(\hat{T}) = B(\hat{T})^T P B(\hat{T}) + \sum_l \lambda_l h_l(\hat{T}) \]  

(14)

Partial differentiation of Eq. (14) with respect to \( t_i \) and \( \lambda_l \) leads to nonlinear algebraic equations to solve for \( t_i \) and \( \lambda_l \).

The system of nonlinear algebraic equations can be solved for the distribution matrix \( \hat{T}(t_i) \). A least square iterative method was used to solve the system of nonlinear algebraic equations, which yields multiple solutions (local optima). Various initial guesses were tested until they converged to the solution within tolerable errors.

Some examples of distribution matrices for the 16-pole heteropolar magnetic bearing with 8 C-core groupings are calculated with the bearing properties of \( \varepsilon_0 (0.508 \text{ mm}), a (602 \text{ mm}^2), n (50 \text{ turns}) \). A typical distribution matrix for an unfailed bearing is:

\[
T = \begin{bmatrix}
1 & 1 & 0 \\
1 & 0 & 1 \\
-1 & 0 & -1 \\
-1 & 1 & 0 \\
1 & -1 & 0 \\
1 & 0 & -1 \\
-1 & 0 & 1 \\
-1 & -1 & 0
\end{bmatrix}
\]  

(15)

The calculated distribution matrix for the 8th coil failed bearing is:
\[
T_8 = \begin{bmatrix}
1.2135 & 1.0078 & 0.5893 \\
0.6907 & 0.7636 & 0.2509 \\
1.0992 & -0.4251 & 1.1080 \\
0.6762 & -0.3080 & 0.0252 \\
1.4549 & -1.1658 & -0.5204 \\
0.2842 & 1.0320 & 0.7136 \\
1.8386 & 0.7919 & -1.5185 \\
0 & 0 & 0
\end{bmatrix}
\]

(16)

and the 7th-8th coils failed bearing is:

\[
T_{78} = \begin{bmatrix}
1.3615 & 2.3645 & -0.6692 \\
-0.0127 & 0.4213 & -2.0599 \\
1.2117 & -0.1963 & 0.3506 \\
1.2158 & -0.3399 & 0.2511 \\
-0.0451 & 2.6952 & -0.5881 \\
1.8523 & 0.8237 & -3.0404 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(17)

and the 6th-7th-8th coils failed bearing is:

\[
T_{678} = \begin{bmatrix}
1.2149 & 0.9906 & 0.0413 \\
0.9973 & 0.1966 & 0.5765 \\
1.2670 & -0.1684 & 0.7603 \\
1.1128 & -0.9911 & 0.2180 \\
1.0113 & 0.1621 & -0.3595 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(18)

and the 2nd-4th-6th-7th-8th coils failed bearing is:

\[
T_{24678} = \begin{bmatrix}
1.8386 & 1.2162 & 0.6319 \\
0 & 0 & 0 \\
1.6047 & -0.4186 & 0.9119 \\
0 & 0 & 0 \\
-0.1762 & 1.2695 & 0.7494 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

(19)

The distribution matrices in Eqs. (15)-(19) are calculated at the rotor off-centered position. The rotor is assumed to be spinning at the equilibrium of 0.2g0. The linearized forces in Eq. (9) with any of the unfailed or failed distribution matrices of Eqs. (15)-(19), and \( v_s \) of 5 result in \( k_{pzz} = -3076200 \ N/m, \)

\( k_{pyy} = -3421300 \ N/m, \)

\( k_{pxx} = k_{pxx} = 0 \ N/m, \)

\( k_{pyy} = 271 \ N/volt, \)

\( k_{pxx} = 293.5 \ N/volt, \) and

\( k_{pyy} = k_{pxx} = 0 \ N/volt. \) This shows that the magnetic forces remain unchanged throughout a failure event even if the distribution matrix for a specific failure is utilized.

The fault-tolerant control scheme with the PD control law is illustrated in Fig. 2. The \( T \) 's can be calculated and stored in the database for all possible combinations of failure. Coil or power amplifier failure can be detected with the current sensors installed on all coils. If the failure status vector is determined from measurements, the corresponding \( T \) will be searched from the data base and replace the existing \( T \).

4. Conclusion

The previous works on the fault-tolerant magnetic bearing utilizes the flux coupling property of heteropolar magnetic bearings. However, flux coupling with independent currents results in an electromagnetic stability problem. To remedy this stability problem, an additional hardware of decoupling choke is required. The fault-tolerant scheme proposed in this paper utilizes flux isolation between adjacent C-core groupings. Fluxes can be isolated for the magnetic bearing when coils are wound in series with a second pole of opposite polarity. No decoupling chokes are required for this proposed fault-tolerant scheme. The fault-tolerant scheme in this
paper also shows that the bearing actuator will preserve the same linearized forces even after some components such as coils or power amplifiers experience failure. The overall load capacity of the bearing is reduced as coils fail. The same magnetic forces are then preserved only up to the load capacity of the failed bearing. Disturbance level from imbalance, runouts, and sideloads should be maintained at low levels to prevent premature magnetic saturation for this fault–tolerant operation of magnetic bearings.

References


