특별직교 이방성 판 이론에 의한 교량 상부구조 설계

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Bridge Superstructures Design by Special Othotropic Plate Theory

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ABSTRACT

The Special orthotropic plate theory is used for analysis of panels made of steel girders and cross-beams, and made of reinforced concrete. The cross-sections of girders and cross-beams are WF types. The result is compared with that of the beam theory. According to the numerical examination given in this paper, the result by the plate theory is 2.43 times stiffer than that of beam theory. The result for the concrete slab in given for the practicing engineers.

Key Word : Special orthotropic plate theory, F.D.M, Navier and Levy Solution.

1. Introduction

There are several means for steel slab system analysis such as

a. beam strip method,
b. composite beam theory between concrete slab and steel beams, and
c. grid analysis method for cross beams and girders.

The 3.1 elevated expressway in seoul, designed and built in 1967, used less than half of steel required by other best design, at that time [5]. The methods used were,

a. grid analysis,
b. composite action,
c. use of welding,
d. use of hybrid materials,
e. use of high tension bolt, and others.

In this reference, several existing design methods are studied and compared. An extensive references are also given.

Many of the bridge and building floor systems, including the girders and cross-beams, and concrete decks behave as the special orthotripic plates which have [0°,90°,0°], fiber orientations.

2. Method of Analysis Used

The equilibrium equation for the special orthotropic plate is :

$$D_1\frac{\partial^4 w}{\partial x^4}+2D_3\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4}=q(x,y) \qquad (1)$$

where $D_1=D_{11}, D_2=D_{22}, D_3=D_{12}+2D_{66}$

The assumptions needed for this equation are:

* The transvers shear deformation is neglected.

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* Specially orthotropic layers are arranged so that no coupling terms exist, i.e.,

  $B_{ij}=0, (\ )_{16}=(\ )_{26}=0.$

* No temperature or hygrothermal terms exist.

The purpose of this paper is to demonstrate, to the practicing engineers, how to apply this equation to the slab systems made of plate girders and cross-beams, and of reinforced concrete.

In case of an orthotropic plate with boundary conditions other than Navier or Levy solution types, or with irregular cross section, or with nonuniform mass including point masses, analytical solution is very difficult to obtain. Numerical method for eigenvalue problems are also very much involved in seeking such a solution. Finite difference method (F.D.M) is used in this paper. The resulting linear algebraic equations can be used for any case with minor modifications at the boundaries, and so on.

3. F.D.M

Since no reliable analytical method is available for the subject problem, F.D.M. is applied to the governing equation of the special orthotropic plates.

The number of the pivotal points required in the case of the order of error $\triangle^2$, where $\triangle$ is the mesh size, is five for the central differences. This makes the procedure at the boundaries complicated. In order to solve such problem, the following three simultaneous partial differential equations of equilibrium with three dependent variables, w, Mx, and My, are used instead of the ordinary partial differential equation for the bending of the special orthotropic plate.

$$D_{11}\frac{\partial^2 Mx}{\partial x^2}-4D_{66}\frac{\partial^4 \omega}{\partial x^2 \partial y^2}+D_{22}\frac{\partial^2 My}{\partial y^2}$$
$$=-q(x,y)+kw(x,y)$$
(1)

$$M_x= -D_{11}\frac{\partial^2 w}{\partial x^2} -D_{12}\frac{\partial^2 w}{\partial y^2} \quad (2)$$

$$M_y= -D_{12}\frac{\partial^2 w}{\partial x^2} -D_{22}\frac{\partial^2 w}{\partial y^2} \quad (3)$$

If F.D.M. is applied to these equations, the resulting matrix equation is very large in sizes, but the tridiagonal matrix calculation scheme used by Kim D. H.[4] is very efficient to solve such equations.

In order to confirm the accuracy of the F.D.M., $[A/B/A]_r$ type laminate with aspect ratio of a/b=1m/1m=1 is considered. The material properties are :

$E_1=67.36\ GPa,\ E_2=8.12\ GPa,$

$\nu_{12}=0.272,\ \nu_{21}=0.0328,\ G_{12}=3.0217\ GPa,$

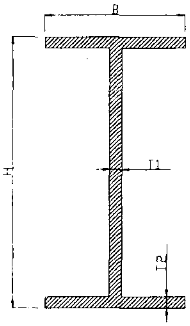
The thickness of a ply is 0.005m. As the r increases, B_{16}, B_{26}, D_{16}, and D_{26} decrease and the equations for special orthotropic plates can be used. For simplicity, it is assumed that $A=0^0$, $B=90^0$, and r=1. Then D(1,1)=18492.902 Nm. Since one of the few efficient analytical solutions of the special orthotropic plate is Navier solution, and this is good for the case of the four edges simple supported, F.D.M. is used to solve this problem and the result is compared with the Navier solution.

The mesh size is Δx=a/10=0.1m, Δy=b/10 =0.1m. The deflection at(x,y), under the uniform load of 100N/m², the origin of the coordinates being at the corner of the plate, is obtained, and the ratio of the Navier solution to the F.D.M solution is 1.005~1.00028.

4. Numerical Example

4.1 Steel Slab

The slab structure under construction is as given in Figure 1. The stiffnesses are given in Table 1.



Es=2040000 kgf/㎠

υ =0.3

S=2m

	Girder(mm)	Cross-Section (mm)
H_L	500	300
B_L	200	150
$T1_L$	15	10
$T2_L$	20	18

Fig 1.1 Cross-Section.

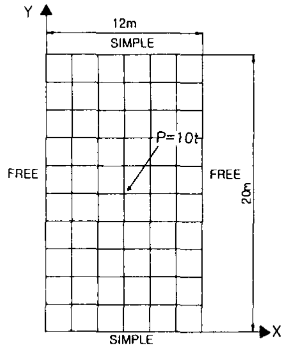


Fig 1.2 Girders and Beams.

Fig.1 Structure under consideration.

D_{ij} (N·m)	Plate	Beam
D_{11}	101199927.65	101199927.65
D_{22}	21757837.94	0.00

Table 1. Stiffness

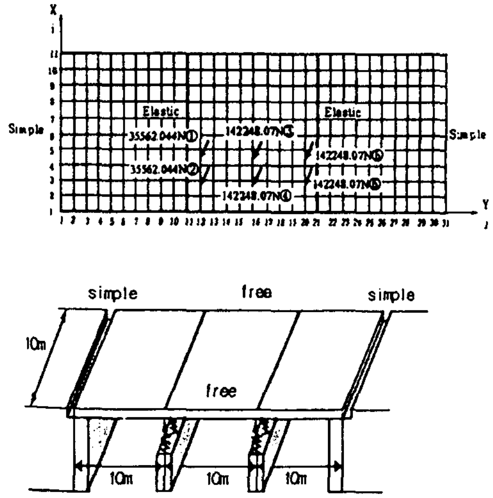
Analysis is carried out and the result is given in Table 2.

	Plate	Beam	Plate/Beam
δ(m)	0.6765E-01	0.1646E+00	2.43

Table 2. Deflection at the center(m)

4.2 Reinforced Concrete Slab

The bridge is as shown in Figure 2 and 3.



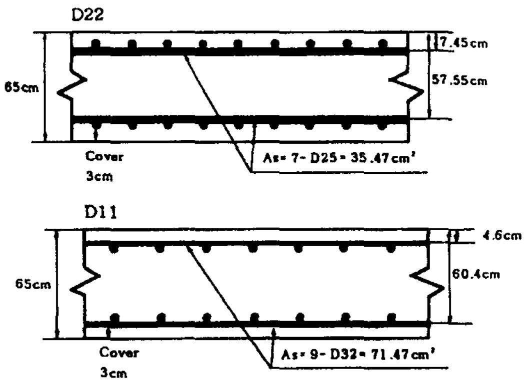


Figure 2. Concrete Slab Bridge.

Figure 3. Cross Section of the Slab with Unit Width.

Figure 3 shows the cross section of the slab with unit width.

$\sigma_{ck} = 210 kg/cm^2 = 20.5942926 MPa$ and

$E_c = 15000\sqrt{\sigma_{ck}} = 21.317118060 GPa.$

Poissons ratio $\nu_{12} = \nu_{21} = 0.18$ for concrete.

The stiffness and deflections are given in Tables 3 and 4.

stiffness	N·m
D_{11}	323428383.7
D_{22}	151828300.8
D_{12}	90690632.4
D_{66}	206573097.2

Table 3. Flexural stiffnesses. (N·m)

load point	dflection (m)
1	0.2955E-03
2	0.2458E-03
3	0.2300E-02
4	0.2054E-02
5	0.4155E-03
6	0.3504E-03

Table 4. The deflections at wheel loading points(m).

5. Conclusion

In this paper, results of analysis for design of both plate girders and reinforced concrete slabs for bridges are presented. It is concluded that the existing design methods with beam strip concept gives us too far off results from the safe and economic bridges.

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