Ultrasonic Phase Velocity and Attenuation Coefficient Predicted by Biot's Theory and the MBA Model in Cancellous Bone

Kang Il Lee and Suk Wang Yoon

BK21 Physics Research Division and Institute of Basic Science,

Department of Physics, SungKyunKwan University, Suwon 440-746, Republic of Korea

acustica@skku.edu

Abstract

Biot's theory and a modified Biot-Attenborough (MBA) model are applied to predict the dependences of acoustic characteristics on frequency and porosity in cancellous bone. The phase velocity and the attenuation coefficient predicted by both theories are compared with previous *in vitro* experimental measurements in terms of the mixed, the fast, and the slow waves. Biot's theory successfully predicts the dependences of phase velocity on frequency and porosity in cancellous bone, whereas a significant discrepancy is observed between predicted and measured attenuation coefficients. The MBA model is consistent with reported measurements for both dependences of phase velocity and attenuation coefficient on frequency and porosity. Based on the theoretical predictions from the MBA model, it is suggested that the attenuation coefficient of the mixed wave is dominated by the fast wave in the low-porosity region while it is dominated by the slow wave in the high-porosity region. This provides a qualitative explanation for the nonlinear relationship of attenuation of the mixed wave with porosity in cancellous bone.

1. Introduction

The theoretical modeling of acoustic wave propagation in cancellous bone is very important in understanding interactions between acoustic wave and cancellous bone and in improving quantitative ultrasound techniques for the diagnosis of osteoporosis. Without the help of such models, quantitative ultrasonic assessment of cancellous bone will remain an empirical pursuit, founded on observation rather than theory. Recently, Lee *et al.* [1] applied the MBA model to cancellous bone and found excellent agreement between measured and predicted ultrasonic properties in bovine cancellous bone specimens with trabeculae aligned in the perpendicular direction to propagation at normal incidence. Since the fast and the slow waves were completely overlapped in this perpendicular direction and were observed as if one longitudinal wave propagates, theoretical predictions were performed in terms of the mixed wave overlapped with two waves. The MBA model certainly shows promise and needs to be explored in greater depth both theoretically and experimentally. More works are required to compare its predictions with other modeling approaches and to determine more exact values for the parameters used within the theory. There should be also further work in terms of the fast and the slow waves to increase our understanding of the acoustic behavior of cancellous bone.

To this purpose, Biot's theory and the MBA model are applied to predict the dependences of acoustic characteristics on frequency and porosity in cancellous bone. The phase velocity and the attenuation coefficient predicted by both theories are compared with previous *in vitro* experimental measurements in terms of the mixed, the fast, and the slow waves.

2. Results

Figure 1 shows the experimental and theoretical phase velocities of the mixed wave in the perpendicular direction to the trabeculae at a porosity of 0.81 as functions of frequency. Theoretical predictions were performed in terms of the mixed wave overlapped with the fast and the slow waves using Biot's theory and the MBA model, respectively. The frequency-dependency of phase velocity is essentially the same for both theories. For comparison, the measurement in one bovine cancellous bone specimen with a porosity of 0.81 reported by Lee *et al.* [1] is depicted in Fig. 1. Both experimental and theoretical phase velocities of the mixed wave in cancellous bone can be approximated as nondispersive in the frequency range of 0.5-1 MHz.

The experimental and theoretical phase velocities of the fast and the slow waves in the parallel direction to the trabeculae at a porosity of 0.81 are shown as functions of frequency in Fig. 2. The measurement was performed on one bovine cancellous bone specimen with a porosity of 0.81 (bone volume fraction of 0.19) by Hosokawa and Otani [2]. The phase velocities of both fast and slow waves are almost constant and nondispersive in the frequency range of 0.5-1 MHz.

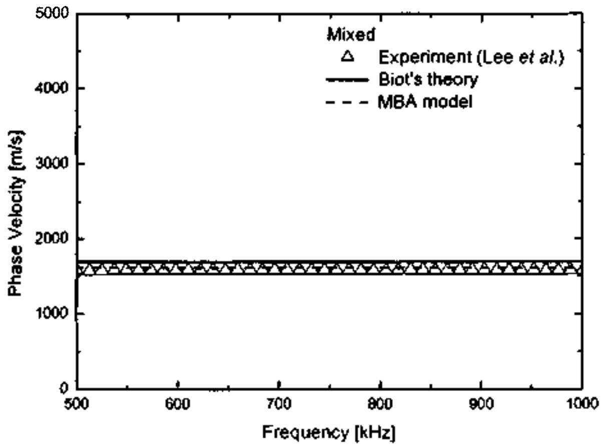

Figure 1. Experimental and theoretical phase velocities of the mixed wave in the perpendicular direction to the trabeculae at a porosity of 0.81 as functions of frequency.

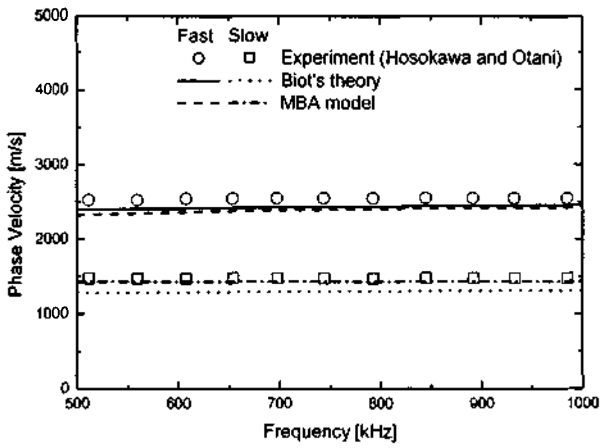

Figure 2. Experimental and theoretical phase velocities of the fast and the slow waves in the parallel direction to the trabeculae at a porosity of 0.81 as functions of frequency.

Figure 3 shows the experimental and theoretical phase velocities of the mixed wave in the perpendicular direction to the trabeculae at a frequency of 1 MHz as functions of porosity. Experimental data were measured at 1 MHz with 10 bovine cancellous bone specimens by Hosokawa and Otani [3], and with 12 specimens by Lee *et al.* [1]. The theoretical phase velocity of the mixed wave varies from 4000 to 1450 m/s as the porosity increases. No data for the slow wave were obtained for propagation in the perpendicular direction because the fast waveform completely overlaps the slow waveform.

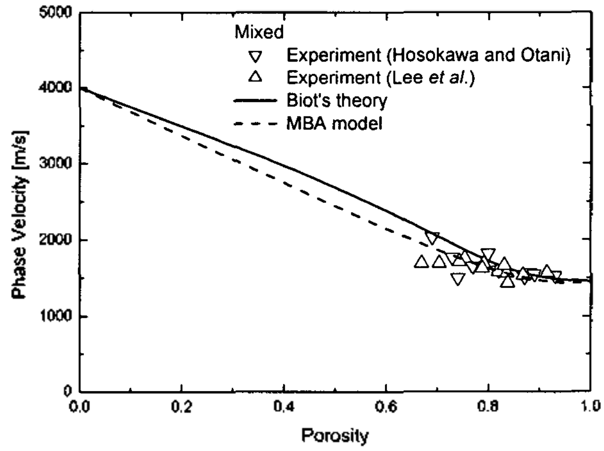

Figure 3. Experimental and theoretical phase velocities of the mixed wave in the perpendicular direction to the trabeculae at a frequency of 1 MHz as functions of porosity.

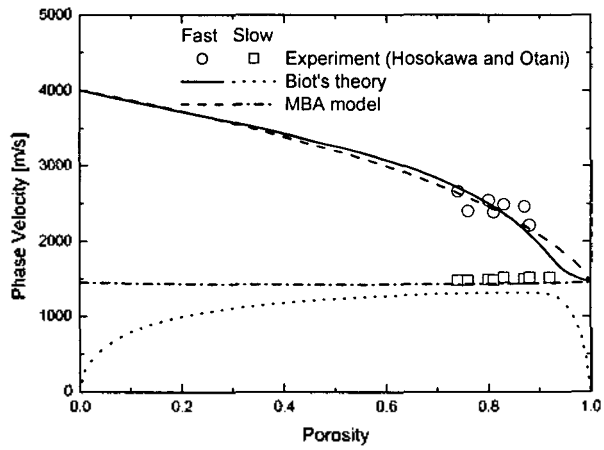

Figure 4. Experimental and theoretical phase velocities of the fast and the slow waves in the parallel direction to the trabeculae at a frequency of 1 MHz as functions of porosity.

The experimental and theoretical phase velocities of the fast and the slow waves in the parallel direction to the trabeculae at a frequency of 1 MHz are shown as functions of porosity in Fig. 4. The experimental measurement was reported by Hosokawa and Otani [2]. Both experimental and theoretical phase velocities of the fast wave decrease as the porosity increases. The experimental phase velocity of the slow wave remains almost constant at about 1500 m/s as predicted by the MBA model, which is close to the sound speed of 1450 m/s in bone marrow. On the other hand, the phase velocity of the slow wave predicted by Biot's theory slowly increases with porosity and rapidly decreases after the porosity of 0.9. The phase velocity 2200-2700 m/s of the fast wave is much slower than the sound speed 3400-4200 m/s of cortical bone (solid bone). This can be explained by the fact that the cancellous bone is not solid but has a porous network structure. In the porosity range of 0.7-0.9, the phase velocity of the fast wave propagating in the parallel direction is much faster than that in the perpendicular direction.

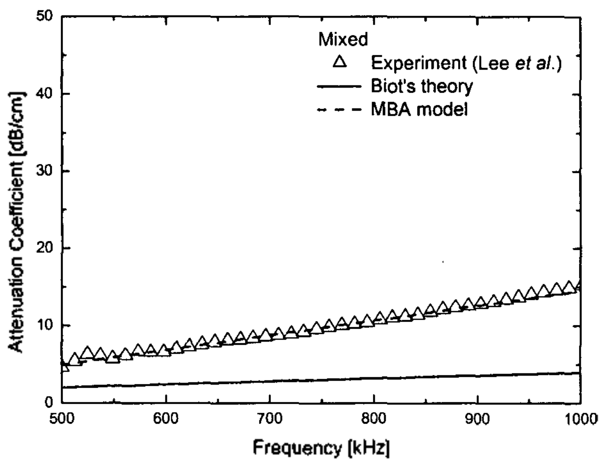

Figure 5. Experimental and theoretical attenuation coefficients of the mixed wave in the perpendicular direction to the trabeculae at a porosity of 0.81 as functions of frequency.

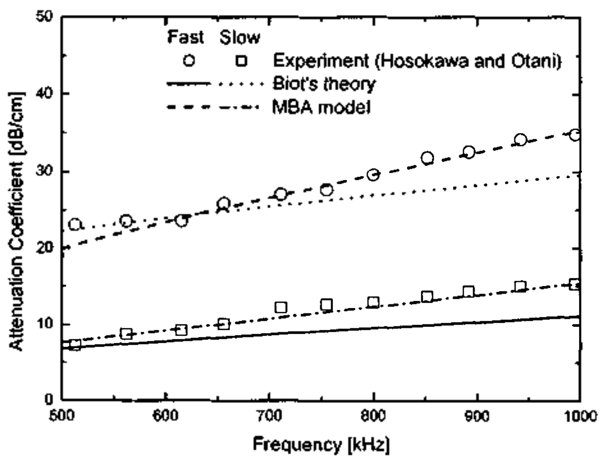

Figure 6. Experimental and theoretical attenuation coefficients of the fast and the slow waves in the parallel direction to the trabeculae at a porosity of 0.81 as functions of frequency.

Figures 5 and 6 show the experimental and theoretical attenuation coefficients of the mixed wave in the perpendicular direction to the trabeculae at a porosity of 0.81 as functions of frequency and those of the fast and the slow waves in the parallel direction, respectively. In the MBA model, the frequency variation of attenuation coefficient for the solid bone has been theoretically modeled with a simple linear relationship between attenuation and frequency, $\alpha(f) = \alpha_0 + \alpha_1 f = -50.7 + 2.1\times10^{-4} f$ over the frequency range of 0.5-1 MHz. The constant α_0 has been introduced to account for the transmission loss and to avoid any assumption about the frequency variation of attenuation outside the measured frequency bandwidth. As can be seen from Figs. 5 and 6, the experimental attenuation coefficients of the three waves linearly increase with frequency in the frequency range of 0.5-1 MHz. For the mixed wave in Fig. 5, although the trends predicted by both theories are similar to the experimental observation of Lee *et al.* [1], the theoretical values predicted by Biot's theory are considerably lower than the experimental values. Furthermore, in Fig. 6, the attenuation coefficient of the fast wave predicted by Biot's theory is much lower than that of the slow wave, which shows the opposite trend to the measurement of Hosokawa and Otani [2]. This could be because Biot's theory considers the absorption due to only viscous loss at internal interfaces, but the experimental measurements record the signal loss due to other mechanisms as well.

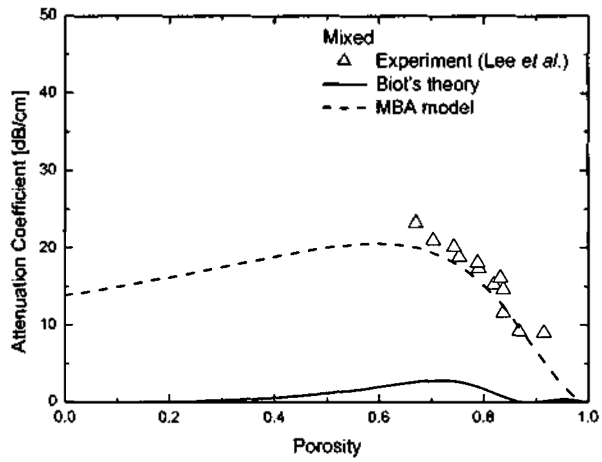

Figure 7. Experimental and theoretical attenuation coefficients of the mixed wave in the perpendicular direction to the trabeculae at a frequency of 1 MHz as functions of porosity.

Figure 7 shows the experimental and theoretical attenuation coefficients of the mixed wave in the perpendicular direction to the trabeculae at a frequency of 1 MHz as functions of porosity. The experimental measurement of Lee *et al.* [1] is in good agreement with the theoretical prediction from the MBA model, but not with ones from Biot's theory. The MBA model predicts the nonlinear dependence of attenuation on porosity in cancellous bone. As the porosity increases, the attenuation coefficient of the mixed wave predicted by the MBA model slowly increases and rapidly decreases after reaching maximum at the porosity of 0.6. It is well known that this nonlinear behavior is the general feature of fluid-saturated porous media.

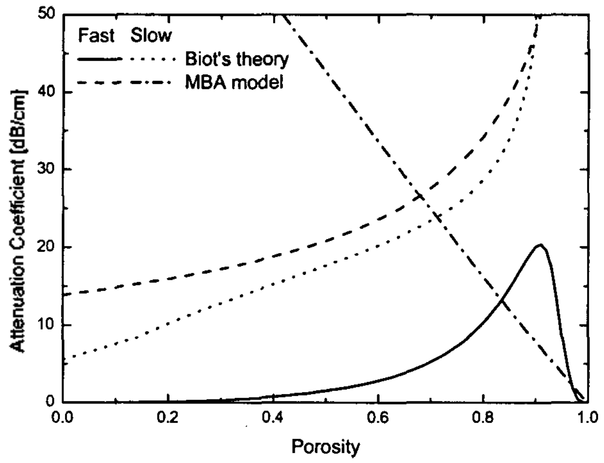

Figure 8. Theoretical attenuation coefficients of the fast and the slow waves in the parallel direction to the trabeculae at a frequency of 1 MHz as functions of porosity.

The theoretical attenuation coefficients of the fast and the slow waves in the parallel direction to the trabeculae at a frequency of 1 MHz are shown as functions of porosity in Fig. 8. Surprisingly, there is significant difference between theoretical predictions from Biot's theory and the MBA model for both fast and slow waves throughout the porosity range. Biot' theory predicts that the attenuation coefficient of the fast wave will slowly increase with porosity and rapidly decrease after reaching maximum at the porosity of 0.9, whereas it rapidly increases after the porosity of 0.9 for the slow wave. On the other hand, the attenuation coefficient of the fast wave predicted by the MBA model increases as the porosity increases, whereas it decreases with porosity for the slow wave. Based on the theoretical predictions from the MBA model in Figs. 7 and 8, it is suggested that the attenuation coefficient of the mixed wave is dominated by the fast wave in the low-porosity region while it is dominated by the slow wave in the high-porosity region. This provides a qualitative explanation for the nonlinear relationship of attenuation of the mixed wave with porosity in cancellous bone.

3. Conclusions

Biot's theory and the MBA model were applied to predict the dependences of acoustic characteristics on frequency and porosity in cancellous bone. The phase velocity and the attenuation coefficient predicted by both theories were compared with previous *in vitro* experimental measurements in terms of the mixed, the fast, and the slow waves. Biot's theory successfully predicted the dependences of phase velocity on frequency and porosity in cancellous bone. However, a significant discrepancy was observed between predicted and measured attenuation coefficients. This is partially due to the incomplete knowledge of the parameters in the theory, resulting in highly tunable implementation with inherent inconclusive validation. On the other hand, the MBA model is consistent with reported measurements for both dependences of phase velocity and attenuation coefficient on frequency and, in particular, on porosity. Both of these physical phenomena are closely related to diagnostic measurements in the current diagnosis of osteoporosis. Although the MBA model relies on phenomenological parameters that are themselves derived from the experimental data rather than the physics-based modeling, its approach to cancellous bone can be usefully employed in the field of clinical ultrasonic bone assessment.

Acknowledgements

This work was supported by the Postdoctoral Research Program of SungKyunKwan University (2004).

References

1. K. I. Lee, H. S. Roh, and S. W. Yoon, "Acoustic wave propagation in bovine cancellous bone: application of the modified Biot-Attenborough model," J. Acoust. Soc. Am. **114**, 2284-2293 (2003).
2. A. Hosokawa and T. Otani, "Ultrasonic wave propagation in bovine cancellous bone," J. Acoust. Soc. Am. **101**, 558-562 (1997).
3. A. Hosokawa and T. Otani, "Acoustic anisotropy in bovine cancellous bone," J. Acoust. Soc. Am. **103**, 2718-2722 (1998).