지하매설 복합재료 파이프의 비선형 해석

김 덕 현* · 이 인 원** · 한 봉 구*

Non-linear Analysis of Underground Laminated Composite Pipes

Duk-Hyun Kim[1], In-Won Lee[2], Bong-Koo Han[1]

Abstract

An analytical study is conducted using the Galerkin technique to determine the behaviour of thin fibre-reinforced composite pipes under soil pressure. Geometric nonlinearity and material linearity are assumed. It is assumed that the vertical and lateral soil pressures are proportional to the depth and the lateral displacement of the pipe respectively. It is also assumed that the radial shear stress is negligible because the ratio of the thickness to the radius of the pipe is very small. The calculation results are compared with the finite element analysis result.

Key Words: Composites pipes, Underground pipes, Nonlinearity

1. Introduction

Compared with metal pipes, composite pipes have the following advantages.

Corrosion resistance - Maintenance free
Lightweight - Economical to install
Excellent flow properties - Energysave
Paraffin build-up resistance
Scale build-up resistance
Nonconductive
Low thermal conductivity

In case of pipe construction, the cost of transporting pipes can be significant : it could be as mush as or more than the cost of pipes, depending on the length of the pipeline. This transportation effect could become the major concern of the project if the size of the pipes is large. Because of its lightweight nature, composite pipes can be extensively used for pipeline construction. When the pipe is buried, the surrouding soil and the pipe interact structurally, and the forces acting on the pipe are functions of the medium that surrouds it as well as the stiffness of the pipe itself. The contribution of the surrounding soil in resisting external loads can be very important and can provide considerable saving in pipe material. The conventional recommendations for design and construction of underground pipes are based on the extensive work of Marston and Spangler[1]. However, the use of this method for flexible pipes may cause high safety factors[2]. Earlier works on this subject by others include these of Meyerhof, Timmers, Brockenbrough, Valentine, et al. Costes suggests to use arching theory to reduce the soil load directly above the pipe. Richards and Agrawal applied Barjansky's tunnel solution by transforming the pipe section to an equivalent ring of soil. The loading on the cross section of the buried pipes is assumed to come from two sources: (A) The weight of backfill soil and any top loading present. The magnitude of this vertical

1. 서울산업대학교 구조공학과
2. 한국과학기술원

forces, q_v, is the weight of the backfill soil, modified by the effect of shear stress due to settlement of this backfill, plus the pressure due to any line load present, usually calculated by Boussinesq solution. (B) The force of the soil against the pipe sides which tends to prevent deformations caused by the (A) type forces (see Figure 1). The assumption generally made here is that this force, q_h, is proportional to the horizontal pipe deflection, δ_h, and can be expressed as $K\delta_h$ where K is called as subgrade reaction coefficient or modulus of the foundation. Defining the value of K correctly is rather complex. Rubin, using inextensional cylindrical shell theory, and assuming that the vertical deformation is approximately equal to the negative of the horizontal deformation, obtained the value of K as the function of rigidity and diameter of the pipe. However, according to Valentine's experiment, the magnitude of the horizontal deformation is between 0 to 80 percent of the vertical deformation. Furthermore, it has to be noted that K value is related to soil properties. Molin adds the horizontal earth pressure at rest to the Spangler's concept. This may be acceptable if the surrounding media is clay. However, it is general practice to put granular material around the pipes and to compact it. If the pipe deflects to "near" maximum under above condition, when the horizontal thrust from soil becomes significant, the $K\delta_h$ value can be assumed to be "close" enough to the passive earth pressure of the soil. Szyszkowski and Glockner reported the result of nonlinear analysis of buried aluminum tubes. The model used is similar to that of Spangler with $K\delta_h$ value as passive earth pressure. In this paper, underground pipes made of laminated composites are considered. Because of the flexible nature of thin composite walls, the problem is geometrically nonlinear. Assuming the pipe section under consideration is sufficiently "far" from the end or the bent, the problem is considered as that of plane strain. Equilibrium equations are obtained from the deformed shape and are solved by Galerkin's method. It is assumed that the vertical load on the pipe is proportional to the depth of the backfill and horizontal load as proportional to the horizontal deformation. Any modification of loading can be made easily depending on actual soil condition and live loads, if necessary. Given constant value of such surrounding condition, a composite pipe has different stiffness depending on number of layers and fiber directions and so on. This results in different amount of deflection and different soil-structure interaction. This paper presents a method of nonlinear analysis of underground composite pipes and the effect of the variable factors, such as fiber orientations and different values of subgrade reaction coefficients, on soil-structure interaction.

2. Analysis of Pipes

2.1 Strain and Displacement Relation

Equations of strain and displacement relation with respect to orthogonal curvilinear coordinates are as follows.

$$\varepsilon_x = \frac{1}{\alpha}\Big[u_x + \frac{\alpha_y v}{\beta} + \frac{\alpha_z w}{\gamma} + \frac{1}{2\alpha}\Big(u_x + \frac{\alpha_y v}{\beta} + \frac{\alpha_z w}{\gamma}\Big)^2 + \frac{1}{2\alpha}\Big(v_x - \frac{\alpha_y u}{\beta}\Big)^2 + \frac{1}{2\alpha}\Big(w_x - \frac{\alpha_y u}{\gamma}\Big)^2\Big]$$

$$\varepsilon_y = \frac{1}{\beta}\Big[v_y + \frac{\beta_z w}{\gamma} + \frac{\beta_x u}{\alpha} + \frac{1}{2\beta}\Big(v_y + \frac{\beta_y w}{\gamma} + \frac{\beta_x u}{\alpha}\Big)^2 + \frac{1}{2\beta}\Big(w_y - \frac{\beta_z v}{\gamma}\Big)^2 + \frac{1}{2\beta}\Big(u_y - \frac{\beta_x v}{\alpha}\Big)^2\Big] \qquad (1)$$

$$\varepsilon_z = \frac{1}{\gamma}\Big[w_z + \frac{\gamma_x u}{\alpha} + \frac{\gamma_y v}{\beta} + \frac{1}{2\gamma}\Big(w_z + \frac{\gamma_y u}{\alpha} + \frac{\gamma_y v}{\beta}\Big)^2 + \frac{1}{2\gamma}\Big(u_z - \frac{\gamma_x w}{\alpha}\Big)^2 + \frac{1}{2\gamma}\Big(v_z - \frac{\gamma_y w}{\beta}\Big)^2\Big]$$

where $u, v,$ and w are displacements with repect to axes and α, β, γ are Lame's coefficients. When all square terms except the ones about w are ignored and these are transformed to cylinderical coordinate system, the following relation between circumferential strain and displacement can be obtained.

$$\varepsilon_{\phi}=\frac{1}{R}[(\frac{dv}{d\phi}-w)-\frac{1}{2R}(\frac{dw}{d\phi})^{2}] \quad (2)$$

where v, w are tangential and radial displacements. The positive of w is defined to be toward the center. According to inextentional deformation theory, the displacements due to extentional of the center line of a cylinder section are very samll in comparision with the displacements due to bending and usually can be neglected.

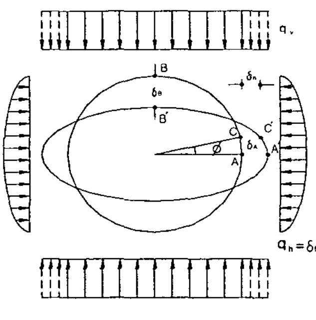
$$v=\int[w+\frac{1}{2R}(\frac{dw}{d\Phi})^{2}]d\Phi \quad (3)$$

2.2 Moment Equations

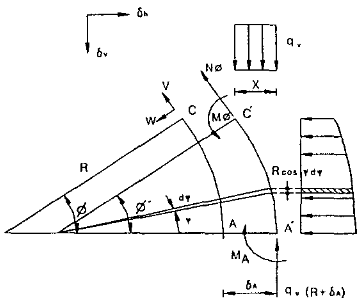
If the pipe section is made such that it is exactly symmetric about its middle surface, the equations for the moments are

$$\begin{aligned} M_{11}&=D_{11}k_{11}+D_{12}k_{22}\\ M_{22}&=D_{12}k_{11}+D_{22}k_{22}\\ M_{12}&=D_{33}k_{12}\end{aligned} \quad (4)$$

2.3 Equilibrium Equation



(a)



(b)

Fig.1 Definition of loading and sign convention for conduit

$$\begin{aligned}\delta_h &= -v\sin\phi-w\cos\phi\\ \delta_v &= -v\cos\phi+w\sin\phi\\ x &= (R+\delta_A)-R\cos\phi-\delta_h\\ &= R(1-\cos\phi)+\delta_A+v\sin\phi+w\cos\phi\end{aligned} \quad (5)$$

From Fig.1(b), equilibrium equation on deformed shape is as follows.

$$\Sigma M_{c'}=0$$

$$-M_{\phi}+M_A+q_v x\frac{x}{2}-q_v(R+\delta_A)x$$

$$+\int_0^{\phi}\delta_h K(Rd\varphi\cos\phi)[R(\sin\phi-\sin\varphi)]=0 \quad (6)$$

Substituting Eq.(5) into Eq.(6)

$$-M_{\phi}+M_A-\frac{q_v}{2}[R(1-\cos\varphi)+\delta_A+v\sin\varphi+w\cos\phi]$$

$$[R(1+\cos\varphi)+\delta_A-v\sin\phi-w\cos\phi]$$

$$+\int_0^{\phi}\delta_h KR^2(\sin\varphi-\sin\phi)]\cos\varphi d\varphi=0 \quad (7)$$

Since the problem is assumed to be that of plane strain, the following relation can be obtained.

$$M_{\phi}=Dk_{\phi}$$

where $$k_{\phi}=\frac{1}{R^2}(\frac{dv}{dw}-\frac{d^2w}{d\phi^2}) \quad (8)$$

$$D=D_{22}=\frac{1}{3}\sum_{k=1}^{n}(\overline{Q}_{22})_k(h^3_{k+1}-h^3_k)$$

Since Eq.(7) is the differential equation about w, Galerkin's method is used to obtain the solution.

$$\bar{w}=\frac{w}{R}=A_1\cos2\phi+A_2\cos4\phi \qquad (9)$$

$$\int_0^{\frac{\pi}{2}} X\cos2n\phi=0 \qquad n=1,2 \qquad (10)$$

X is the left side of Eq. (7). Substituting Eq. (3)(7)(8)(9) into (10), we obtain the following nonlinear simultaneous equations.

$$\bar{q}\,[(1-A_1)^2-\frac{5}{4}A_2-\frac{3}{16}(\frac{A^2_1}{3}-6A_1A_2-\frac{5}{2}A^2_2)]$$
$$=-12A_1(1+\frac{38.3}{210}\frac{\beta}{\pi})-A_2\frac{96.4}{360}\beta$$
$$\bar{q}\,[\frac{A_1}{8}-2A_2+\frac{3}{32}A^2_1+\frac{3}{8}A_1A_2] \qquad (11)$$
$$=15A_2(1+\frac{11.9}{1920}\frac{\beta}{\pi})+\frac{12.6}{385}A_1\frac{\beta}{\pi}$$

where $\bar{q}=\frac{q_vR^3}{D}$; $\beta=\frac{KR^4}{D}$

Talyor's series is used to solve above equations to obtain the continuous displacement function expressed in trigonometic forms.

3. Numerical Example

A composite pipe made with Glass/Epoxy which has the following material property and geometry

$E_1=0.55\times10^6\,kg/cm^2$ $R=12.7cm$

$E_2=0.18\times10^6\,kg/cm^2$ 4 laminae with same thickness

$G_{12}=0.091\times10^6\,kg/cm^2$ total thickness $=0.203cm$

$\nu=0.25$ soil unit weight $(\gamma)=1995kg/m^3$

is analysed. Fig. 2 shows the effect of soil pressure against the pipe and nonlinearity of the displacement according to the depth. Fig. 3 shows the comparison of the results obtained by the method of this paper and FEM.

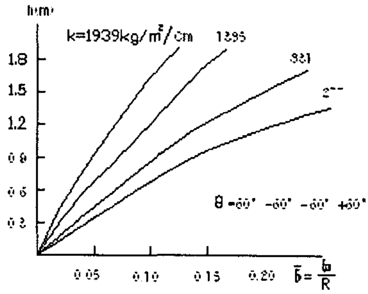


Fig. 2 Displacement according to depth and soil

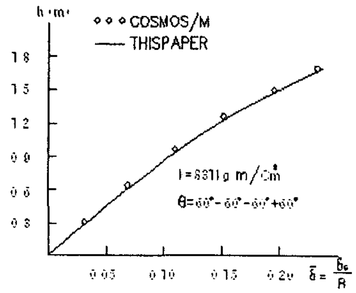


Fig. 3 Comparision with FEM

4. Conclusion

The project size of underground pipe construction is huge : 300 BUSD/year for water related projects only, excluding the oil and gas pipelines. Unfortunately, these have been constructed without proper design and analysis. In this paper the method of geometric nonlinear analysis of underground composite pipes is briefly given for junior practicing engineers.

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