Synthesis of Nonlinear Model Matching Flight Control System for Tilt Rotor Aircraft

Yasuhiro ASADA*, Yasuhiro OSA**, Shigeru UCHIKADO*** and Kanya TANAKA****

* The Advanced Course of Mechanical Systems Engineering, Kobe City College of Technology, Kobe, Japan
** Department of Mechanical Engineering, Kobe City College of Technology, Kobe, Japan
*** School of Science and Engineering, Tokyo Denki University, Saitama, Japan
**** Faculty of Engineering, Yamaguchi University, Ube, Japan

Abstract: In this study, we suggest a tilt rotor aircraft and attempt to apply a nonlinear model matching control method for its maneuver. The proposed method is very simple and useful to construct the control law for the complicated nonlinear system such as aircraft motion.

1. Introduction

Tilt Rotor Aircraft (TRA) is one of VTOL that has two rotor engine nacelles, which provide the lift and/or the thrust, at the both wing edges. [1, 2] This aircraft possesses many merits based on its high maneuverability. For example, of course it can take off and land vertically and does not need the runway like helicopters. And it makes higher speed transportation possible in comparing with helicopters. Therefore TRA will become a very useful and popular vehicle in future. TRA is used versatile and used for following purposes.

(1) Sea Rescue
(2) Handling on emergency case
(3) Commuter transport at two points
(4) Police units
(5) Immigration control, etc

Up to the present linear control laws like PI controller with gain scheduler have been used for the flight control systems of passenger planes, etc. But originally aircraft motion is expressed with six-degree-of-freedom nonlinear equations, and the equations include several terms of the products and the trigonometric functions with respect to the state-space variables. In addition, on the transient flight state from the hovering to the steady state flight, the equations of TRA motion become considerably complicated because it changes the body form by turning the rotor engine nacelles. Hence the enough control performance can not be obtained by a linear control law such as PI controller. Moreover it implies a problem that the thrust acts like a variable vector. The input terms are expressed with the products of the magnitude of thrust and the trigonometric function of nacelle angle. It means that the control law can not be determined uniquely. Here we have a question, whoever or whatever turns the nacelles forward on taking off and makes the aircraft go forward? We think the answer is only the expert pilots at present time.

To solve the above problems, we propose a nonlinear model matching control method and attempt to apply it for the maneuver of TRA. [3, 4] First the nonlinear longitudinal equations of TRA motion are given. [5] Where forward velocity, vertical velocity, pitch angle and pitch rate are considered as the state variables. More nacelle angle is added to them to determine the control law uniquely. And vertical velocity, pitch angle and nacelle angle are set as the outputs, and magnitude of thrust, piloted signal for nacelle angle and hub moment are used as the inputs. Next formulation of the problem is described. The objective of the control system is to match the outputs of TRA to the reference model outputs. Next, the control system is constructed with our nonlinear model matching method. This method facilitates easy determination of the control law using the relationship, between the outputs and the inputs, which is obtained by the time shift of each output signal. At the end of paper, the proposed control system is applied to a TRA and numerical simulations are shown to investigate the feasibility of the proposed approach.

2. Longitudinal Nonlinear Equations of Aircraft Motion

In this section, the longitudinal nonlinear equations of aircraft motion [3] for TRA are shown as the controlled system. At first the nonlinear ones are constructed in continuous time, secondly they are transformed to the discrete form. [4]

2.1 General Longitudinal Equations of Aircraft Motion
The body axes of TRA can be set as in Fig. 2.
Then the longitudinal nonlinear equations of TRA motion are expressed as follows:

\[
\begin{align*}
\dot{X} &= \Theta - \sin(\Theta) - \Phi \cos(\Phi) \\
\end{align*}
\]

\[
\begin{align*}
\dot{Y} &= -\Phi - \theta \\
\dot{Z} &= -\Phi - \theta \\
\end{align*}
\]

(1)

Where
- \( U \): forward velocity [m/s]
- \( V \): lateral velocity [m/s]
- \( W \): vertical velocity [m/s]
- \( P \): roll rate [rad/s]
- \( Q \): pitch rate [rad/s]
- \( R \): yaw rate [rad/s]
- \( X \): longitudinal force (aerodynamic force in direction of X body axis) [N]
- \( M \): aerodynamic moment about Y body axis [Nm]
- \( \Theta \): pitch angle [rad]
- \( \Phi \): roll angle [rad]

2.2 Aerodynamic Force and Moment
The longitudinal model of TRA motion can be considered such as Fig. 3, Fig. 4.[3]

\[
\begin{align*}
\rho \left( \frac{\partial}{\partial t} X + V \frac{\partial X}{\partial x} + W \frac{\partial X}{\partial \tilde{z}} \right) &= 0 \\
\end{align*}
\]

(2)

Where
- \( \rho \): air density [kg/m³]
- \( X \): moment of inertia about X body axis [kgm²]
- \( V \): resultant linear velocity [m/s]
- \( \Theta \): non-dimensional longitudinal-force coefficient about nacelle angle
- \( \Theta \): non-dimensional longitudinal-force coefficient about nacelle angle
- \( T \): Trust [N]

Vertical force

\[
\begin{align*}
\dot{Z} &= 2 \left( \rho \Theta \right) \cos(\Theta) - \sin(\Theta) \cos(\Theta) \\
\end{align*}
\]

(3)

Where
- \( \rho \): air density [kg/m³]
- \( \Theta \): non-dimensional vertical-force coefficient
- \( \Theta \): non-dimensional vertical-force coefficient about nacelle angle

Pitching moment

\[
\begin{align*}
\dot{\Theta} &= 2 \left( \rho \Theta \right) \cos(\Theta) - \sin(\Theta) \cos(\Theta) \\
\end{align*}
\]

(4)

Where
- \( \rho \): mean aerodynamic chord [m]
- \( \Theta \): length from center of gravity to center of nacelle [m]
- \( hM \): Hub moment [Nm]

Moreover the lateral-directional state variables \( V, P, R \) and \( \Phi \) in Eqs.(2)-(4) can be given as 0 respectively. As a result, the following equations can be obtained.
Forward velocity
\[ \dot{v} = - \sin \theta + \frac{1}{2} \rho^2 \left( \cos \Theta + \frac{1}{4} \rho^2 \right) \]
\[ + \frac{1}{2} \sin \theta \cos \Theta \]  
(5)

Vertical velocity
\[ \dot{v} = \sin \theta + \frac{1}{2} \rho^2 \left( \cos \Theta - \frac{1}{4} \rho^2 \right) \]
\[ - \frac{1}{2} \cos \theta \cos \Theta \]  
(6)

Pitch angle
\[ \dot{\Theta} = \]  
(7)

Pitching moment
\[ \dot{\Theta} = \frac{1}{2} \rho^2 \left( \cos \Theta + \frac{1}{4} \rho^2 \right) \]
\[ - \frac{1}{2} \sin \theta \cos \Theta + \]  
(8)

Now we have a problem for the control inputs. As you see the products of the thrust and the trigonometric function of the nacelle angle exist in the above equations, the control law can not be determined uniquely. Because both thrust and nacelle angle can be considered as the control inputs. Hence consider the nacelle angle as the state variable, the following first order system is added to the above equations.

Nacelle angle
\[ \dot{\Theta} = - \frac{1}{T_c} \Theta + \frac{1}{1} \]  
(9)

\[ T_c: \text{time constant of nacelle actuator} \]

2.3 Continuous Time Controlled System

Based on Eq. (5)-(9) the following state-space equation can be obtained. Also as the outputs the vertical velocity, the pitch angle and the nacelle angle are selected for making the vertical taking off and landing possible and keeping the constant longitudinal body attitude.

\[ ( ) = ( ) + ( ) ( ) \]
(10)

Where
\[ ( ) = ( ), ( ), \Theta( ), ( ), \Theta( ) \]
\[ ( ) = ( ), ( ), ( ) \]
\[ ( ) = ( ), \Theta( ), \Theta( ) \]
\[ ( ) = ( ) \]

3. Formulation of the Problem

Normally we have no strict transform method for nonlinear equations between continuous time and discrete time. Then apply the following first order approximation to the differentiation terms with respect to state variables in left hands of general state-space equation \( \Delta \) is the sampling time [s].

\[ ( ) = \left( ( + 1) - ( ) \right) \frac{1}{\Delta} \]
(11)

The discrete time nonlinear state-space equation and the output equation can be given as follows, and consider them as the controlled system.

System \( \Sigma \)
\[ ( + 1) = ( ) + ( ) ( ) \]
(12)

Where
\[ \dot{F}(x) = [f_1(x), f_2(x), \ldots, f_n(x)]^T, f_i(x) : R^n \rightarrow R \]
\[ \dot{B}(x) = [b_1^T(x), b_2^T(x), \ldots, b_n^T(x)]^T, b_i^T(x) : R^n \rightarrow R^p \]
\[ C = [c_1^T, c_2^T, \ldots, c_p^T] \in R^{q \times n} \]
\( \mathbf{x}(k) = [x_1(k), x_2(k), \ldots, x_q(k)]^T \in \mathbb{R}^q \)
\( \mathbf{y}(k) = [y_1(k), y_2(k), \ldots, y_p(k)]^T \in \mathbb{R}^p \)
\( \mathbf{u}(k) = [u_1(k), u_2(k), \ldots, u_q(k)]^T \in \mathbb{R}^q \)

\( \mathbf{x}(k) \in \mathbb{R}^q \), \( \mathbf{y}(k) \in \mathbb{R}^p \) and \( \mathbf{u}(k) \in \mathbb{R}^q \) are the state variable vector, the output signal vector and the input signal vector, and the above inverse system is assumed to be stable. Let \( F(\mathbf{x}) \) and \( B(\mathbf{x}) \) be the real polynomial functions with respect to the state variable.

On the other hand, consider the following equation as a reference model which the system designer sets arbitrarily.

**Reference System \( \Sigma \)**

\[
\begin{align*}
( + 1) &= A ( ) + ( ) \\
( ) &= ( ) \quad (13)
\end{align*}
\]

where
\[
\begin{align*}
\mathbf{x}_0(k) &= [x_{01}(k), x_{02}(k), \ldots, x_{0q}(k)]^T \\
\mathbf{y}_0(k) &= [y_{01}(k), y_{02}(k), \ldots, y_{0p}(k)]^T \\
\mathbf{u}_0(k) &= [u_{01}(k), u_{02}(k), \ldots, u_{0q}(k)]^T \\
C_M &= [c_{M1}, c_{M2}, \ldots, c_{Mp}]^T \\

x_0(k) \in \mathbb{R}^q \text{ is the reference state variable, } u_0(k) \in \mathbb{R}^q \text{ and } y_0(k) \in \mathbb{R}^p \text{ are the bounded reference input and output.}
\]

The objective of this study is to design a model matching control system which forces the output of the state vector nonlinear system \( y(k) \) to match the reference model output \( y_0(k) \).

And here the model matching is defined as follows:

**Definition**

For the following condition of System \( \Sigma \) and Reference Model \( \Sigma_M \)

\[
x_0 = 0, \quad x_{M0} = 0, \quad F(x_0) = 0, \quad B(x_0) = 0
\]

When,

\[
( ) = ( ) \quad (14)
\]

can be achieved, it is called that System \( \Sigma \) can be model-matched to Reference Model \( \Sigma_M \).

4. Nonlinear Model Matching

In this section, for System \( \Sigma \) and Reference System \( \Sigma_M \), the dynamic model matching control system based on Hirschorn's algorithm [6] extended with Silverman's structure algorithm [7] is proposed. For System \( \Sigma \) and Reference System \( \Sigma_M \), perform the following procedure.

**Step1** Consider the time shift signals of the output \( y_1(k) \) and \( y_{01}(k) \), left-multiply them by \( z \), the following equations are obtained with Eq.(11) and (12).

\[
\begin{align*}
1_1( ) &= 1_1( ) + 1 ( ) ( ) \\
1_1( ) &= 1_1 A ( ) + 1 ( ) ( ) \\
1_1( ) &= 1_1 A ( ) + 1 ( ) ( ) \\
\end{align*}
\]

Where \( z \) is time-shift operator.

Next formally replace the above equations with as follows:

\[
\begin{align*}
1_1( ) &= 1_1 ( ) + 1 ( ) ( ) \\
1_1( ) &= 1_1 A ( ) + 1 ( ) ( ) \\
1_1( ) &= 1_1 A ( ) + 1 ( ) ( ) \\
\end{align*}
\]

Where the left "1" of sub index "11" means the 1st output, the right "11" means the 1st power of \( z \). And generally \( C_{M1}(x) \), \( C_{MA1}(x) \), \( D_{M1}(x) \) and \( D_{MA1}(x) \) are polynomial functions and matrices with respect to \( x \).

In the above equations, when if \( Da_0(x) \neq 0 \), replace subscripts "11" of \( f_1 \), \( C_{M1} \), \( D_{M1} \) and \( D_{MA1} \) with "1" and go to the next step. When if \( Da_0(x) = 0 \), the following equations are obtained by repeating the time shift

\[
\begin{align*}
1_1( ) &= 1_2 ( ) + 1 ( ) ( ) \\
1_1( ) &= 1_2 ( ) + 1 ( ) ( ) \\
\end{align*}
\]

where \( Da_0(x) \neq 0 \) and it is assumed that "1" which satisfies the above equations exists. Likewise, replace the sub indices "11" of \( f_1 \), \( C_{M1} \) and \( D_{M1} \) in the above equations with "1" and go to the next step.

**Step2** Do the same procedure as Step 1 for the output \( y_2(k) \), the following equations are obtained.

\[
\begin{align*}
2_1( ) &= 2_2 ( ) + 2 ( ) ( ) \\
2_1( ) &= 2_2 ( ) + 2 ( ) ( ) \\
\end{align*}
\]

**Step3** When if \( Da_0(x) \neq 0 \), replace the sub indices "22" with "2" and do the same procedure from Step 2 for \( y_3(k) \). Where, \( \alpha_{21}(x) \) is a polynomial function with respect to \( x \).

When if \( Da_0(x) = 0 \), consider the new outputs as follows:

\[
\begin{align*}
-\alpha_{21}( ) &= 2_1^{( ) + 2 2 ( )} \\
-\alpha_{21}( ) &= 2_1^{( ) + 2 2 ( )}
\end{align*}
\]

and do the same procedure from Step 2.

**Step 4** By repeating the above procedure to the outputs \( y_4(k) \) and \( y_{04}(k) \), the following equations can be obtained

\[
\begin{align*}
( ) &= ( ) + ( ) ( ) \\
( ) &= ( ) ( ) + ( ) ( ) \\
\end{align*}
\]

Where \( Na(z, x) \) is a lower triangular matrix in which the diagonal entries are \( z^0 \), and \( C_{a}(x) \) and \( D_{a}(x) \) are respectively

\[
\begin{align*}
( ) &= \begin{bmatrix} 1 \end{bmatrix} ( ) \\
( ) &= \begin{bmatrix} 1 \end{bmatrix} ( ) \\
( ) &= \begin{bmatrix} 1 \end{bmatrix} ( ) \\
( ) &= \begin{bmatrix} 1 \end{bmatrix} ( ) \\
\end{align*}
\]

Then it is clear that \( Na(z, x) \) is a lower triangular matrix because of the procedure in Step 3 which makes out the relations between \( y(k) \) and \( u(k) \) with the time shift form of \( y(k) \).

Using the above relation the following theorem can be obtained.
Theorem
If the following condition is satisfied
\[
\begin{bmatrix}
  ( ) \\
  ( )
\end{bmatrix} = 0 \quad \forall \quad ( ) \in \mathbb{R}
\]
System \( \Sigma \) can be model-matched to Reference System \( \Sigma_M \) by the control law \( u(k) \) as
\[
( ) = ( ) + ( ) + ( ) ( )
\]
(21)

(Proof) Define the output error \( e(k) \) as
\[
( ) = ( ) - ( )
\]
the following relation can be obtained using Eqs.(20)–(21)
\[
( , ) ( ) = 0
\]
(23)
Where notice the form of \( Na(z, x) \), especially the diagonal entries which have the stable polynomials, for the condition : \( x(0)=0, \ x_o(0)=0, \ F(x_o)=0, \ B(x_o)=0, \) the following relation can be obtained
\[
( ) = ( ) \quad \text{for} \quad \geq 0
\]
and it means that the model matching can be achieved. Also because of Eq.(22), for the arbitrary initial values, the following relation can be obtained
\[
( ) \rightarrow ( ) \quad \text{for} \quad \rightarrow \infty
\]

Comment As a result, by replacing \( Na(z, x) \) with an interactor matrix [5] of a system, we can understand that this method is an extension of the linear model matching control system proposed by Wolovich [8]

5. Application to Lift Cruise Fan Aircraft
In this section, we attempt to apply the proposed method to the flight control system for a TRA, and investigate the feasibility by numerical simulations. First from the state of the hovering, make it rise up while changing the nacelle angle from 0 deg to 25 deg. Next make it go forward with \( W=0 \) [m/s], changing the nacelle angle to 50 deg.

The data of TRA [1], flight condition [9] and reference models are given as follows. Yet many data of another aircraft are included in them.

Data of TRA
\( m=5195 \) [kg], \( g=9.81 \) [m/s²], \( \Delta=1.50 \) [m], \( \Delta=0.05 \) [s], \( S=39.56 \) [m²], \( C=4.89 \) [m], and \( I_Y=178457 \) [kgm²]

Flight Condition
\( \rho=0.5495 \) [kg/m³], \( C_X=0.0325, C_{200}=0.016, C_{200}^2=0.851, C_{200}^3=0.34, C_{200}=0.0373, C_{200}^2=0.052, C_{200}^3=6.0, T_c=0.05 \) [s]

Reference Model
2nd order transfer functions with damping ratio \( \zeta = 0.9 \) and natural frequency \( \omega_n = 5.2 \) [rad/s] are given for each output. The reference inputs are given as \( u_M(k) = \{0 \text{[m/s]}, 0 \text{[deg]}, 0 \text{[deg]} \} \) from the beginning to 1 sec, \( u_M(k) = \{1 \text{[m/s]}, 0 \text{[deg]}, 25 \text{[deg]} \} \) from 1 sec to 5 sec, \( u_M(k) = \{0, 0, 50\} \) from 5 sec to 10 sec.
Evaluation

The results in Fig.5 show that each output perfectly matched to the reference model outputs and the smooth vertical translation keeping the pitch angle 0 deg could be accomplished in spite of the change of nacelle angle. Fig.6 shows that the thrust may be made within the range of the available power. Also Fig.7 shows that at 10 sec the forward velocity reaches approximately 60 km/h with keeping nacelle angle 50 deg.

Yet it is necessary to consider about the longitudinal stability of body at the steady state for the zero hub moment.

Conclusions

In this paper, we suggested the longitudinal non-linear motion of Tilt Rotor Aircraft and attempted to apply our nonlinear model matching control method for it. And we showed the feasibility of proposed method with numerical simulations.

However it is necessary to consider the following premises or assumptions used in the flight control system.

In its hardware, 1 All state variables of the aircraft can be measured.
2 Due to the special form with the rotor nacelle, it is always necessary to let make the rotor produce the hub moment to maintain the longitudinal stability. This situation is inefficient from the aerodynamic point of view. Therefore we should consider any mechanisms to improve this problem.

In its software, 1 The plant parameters such as non-dimensional aerodynamic derivatives are fixed. [4, 10] 2 Generally stability of non-linear control system has not been proved, yet.

Hence these are remained as the theme which should be improved.

References
