1. Introduction

The composite thin-walled beam structure which has been used in modeling and analysis of the various mechanical systems are attractive to many fields of advanced technology because of its structural high performance and efficiency. According to its growing interest, many studies has going on to make this type of aircraft wing stable in certain circumstances (1-3).

This paper examines the structural dynamic response of the composite thin-walled beam structure induced by unsteady aerodynamics based on the mode expansion theorem and assumed mode shape, etc. In addition, it was presented that the control simulation results of its unstable dynamical phenomenon by using the sliding mode methodology.

2. System Modeling

2.1 Beam Structure

In this paper, fiber-reinforced, closed-section, a single-cell thin-walled beam is used in the modeling of the system (see Fig.1).

![Fig. 1. Geometric configuration of the structure](image)

For the chosen local and global coordinate systems, we can define the 3-D displacement quantities as follows (3):

\[
\begin{align*}
  u(x, y, z, t) &= u_0(y, t) + z \cdot \phi(y, t) \\
  w(x, y, z, t) &= w_0(y, t) - x \cdot \phi(y, t) \\
  v(x, y, z, t) &= v_0(y, t) + \left[ x(s) - n \cdot \frac{dz}{ds} \right] \cdot \theta_z(y, t) \\
  &+ \left[ z(s) + n \cdot \frac{dx}{ds} \right] \cdot \theta_y(y, t) - \left[ F_w(s) + n \cdot a(s) \right] \cdot \phi(y, t)
\end{align*}
\]

where \( \theta_x, \theta_y \) and \( \phi \) denote rotations of the plane of the cross-section about the axes \( x, z \) and twist about the \( y \) axis, respectively.

Furthermore, the strains that contribute to the potential energy are easily derived as (3):

- **Sapwise:** \( \varepsilon_{yy}(n, s, y, t) = \varepsilon_{yy}^0(s, y, t) + n \cdot \varepsilon_{yy}^0(s, y, t) \)
- **Tangential Shear:** \( \gamma_{yu}(s, y, t) = \gamma_{yu}^0(s, y, t) + \psi(s) \cdot \phi(y, t) \)
- **Transverse Shear:** \( \gamma_{uy}(s, y, t) = -\gamma_{uy}^0 \frac{dz}{ds} + \gamma_{uy} \frac{dx}{ds} \)

\[
\begin{align*}
  &= -\left[ u_0' + \theta_u \right] \frac{dz}{ds} + \left[ w_0' + \theta_w \right] \frac{dx}{ds}
\end{align*}
\]

2.2 Subsonic Aerodynamic Loads

The indicial function approach gives a general and convenient way to describe the compressible unsteady aerodynamics in linear aerodynamic theory.

From the indicial lift and moment functions due to the unit step change of the vertical translation velocity at the leading edge, the indicial lift \( L_y^* \) and aerodynamic moment \( T_y^* \) about the mid-chord are defined. Also the indicial lift \( L_y^* \) and aerodynamic moment \( T_y^* \) are derived from the indicial lift and moment functions due to the unit step change of the pitching rate at the leading edge.

As a result, the total aerodynamic lift and moment about the mid-chord are (4):

- **Total Aerodynamic Lift:** \( L_\alpha(\eta, \tau) = L_y^*(\eta, \tau) + L_y^*(\eta, \tau) \)
- **Total Aerodynamic Moment:** \( T_\alpha(\eta, \tau) = T_y^*(\eta, \tau) + T_y^*(\eta, \tau) \)

Due to the limited space, the details are omitted.
2.3 Governing Equations

Extended Hamilton’s Principles are usually used to get the governing equations and boundary conditions at one time. 

\[ \int_0^T (\delta T - \delta V + \delta W_x) dt = 0 \]

with \( \delta u_0 = \delta v_0 = \delta w_t = \delta \theta_t = \delta \phi = 0 \)

The structure features circumferentially asymmetric stiffness lay-up configuration. Hence the equations of motion and the boundary conditions are completely separated by two parts. We are mainly concerned about vertical bending, twist and vertical transverse shear part. So, the governing equation and the boundary condition about the vertical bending is\(^{(5)}\):

\[
\begin{align*}
\delta w_0 : a_{35}(w_0^* + \theta_t^*) + a_{36}\phi^* + p_\varepsilon - b_1 \ddot{w}_0 &= 0 \\
\text{at } y = 0, & w_0 = 0 \\
\text{at } y = L, & \delta w_0 : a_{35}(w_0^* + \theta_t^*) + a_{36}\phi^* = 0 \\
\end{align*}
\]

3. Sliding Mode Methodology

3.1 Sliding Mode Observer

We use the sliding mode observer to estimate of states, to require robustness against disturbance and observation spillover from the unmeasured states. The sliding mode observer has the form\(^{(6)}\):

\[ \dot{x}(t) = A \cdot x(t) + B \cdot u(t) + L \cdot (y(t) - C \cdot x(t)) + N \cdot v(t) \]

where \( L, N \) is the linear and nonlinear observer gain, while \( v(t) \) is a discontinuous vector defined as\(^{(7)}\):

\[ v(t) = \begin{cases} 
-\mu \left[ \frac{P \cdot C e}{P_t \cdot C e} \right] & \text{if } C e \neq 0 \\
0 & \text{otherwise}
\end{cases} \]

Herein \( \mu \) denotes the positive scalar function and \( P_t \) is a positive definite symmetric matrix.

3.2 Sliding Mode Control

This type of control law is proper method for the uncertain dynamical model, because it has the nonlinear control part as follows\(^{(8)}\):

\[ u(x) = L_x x(t) + \rho \frac{N \cdot x(t)}{\left\| M \cdot x(t) \right\|} \]

The linear control law \( L_x \) that is state feedback control converges on equilibrium point through the hyperplane.

And the second term in the RHS of an equation plays a role suppressing the effect of model uncertainty.

4. Conclusion

Aeroelastic response of the composite thin-walled beam structure is revealed by the simulation results. Its dynamic response is determined through the ply angle, sweep angle and the flight speed. Given conditions, the unstable dynamic phenomenon occurs near to a Mach number of 0.7. Sliding mode controller based on sliding mode observer effectively decreases the plunging and pitching displacements of the wing structure.

Acknowledgment

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0001642).

References