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Dynamic Model for Open-loop Control of Spherical Motor

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1. Introduction

Many mobile vehicles such as wheels for mobile vehicles, propellers for boats, helicopter or underwater vehicle, robotic joint, and machine tools require orientation control of the rotating shaft. Existing designs are typically single-axis devices; thus, orientation control of their rotating shafts must be manipulated by an external mechanism. These multi-axe spinners are generally bulky, slow in dynamic response, and lack of dexterity in negotiating the orientation of the rotating shaft. This paper presents a spherical wheel motor (SWM); an alternative design built upon the concept of a VRSM originally conceptualized in [1]. The SWM, much like the VRSM capable of offering three-DOF in a single joint, is essentially a ball-joint-like, brushless, direct-drive actuator. However, unlike a VRSM which has been mainly designed to control its three-DOF angular displacements, the SWM discussed here offers a means to control in open-loop (OL) the orientation of a rotating shaft in the single spherical joint.

The interest to develop an open-loop stable spherical motor has led us to the concept of a SWM operated on a push-pull principle and the distributed multi-pole (DMP) method to model the magnetic field of a permanent magnet in closed-loop [2]. Illustrations of the DMP method for deriving the torque model of a spherical motor can be found in [3]. In this paper, we extend the application of the DMP method to the design of a model-based controller for operating the SWM in open-loop.

2. Dynamic model of SWM

A dynamic analysis of a spherical wheel motor including a torque computation is essential for optimizing the design and performance in controlling precise motion. Figure 1 illustrates the schematic design of the SWM totally consisting of m_r permanent magnets (PMs) in the rotor, m_s electromagnets (EMs) in the stator and a universal ball bearing at a centre of the rotor, which supports the rotor and enables three DOFs motion. Despite its simple structure, there are a number of difficulties in controlling orientation and analyzing a magnetic field for torque computation.

As shown in Fig. 1, the rotor and the stator of the SWM are spherically symmetric with respect to both electrical and mechanical configurations in the design. Both PMs and EMs are equally spaced on four circular planes accordingly (*i=1,2,3...,m_r and j=1,2,3,...,m_s*). In addition to the planes along with the z-axis, the PMs and EMs are grouped in pairs and every two pairs form a plane, and their magnetization axes pass radially through the centre with opposite polarities. Once an electric current input is applied, a pair of EMs is energized at the same time and thus, twice of a torque is generated accordingly. The torques generated by the PMs and EMs enable to control the rotor in a desired orientation while it maintains spinning.

The dynamic equations of motion including torques can be derived using the Lagrangian formulation in terms of the ZYZ Euler angles (α, β, γ) shown in Fig. 1, which has the following form in (1).

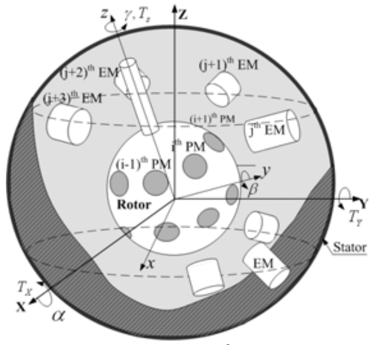


Fig. 1 Design of SWM

$$\left[\mathbf{M}\right]\dot{q}_2 + \mathbf{C}(q_1, q_2) + \mathbf{C}_f = \mathbf{Q} + \mathbf{T_{ext}} \quad (1)$$

where
$$\mathbf{M} = \begin{bmatrix} I_t C_\beta^2 + I_a S_\beta^2 & 0 & -I_t S_\beta \\ 0 & I_t & 0 \\ -I_t S_\beta & 0 & I_a \end{bmatrix}; \quad (1a)$$

$$C(\dot{q}, q) = \begin{bmatrix} 2(I_a - I_t) S_\beta C_\beta \dot{\alpha}\dot{\beta} - I_a C_\beta \dot{\beta}\dot{\gamma} \\ (I_t - I_a) S_\beta C_\beta \dot{\alpha}^2 + I_a C_\beta \dot{\alpha}\dot{\gamma} \\ -I_a C_\beta \dot{\alpha}\dot{\beta} \end{bmatrix}; \quad (1b)$$

$$\mathbf{Q} = \begin{bmatrix} -S_\beta C_\gamma & S_\beta S_\gamma & C_\beta \\ S_\gamma & C_\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} T_X \\ T_Y \\ T_Z \end{Bmatrix}; \quad (1c)$$

$q_1 = [\alpha \;\; \beta \;\; \gamma]^{\mathrm{T}}$; $q_2 = \dot{q}_1$; $\mathbf{T_{ext}}$ and $\mathbf{C}_f$ are the torques imposed by external (or load) and mechanical bearing frictions respectively. In (1a, b), $I_a = I_{zz}$; $I_t = I_{xx} = I_{yy}$ due to the symmetry of the rotor along the Z axis and the rotor center of gravity is assumed to coincide with the rotation center. In (1c), **Q** represents the applied (magnetic) torque to the generalized moments in the rotor coordinates. The torque vector can be computed from distributed multi-pole (DMP) method in [3].

Since the inertia matrix [M] is positive-definite in the inclination range of control, –20°≤(α, β)≤ 20°, the nonlinear dynamics (8) can be expressed in the standard state-space form:

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ & f(q_1, q_2) \end{bmatrix} + \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix}\mathbf{Q} \quad (2)$$

where $f(q_1, q_2) = \left[\mathbf{M}\right]^{-1}\mathbf{C}(q) \in R^{3\times1}$ is given by

$$f(q) = \frac{1}{I_t}\begin{bmatrix} \dot{\beta}\sec\beta\left(I_a\dot{\gamma} + (2I_t - 3I_a)\dot{\alpha}S_\beta\right) \\ \dot{\alpha}C_\beta\left(-I_a\dot{\gamma} + (I_t - I_a)\dot{\alpha}S_\beta\right) \\ \dot{\beta}\left\{-I_t\dot{\alpha}C_\beta + \left[I_a\dot{\gamma} + (2I_t - 3I_a)\dot{\alpha}S_\beta\right]\tan\beta\right\} \end{bmatrix} \quad (3)$$

The equations of motion in (2) and (3) can be linearized around the desired state and expressed in (4), which will be used for designing a closed-loop control system later.

$$\dot{q} = [\mathbf{A}]q + \begin{bmatrix} 0_{3\times3} \\ I_{3\times3} \end{bmatrix} \mathbf{Q} \quad (4)$$

$$[\mathbf{A}] = \left(\frac{\partial f_j}{\partial q_i} \right) \Bigg|_{q=q_d} = \frac{1}{I_t} \begin{bmatrix} [\mathbf{A}_1] & [\mathbf{A}_2] \\ [\mathbf{A}_3] & [\mathbf{A}_4] \end{bmatrix}_{q=q_d} \quad (4a)$$

where $[\mathbf{A}_1] = [0_{3\times3}]$; $[\mathbf{A}_2] = I_t[\mathbf{I}_{3\times3}]$; $\mathbf{I}$ is the identity matrix;

$$[\mathbf{A}_3]_{q=q_d} = \begin{bmatrix} 0 & A_{42} & 0 \\ 0 & A_{52} & 0 \\ 0 & A_{62} & 0 \end{bmatrix}_{q=q_d} ; \quad [\mathbf{A}_4] = \begin{bmatrix} A_{44} & A_{45} & A_{46} \\ A_{54} & 0 & A_{56} \\ A_{64} & A_{65} & A_{66} \end{bmatrix}_{q=q_d} ;$$

$A_{42} = q_5(2I_t q_4 - 3I_a q_4 + 3I_a q_6 \sin q_2)\sec^2 q_2$;

$A_{52} = q_4[(I_1 - I_3)q_4 \cos 2q_2 + I_3 q_6 \sin q_2]$;

$A_{62} = q_5[I_a q_6 \sec^2 q_2 + 3(I_t - I_a)q_4 \sin 2q_2 + (2I_t - 3I_a)q_4 \sec q_2 \tan q_2]$;

$A_{44} = (2I_t - 3I_a)q_5 \tan q_2$; $A_{54} = [2(I_1 - I_3)q_4 \sin q_2 - I_3 q_6]\cos q_2$;

$A_{64} = q_5[(2I_t - 3I_a)\sin q_2 \tan q_2 - I_t \cos q_2]$;

$A_{45} = [I_a q_6 + (2I_t - 3I_a)q_4 \sin q_2]\sec q_2$;

$A_{65} = [I_a q_6 + (2I_t - 3I_a)q_4 \sin q_2]\tan q_2 - I_t q_4 \cos q_2$; and

$A_{46} = I_a q_5 \sec q_2$; $A_{56} = -I_3 q_4 \cos 2q_2$; and $A_{66} = I_a q_5 \tan q_2$

3. Open-loop (OL) control of SWM

The SWM features its orientation control with or without continuously spinning of a rotor. The feasibility of an open-loop control for the SWM which decouples the orientation control from the spin motion has been shown in [2]. The open-loop controller consists of two parts; torque model-based inclination (α, β) control and switching spin rate ($\dot{\gamma}$) control so that two motions can be controlled independently. Namely, the magnitude of current inputs is controlling the orientation of the rotor while the frequency of them the spinning rate. Although the control system has been successfully demonstrated both in simulations and experiments, the performance (in particular for transient periods such as oscillation, delay, and overshoot, etc) needs to be improved using either a feedback control system or a model based open loop control one of which is input shaping technique to shape an input of the system so as to minimi ze undesired effects in outputs.

Figure 2 shows an open-loop controller of the orientation of the SWM. The inclination current inputs are governed by the driving torque $\Delta\mathbf{T}$ directly. The torque required to maintain a desired orientation at (α_d, β_d) can be given by

$$\left[\tilde{\mathbf{T}}(\boldsymbol{\alpha}, \boldsymbol{\beta})\right]\mathbf{u}_{\alpha\beta} = \Delta\mathbf{T}_\mathbf{d} \quad (5)$$

where $\Delta\mathbf{T}_\mathbf{d}$ can be computed from the inverse of (1) including the dynamics of the rotor and $\tilde{\mathbf{T}}$ is the orientation-dependent torque constants in the forms of Lorentz equation or Maxwell stress tensor.

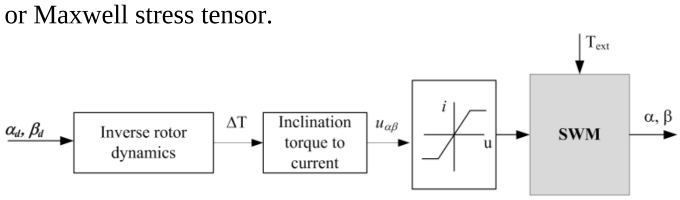


Fig. 2 Open-loop controller of SWM

In general, the orientation-dependent torque constants in (5) must be volume integrated numerically in real time. To reduce the computation to a tractable form, we take advantages that the torque is a linear function of to currents and apply the principle of superposition to compute the total magnetic torque acting on the rotor:

$$\tilde{\mathbf{T}} \approx \left[\hat{\mathbf{K}}_\mathbf{1} \cdots \mathbf{K}_\mathbf{j} \cdots \mathbf{K}_{\mathbf{m}_s}\right]\mathbf{u} ; \quad (6)$$

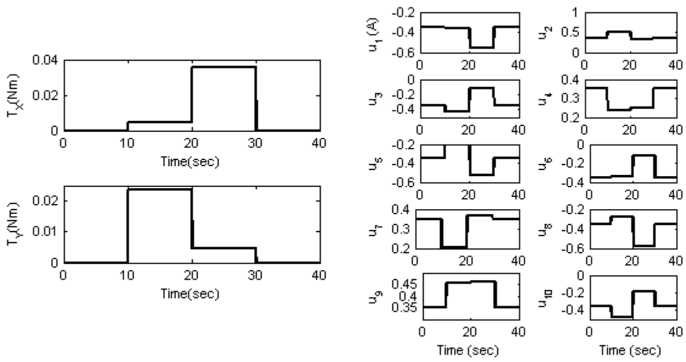
where
$$\hat{\mathbf{K}}_\mathbf{j} = \begin{cases} -\sum_{k=1}^{m_r} \left\{ \hat{f}(\varphi)\Big|_{\varphi=\varphi_{jk}} \dfrac{\mathbf{s}_j \times \mathbf{r}_k}{|\mathbf{s}_j \times \mathbf{r}_k|} \right\} & \text{if } \mathbf{s}_j \times \mathbf{r}_k \neq 0 \\ 0 & \text{if } \mathbf{s}_j \times \mathbf{r}_k = 0 \end{cases} ; \quad (6a)$$

$\mathbf{s}_j$ and $\mathbf{r}_k$ are the position vectors of the j^{th} EM and the k^{th} PM respectively; $\hat{f}(\varphi)$ is a curve-fitting function of the torque between the k^{th} PM and the j^{th} EM in terms of the separation angle φ:

$$\varphi_{jk} = \cos^{-1}\left(\mathbf{s}_j \bullet \mathbf{r}_k\right) / \left(|\mathbf{s}_j||\mathbf{r}_k|\right) \quad (7)$$

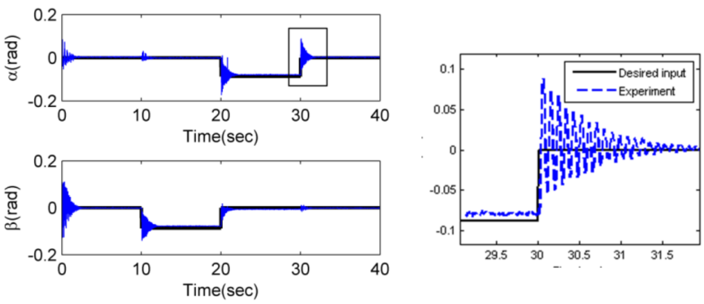
The current vector to generate this torque is given by the inverse model in (8). Figure 7 and 8 present the experimental data of the OL control using the step inputs given in (8). Although they have large oscillation and overshoot in the transient period, the results show feasibility of the OL control of the SWM.

$$\mathbf{u}_{\alpha\beta} = \left[\tilde{\mathbf{T}}\right]^\mathrm{T} \left(\left[\tilde{\mathbf{T}}\right]\left[\tilde{\mathbf{T}}\right]^\mathrm{T} \right)^{-1} \Delta\mathbf{T}_d \quad (8)$$



(a) Inclination torques (b) Applied current inputs

Fig. 7 Open loop control using step input



(a) Step response in OL control (b) Transient response

Fig. 8 Step response of the open-loop control

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