

# 구형 노치 관절의 자유도와 강성 해석 Degree of Freedom and Stiffness Analysis of Spherical Notch Joint

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## 1. Introduction

Flexible joints (flexure hinges) offer an alternative to traditional mechanical joints that alleviates many of their disadvantages, such as eliminating the presence of friction, backlash, and wear, producing sub-micron accuracy due to their continuous monolithic construction, simplifying production as monolithic construction, enabling low-cost fabrication. However, the motion of flexure hinge is not pure because axial shearing, torsion loading and bending exert a profound influence on the deformation of a flexure, which will lead to a puzzle when we design a parallel mechanism based on the flexure hinge. If the position of one end relative to the opposite one, the degree of freedom (DOF) of a three dimensional flexure hinge is six. When we use the pseudo-rigid-body model approach to model and analyze the compliant parallel mechanisms, it is very difficult to confirm the whole systems DOF and mobility, more difficult to arrange the actuators.

In the [1], a single-axis flexure hinge of constant-width and rectangular cross-section are modeled as have a revolute joint and a prismatic joint (2-DOF) instead of 1-DOF revolute joint, which increase the position accuracy of the system. In the [2, 3, 4], circular cross section profile of multi-axis flexure hinge is modeled as a spherical joint (3-DOF). N. Lobontiu [5] analyzed the circular cross-section corner-filletted flexure hinges of three-dimensional compliant mechanism, which considered the DOF of spherical notch joint is 6. But on the application of the practice, spherical notch hinge joint may be a universal joint (2-DOF) or a spherical joint, depending on the thinness of the narrowest cross-section and other geometric parameters. Pseudo-rigid-body models of spherical notch hinge joint are shown in Fig. 1.

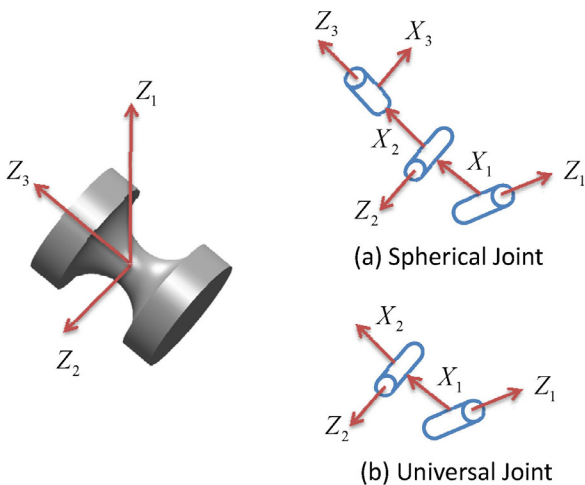


Fig. 1 Pseudo-rigid-body model of spherical notch hinge joint (a) spherical joint, (b) universal joint

This paper focus on the DOF analysis of the right-circular cross section of multi-axis flexure hinge. The circular cross section profile of multi-axis flexure hinge is described as Fig. 2. (a) is the corner-filletted and (b) is elliptical, right-circular can evolve from

the (a) or (b) suppose that  $l=0$  in (a), and  $r_x=r_y=r$  in (b), respectively.

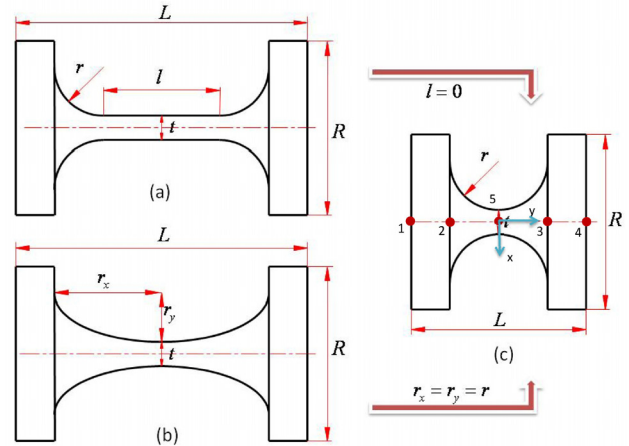


Fig. 2 Circular cross section profile of multi-axis flexure hinge (a) corner-filletted with radius  $r$  and length  $l$ , (b) elliptical with  $r_x$  and  $r_y$  semi-axis, (c) right-circular with radius  $r$

## 2. Degree of freedom and stiffness analysis

In mechanics, DOF is the set of independent displacements and/or rotations. The three dimensional flexure hinge is a 6-DOF element, without exception for spherical notch hinge. When we model the spherical notch joint, we must know how to ignore or pay attention to the displacement or rotation at certain direction. In the conventional operation, comparing the quantity of displacements/rotations under the certain force/ torque to evaluate the DOF of the element or system. However the quantity of displacements/rotations are related to the compliance (or spring rate) of the flexure hinge, and compliance is determined by the geometric parameter of the flexure hinge.

In the [3], closed-form compliance expressions of generic circular cross-section corner-filletted flexure hinge are obtained. Analytic compliance factors of point 3 which denote at Fig. 2 (c) are calculated and shown in Table 1. From the Table 1, it is clear to see that compliance factors increase sharply along with the non-dimensional parameter  $\alpha (=r/t)$ , especially for the rotational compliance factors.

Table 1 Parameters for simulation and analytic compliance factors

No.	$r$ (m)	$r$ (m)	$\alpha$	$C_{x,F_x}$ ( $\times 10^{-9}$ )	$C_{y,F_y}$ ( $\times 10^{-8}$ )	$C_{y,M_z}$ ( $\times 10^{-6}$ )	$C_{\theta,M_x}$ ( $\times 10^{-4}$ )
1	0.005	0.01	0.5	3.2	1.4	4.6	5.8
2	0.005	0.005	1	11.7	19.6	67.6	85
3	0.005	0.002	2.5	65.5	563.4	2400	2958

$R = \phi 0.03m, L = 0.1m, E = 70 \times 10^9 N/m^2, \nu = 0.25$

### 3. FEA modeling of right-circular spherical notch hinge joint

Three-dimensional model is built by using ANSYS, which use 4-node, three-dimensional, 6-DOF per node elements (SOLID 73). The parameters of modeled flexure hinge are shown in Table 1. A "smart meshing" technique is used that automatically refined itself in higher stress concentration circular regions.

Constraints and forces are applied at the opposite end of the flexure, where constraints are applied at the plain in possession of point 1 and force in the  $x$ -,  $y$ -,  $z$ - direction and moment along  $x$ -,  $y$ -,  $z$ - direction is applied at the point 4, respectively. Fig. 3 (a) show the model which added the force and constraints, and (b) is the results after deformed.

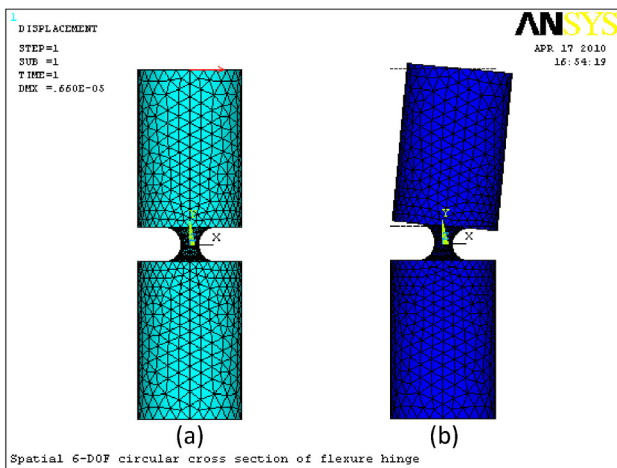


Fig. 3 FEA model (a) model added the force and constraints, (b) results after deformed

In the previous description, analytical design equations [3, 5] were derived to calculate compliances at point 3. But for nodal deformations list from FEA results at point 3 were the total deformation with respect to the section which contain the point 1. But we just concern on the deformation of hinge joint section, so the pure deformations caused by the hinge is the results of nodal deformation of point 3 subtracting the deformation of point 2. Fig. 4 showed the pure deformation which caused by force or moment at point 3.

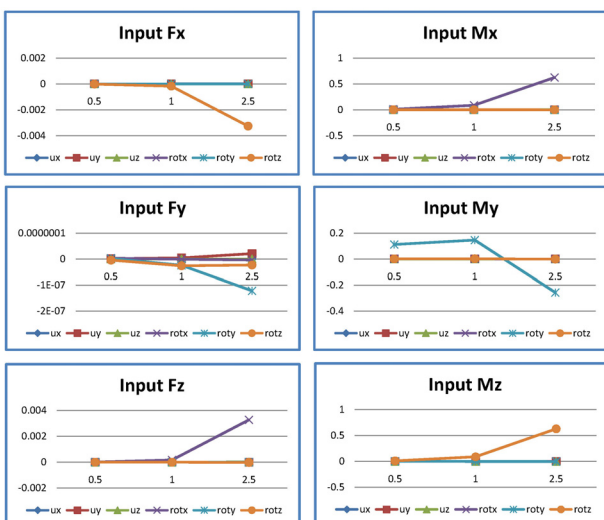


Fig. 4 Comparisons of deformation results of FEA

In the Fig. 4, it is clearly shown that rotational deformation is more distinct and active than the linear deformation at the point 3 under the every force or moment. That is to say spherical notch hinge joint can be assumed to 3 rotational joints instead of 3 linear joints and 3 rotational joints when we model it by using pseudo-rigid-body method. In this situation, spherical notch hinge joint should be a spherical joint. At the same time, from the Fig. 4, deformation of point 4 increases along with the non-dimensional parameter  $\alpha$  ( $= r/t$ ) obviously.

On the other hands, the smaller of non-dimensional parameter  $\alpha$ , rotational deformation is more inconspicuous. And through analyzing the FEA data, if  $\alpha = 1$ , two of three rotational deformations are larger than another one. However,  $\alpha = 0.5$ , two of three rotational deformations are almost same, in this situation, spherical notch hinge joint should be a universal joint.

### 4. Conclusions

The paper introduces the right circular cross-section flexure hinges for three-dimensional compliant mechanisms. Three special non-dimensional parameters are used to analyze the flexure hinge joint. Firstly, the Analytic compliance factors are calculated by using closed-form compliance equations. And find that compliance factors increase along with the non-dimensional parameter  $\alpha$ , especially for the rotational compliance factors. Secondly, finite element simulation results certificate the theoretical computation. Another purpose of the paper is to find an approach to model the spherical notch hinge joint to different DOF by using pseudo-rigid-body. Spherical notch hinge joint can be replaced by a universal joint when non-dimensional parameter  $\alpha = 0.5$ , and by a spherical joint when non-dimensional parameter  $\alpha = 2.5$ , depending on the thinness of the narrowest cross-section.

Because three special  $\alpha$  are used to analyze the hinge joint, it is difficult to find out the critical joint of non-dimensional parameter  $\alpha$ .

### Future work

In the future, a continuous non-dimensional parameter  $\alpha$  should be used to analyze the spherical notch joint. And stress and DOF analysis should obtained via both FEA and experimental measurements.

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