Application SVD–Least Square Algorithm for solving astronomical ship position basing on circle of equal altitude equation

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Abstract: This paper presents an improvement for calculating method of astronomical vessel position with circle of equal altitude equation based on using a virtual object to sun and two stars observation. In addition, to enhance the accuracy of ship position achieved from solving linear matrix system, and surmount the disadvantages on rank deficient matrices situation, the authors used singular value decomposition (SVD) in least square method instead of normal equation and QR decomposition, so, the solution of matrix system will be available in all situation. As proposal algorithm, astronomical ship position will give more accuracy than previous methods.

Key words: Celestial navigation, circle of equal altitude, vector–matrix method, SVD decomposition

1. Introduction

• Nowadays, with development of high technology have created many modern equipment to compute ship position accurately, such as GPS, INS. But, nothing can ensure that there are not any accidents to these. So, we should maintain the methods to backup when these modern equipment can’t use.

• In 2010, at the IMO conference in Manila, celestial navigation received sustained emphasis Manila amendment, there is an explicit encouragement for the usage of electronic nautical almanac and celestial navigation calculation software in Section B-II/1 of Chapter 2, “Guidance regarding the master and the deck department”

• Previous research use Line of position (LOP) to compute ship position, but LOP is not exact locus of ship. Actually, the exact locus of ship lie on Circle of position (COP), the other use COP, but there are some problems as complex procedure, not consider to the movement of ship position

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With above inference, we researched and developed celestial navigation method, it can use to backup. In this research, we use COP – vector matrix method and connect Singular Value Decomposition-least square method, virtual body to compute astronomical ship position. As using COP, so determined ship position will give more accurate than previous method
2. Establishing circle of equal altitude equation on Descartes coordinate system

The plane/equation of COP
\[ x \cdot (X - a_x) + y \cdot (Y - b_y) + z \cdot (Z - c_z) = 0 \]

\[ O \cdot A = k \cdot O \cdot C \]

\[ A(a_x, b_y, c_z) = O \cdot \left( k \cdot x, k \cdot y, k \cdot z \right) \]

\( k \): scale coefficient of vectors was determined according to formula

\[ k = \frac{O \cdot A}{O \cdot C} = \frac{\left| O \cdot A \right|}{\left| O \cdot C \right|} \]

\[ O \cdot A = k \cdot O \cdot C \]

The equation COP of virtual celestial object
\[ x_1 \cdot X + y_1 \cdot Y + z_1 \cdot Z = \sinh \phi \]

After that, from this system, we convert position which have been found to sphere coordinate system according to following formula

\[ \text{Lat} = \arctan \left( \frac{Z}{\sqrt{(X^2 + Y^2)(1 - e^2)}} \right) \]

\[ \text{Long} = \arctan \left( \frac{Y}{X} \right) \]

3. The SVD-Least square algorithm for solving astronomical ship position and virtual celestial body

Normal decomposition Least square algorithms

Matrix system for ship position
\[
\begin{bmatrix}
  x_1 & y_1 & z_1 \\
  x_2 & y_2 & z_2 \\
  \vdots & \vdots & \vdots \\
  x_n & y_n & z_n
\end{bmatrix}
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
= \begin{bmatrix}
  v_1 \\
  v_2 \\
  \vdots \\
  v_n
\end{bmatrix}
\]

\[ A \cdot x + v = b \]

\[ \min \left\{ (b - A \cdot x)^T (b - A \cdot x) \right\} \]

\[ A^T \cdot A \cdot x = A^T \cdot b \]

Solution
\[ x = (A^T \cdot A)^{-1} \cdot A^T \cdot b \]

Singular Value Decomposition – least square algorithm for ship position

\[ A = U \cdot \Sigma \cdot V^T \]

\[ \Sigma = \begin{bmatrix}
  \sigma_1 & 0 & \cdots & 0 \\
  0 & \sigma_2 & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  0 & 0 & \cdots & \sigma_p
\end{bmatrix} \]

\[ U(m \times m), V(n \times n) \] are orthogonal

Solution
\[ x = V \cdot \Sigma^{-1} \cdot U^T \cdot b \]

Using Singular Value Decomposition (SVD) have most accurate to solve matrix system for ship position

Virtual celestial object

Virtual celestial object is used in 2 stars and sun condition

4. Experiment results

• In this paper, some experiments were performed with numeric database of sun, stars on Do Son sea area – Hai Phong – Vietnam

• least square method can carry out in all situations of celestial bodies observation not fear celestial rank matrix deficient with most ship position accuracy

• In case sun or two stars, It is insufficient equation for solving matrix system, It is necessary to use virtual body. In case, altitude observations are more than 2, we don’t need to use this body. Visual Basic 6.0 was also used to solve this problem
4. Experiment results

• In this paper, some experiments were performed with numeric database of sun, stars on Do Son sea area – Hai Phong – Vietnam
• Least square method can carry out in all situations of celestial bodies observation not fear celestial rank matrix deficient with most ship position? accuracy
• In case sun or two stars, It is insufficient equation for solving matrix system, It is necessary to use virtual body. In case, altitude observations are more than 2, we don’t need to use this body. Visual Basic 6.0 was also used to solve these problems

Case 1: fix by sun

<table>
<thead>
<tr>
<th>Body</th>
<th>UT</th>
<th>Ho</th>
<th>x</th>
<th>y</th>
<th>z</th>
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<td>0.9388</td>
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ALat = 20°39′ N, ALong = 106°50′ E, TC = 980, ZN = +07, (Aug 20, 2011)

Case 2: fix by two stars

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(Apr 1, 2011), ZN = +07, ALat = 20°44′ N, ALong = 107°06′ E, TC = 1120

Case 3: fix by three stars

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5. Conclusion

The authors present an improvement to celestial navigation method to determine ship position basing on: circle of equal altitude equation, virtual body and SVD – least square method algorithms. The experiment results also show that ship position accuracy to be more than traditional methods. This approach has the potential to be integrated with ECDIS

Although there are some improve, celestial navigation is not instead of GPS as low cost, high accuracy and reliability. However, it can be used to confirm and backup for GPS. So, In this paper, we carried out some development about celestial navigation method to be possible to take its become e-navigation system in the future.