Modeling and Analysis of Duty-Cycle-Controlled Buck LED Driver

Marn-Go Kim, Young-Seok Jung and Nam-Ho Kim
Pukyong National University

ABSTRACT

A discrete time domain modeling for the duty-cycle-controlled buck LED driver is presented in this paper. Based on the modeling result, a root locus analysis for the buck LED driver is done to derive the stability boundaries of feedback gains.

1. Introduction

Over the past few years, light-emitting diode (LED) technology has emerged as a promising technology for residential, automotive, decorative and medical applications. This is mainly caused by the enhanced efficiency, energy savings and flexibility, and the long lifetime. Today, LEDs are available for various colors and they are suitable for white illumination. Up to now, numerous attempts have been made to characterize the current-mode control system\(^1\)[14]. However, all mentioned modeling approaches are related to voltage regulated converters. Very little work has been done in the area of dynamic modeling for the current regulated LED driver\(^7\).

In this paper, the systematic discrete time domain approach\(^6\) is adapted to modeling and analysis for the duty-cycle-controlled buck LED driver shown in Fig. 1. Root locus analysis is employed to derive the stability boundaries.

2. Discrete time domain modeling of duty-cycle-controlled buck LED driver

\[
\delta x_{k+1} = A \cdot \delta x_k + B \cdot \delta u,
\]

where

\[
\delta x_{k+1} = [\delta i_{k+1} \ \delta v_{k+1}]^T, \ \delta x_k = [\delta i_k \ \delta v_k]^T,
\]

\[
A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, \ B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix},
\]

\[
a_{11} = 1 - \frac{1}{(1-D)} \left( \frac{k_p + k_{n1}}{k_p + k_{n2}D/2 + S_r} \right), \quad a_{12} = \frac{1}{R_s} \left( \frac{1}{1-D} \right) \left( \frac{k_p + k_{n1}}{k_p + k_{n2}D/2 + S_r} \right),
\]

\[
a_{21} = \frac{1}{(1-D)} \left( \frac{k_{n1}(k_{n2}D/2 + S_r)}{k_p + k_{n2}D/2 + S_r} \right), \quad a_{22} = 1 - \left( \frac{1}{(1-D)} \right) \left( \frac{k_{n1}}{k_p + k_{n2}D/2 + S_r} \right),
\]

\[
b_1 = \frac{1}{R_s} \left( \frac{1}{1-D} \right) \frac{1 + k_p + k_{n2}D}{k_p + k_{n2}D/2 + S_r}, \quad b_2 = -\left( \frac{k_{n1}(1+k_pD/2 + S_r)}{k_p + k_{n2}D/2 + S_r} \right),
\]

\[
D = \frac{V_o}{V_i}, \ k_p = \frac{r_i}{R_s}, \ k_{n1} = k_pT_c, \ S_r = \frac{M_o}{(V_o/1)(1-D)}.
\]

3. Analysis

Bode plots have been commonly used to assess the stability of
the closed-loop system by finding the phase margin, but these plots cannot give information on the dynamic behavior of the individual state variables. On the other hand, root locus analysis can provide the engineer with the stability and the transient performance of the individual state variables related to the location of the roots of the characteristic equation.

To analyze the stability and dynamic characteristics of the closed-loop system, the eigenvalues of the system matrix is evaluated. The eigenvalues of \( \mathbf{A} \) is the solutions of

\[
[\mathbf{A} - z] = 0
\]

where \( I \) is the identity matrix. The following root locus analysis is performed for \( R_y = 1 \).

The root locus as a function of the \( P \) gain \( k_p \) for \( k_{nl} = 0.2 \), \( D=0.45 \), and \( S_P = 0.82 \) is shown in Fig. 4. The eigenvalues \( \lambda_1 \) and \( \lambda_2 \) in the z-plane are plotted. Unlike the peak-current-controlled buck LED driver reported in [8], this duty-cycle-controlled buck LED driver is unstable for \( k_p = 0 \). The eigenvalue \( \lambda_1 \) is dominated by the inductor current state. The transient response of \( \lambda_1 \) after a disturbance is underdamped when \( k_p \) is between 0 and 0.6. At \( k_p = 0.6 \), the system response is critically damped. Then \( \lambda_1 \) moves towards the origin of the unit circle with increasing \( k_p \), which means the inductor current becomes faster. When \( k_p \) is greater than 0.6, the transient response of the inductor current is overdamped. Increasing \( k_p \) much further, the current response is underdamped with a natural resonant frequency equal to \( f_z/2 \) due to the negative real value of \( \lambda_1 \). On the one hand, the eigenvalue \( \lambda_2 \) is dominated by the capacitor voltage state of the error amplifier. The transient response of \( \lambda_2 \) is underdamped when \( k_p \) is between 0 and 0.6 due to the two complex roots. Then, \( \lambda_2 \) moves towards the unit circle with increasing \( k_p \), which means the capacitor voltage becomes slower. The capacitor voltage is overdamped when \( k_p \) is greater than 0.6.

Fig. 5 shows stability boundaries of \( k_{nl} \) as a function of \( D \). When \( k_{nl} \) is between zero and the stability boundary, the system is stable. While the peak-current-controlled buck LED driver can be stable by selecting a proper gain for \( k_p = 0 \) [8], this duty-cycle-controlled buck LED driver is always unstable for \( k_p = 0 \). The stability boundary of \( k_{nl} \) is increasing with increasing \( k_p \) for \( D < 0.5 \). But, this stability boundary of \( k_{nl} \) is decreasing with increasing \( k_p \) for \( D > 0.5 \). The stable range of \( k_{nl} \) is very wide for a fixed \( k_p \) and \( D \). However, the design engineer need to select the optimum \( k_p \) and \( k_{nl} \) for a good transient response instead of the simple stable gains.

![Fig. 4. Root locus as a function of the P gain \( k_p, k_{nl}=0.2, D=0.45, S_P=0.82 \)](image)

![Fig. 5. Theoretical stability boundaries of \( k_{nl} \) as a function of \( D \) \( G_p = \frac{1.4}{1+D} \)](image)

References


