ADDENDA TO "ON THE COMPACTNESS OF THE STRUCTURE SPACE OF A RING"

Wuhan Lee

1. In [1] we gave necessary and sufficient condition for the compactness of the structure space of a ring with the property that

(C′) Every principal two-sided ideal is modular.

If the zero ideal {0} is modular, then the ring A has an identity and conversely if A has an identity then every ideal is modular. This fact leads to that the condition (C′) is simply equivalent to the condition that A has an identity and it is well known that for a ring with an identity the structure space is always compact.

As pointed out by R. Blair in his recent letter to us, our original intention was to state as follows instead of (C′):

(C*) Every nonzero principal two-sided ideal is modular.

Proposition 7 and Theorem 2 in [1] with (C*) instead of (C′), however are still valid under the hypothesis that A is not a radical ring; since $B \neq \{0\}$, B contains a nonzero principal two-sided ideal and hence B is modular.

2. Let consider a ring A with the property that

(C″) No nonzero homomorphic image of A is a radical ring.

According to R. Blair and L.C. Eggan [2], for such a ring the structure space is compact if and only if A is generated as an ideal by a finite number of elements.

We shall show relations among those classses of rings satisfying conditions (C), (C′), (C″), and (C*).

(C) For every U_a of $\{U_a\}$, $a \in A$, DU_a is modular.

Proposition 1. If A is not a radical ring then (C*) implies (C″).

Proof: Let B be a proper ideal in A. If $B=\{0\}$, then $A/B \simeq A$ is not a radical ring; and if $B \neq \{0\}$, then B is modular by (C*), so A/B has an identity and is therefore again not a radical ring.

The above result shows that Proposititon 7 in [1] follows immediately from the Corollary to Theorem 2 in [2].

Proposition 2. Condition (C) implies neither (C″) nor (C*).

Proof: Let B be a simple ring witn an identity element and let R be a nonzero radical ring, and let A be the direct sum $B \oplus R$. Then A has exactly one primitive ideal, namely, R. Thus, for any $a \in A$, either $DU_a = A$ (if $a \in R$) or $DU_a = R$ (if $a \notin R$). In either case, DU_a is modular, so that A satisfies (C). However, $A/B \simeq R$ is a radical ring, so A does not satisfy (C″). Moreover A can have no identity element, so B is not modular. Thus A does not satisfy (C*).

Proposition 3. Neither (C″) nor (C*) implies (C).

Proof: Let A be a simple ring without an identity element. Clearly A satisfies (C″) and (C*). The only primitive ideal is the zero ideal {0}. If $a \in A$ with $a \neq 0$, the $DU_a = \{0\}$. Since A has no identity element, DU_a is not modular, so A does not satisfy (C).

REFERENCES

1. Wuhan Lee, *On The Compactness of The Structure Space of A Ring*, J. Korean Math. Soc. vol. 1 (1964) p. 3-5
2. R. Blair and L.C. Eggan. *On The Compactness of The Structure Space of A Ring*, Proc. Amer. Math. Soc. vol. 11 (1960) pp. 876-879.