NOTE ON HYPERSURFACES WITH (f, g, u, v, λ)- STRUCTURE OF S^{2n+1}

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The odd-dimensional sphere S^{2n+1} has an almost contact structure that is natually induced from the Kaehler structure of Euclidean space E^{2n+2}. Blair [1], Ludden [1], Okumura [8] and Yano [1], [8] showed that a submanifold of codimension 2 of an almost complex manifold and a hypersurface of an almost contact manifold admit an (f, g, u, v, λ)-structure.

Recently Blair, Ludden and Yano [2] proved the following

THEOREM. *Let M^{2n} be a complete orientable hypersurface of S^{2n+1} of constant scalar curvature. If the (f, g, u, v, λ)-structure induced on M^{2n} satisfies $Kf+fK=0$ and $\lambda\neq$ constant where K is the Weingarten map of the embedding, then M^{2n} is S^{2n} or $S^n\times S^n$.*

In the present paper, we study the hypersurface M^{2n} of S^{2n+1} satisfying $\lambda=$constant and $Kf+fK=0$.

§1. Hypersurface of S^{2n+1}

Let S^{2n+1} be the natural sphere of dimension $2n+1$ in Euclidean space E^{2n+2}. Let (ϕ, ξ, η, g) be the mormal almost contact metric structure induced on S^{2n+1} by the Kaehler structure on E^{2n+2}. That is to say, ϕ is a tensor field of type (1, 1), ξ is a vector field, η is a 1-form and g is a Riemannian metric on S^{2n+1} satisfying

$$(1.1)\quad \begin{cases} \phi^2=-I+\eta\otimes\xi, \\ \phi\xi=0, \quad \eta\circ\phi=0, \\ \eta(\xi)=1, g(\phi\tilde{X}, g(\phi\tilde{Y}, \phi\tilde{Y})+\eta(\tilde{X})\eta(\tilde{Y})=g(\tilde{X}, \tilde{Y}) \\ [\phi, \phi]+d\eta\otimes\xi=0, \end{cases}$$

where $[\phi, \phi]$ is the Nijenhuis torsion tensor of ϕ and $\tilde{X}$ and $\tilde{Y}$ are arbitrary vector field on S^{2n+1}

Suppose π: $M^{2n}\longrightarrow S^{2n+1}$ is an embedding of the orientable manifold M^{2n} in S^{2n+1}. The tensor G defined on M^{2n} by

$$G(X, Y)=g(\pi_*X, \ \pi_*Y)$$

is a Riemannian metric on M^{2n}. Here, π_* denotes the differential of the embedding π. If C is a field of unit normals defined on M^{2n} and $\tilde{\nabla}$ is the Riemannian connection of g then the Gauss and Weingarten equations can be written as

(1.2) $\begin{cases}\tilde{\nabla}_{\pi_* X}\ \pi_* Y = \pi_*(\nabla_X Y) + k(X,Y)C, \\ \tilde{\nabla}_{\pi_* X} C = \pi_*(KX).\end{cases}$

Then ∇ is the Riemannian connection of G, K is a symmetric tensor of type (0,2) on M^{2n} and

$$G(KX,Y)=k(X,Y).$$

If we set

$$\phi\pi_* X = \pi_* fX + v(X)C,\ \xi = \pi_* U + \lambda C,$$
$$\phi C = -BV,\ u(X) = \eta(\pi_* X),$$

where, f is a tensor field of type (1,1), U and V are vector fields, u, v are 1-forms and λ is a function. Then M^{2n} admits an $(f, g, U, V, u, v, \lambda)$ -structure [1], [8], that is,

(1.4) $\begin{cases} f^2 = -I + u\otimes U + v\otimes V, \\ u\circ f = \lambda v,\ v\circ f = -\lambda u \\ fU = -\lambda V,\ fV = \lambda U, \\ u(U) = v(V) = 1-\lambda^2,\ u(V) = v(U) = 0, \\ G(fX, fY) = G(X,Y) - u(X)u(Y) - v(X)v(Y) \end{cases}$

Differentiating (1.3) covariantly along M^{2n} and taking account of (1.2)~(1.4), we find [2], [8]

(1.5) $(\nabla_X f)Y = G(X,Y)U - u(Y)X - k(X,Y)V + v(Y)KX,$

(1.6) $\nabla_X U = -fX - \lambda KX,$

(1.7) $\nabla_X V = -\lambda X + fKX$

(1.8) $\nabla_X \lambda = v(X) + k(U, X)$

In the sequel we assume that λ is constant different from $0, \pm 1$ in the hypersurface M^{2n}.

Then, we have from (1,8)

(1.9) $KU = -V.$

§2. Hypersurface with $du=0$.

In this section we assume that in the hypersurface M^{2n} $du=0$, that is, equivalent to

(2.1) $fK + Kf = 0$

by virtue of (1,7).

From (2,1), we have [2], [6]

$$\text{trace } K = 0,$$

by virtue of (1,7). From (2,1), we have [2], [6]

$$\text{trace } k = 0,$$

(2.2) $KU=\alpha U+\beta V$

$KV=\beta U-\alpha V$

Using (1.9), we can see thas $\alpha=0, \beta=-1$. So

(2.3) $KU=-V$,

(2.4) $KV=-U$.

If we apply ∇_X to equation (2.4) use equation (1.6), (1.7) and use the fact $(\nabla_X K)Y=(\nabla_Y K)X$ because of the Codazzi equation, we find that

(2.5) $F(X,Y)-F(KX,KY)=0$,

where $F(X,Y)=G(fX,Y)$.

Replace Y be fY in the equation (2.5) and use (1.4) to obtain

$G(KX,KY)-u(KX)u(KY)-v(KX)v(KY)$

$=G(X,Y)-u(X)u(Y)-v(X)v(Y)$,

from this we see that

$$K^2=\frac{2}{1-\lambda^2}(u\otimes U+v\otimes V)-I,$$

from which, $K=0 \quad (n\geq 2)$.

If $n=1$, then we can see that from the equation of Ganss the scalar curvature is zero and consequently the curvature tensor is zero (cf [6]).

THEOREM. *If M^{2n} is a complete orientable hypersurface of S^{2n+1} satisfying $du=0$ and λ is constant different from* $0, \pm 1$, *then.*

(1) M^{2n} is a great sphere S^{2n} $(n\geq 2)$,

(2) M^2 is locally Euclidean.

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