ON ANTI-COMMUTE (f, g, u, v, λ)-STRUCTURES ON SUBMANIFOLDS OF CODIMENSION 2 IN AN EVEN DIMENSIONAL EUCLIDEAN SPACE

By Jin Suk Pak

§1. Introduction

A structure induced on a submanifold of codimension 2 of an almost Hermitian manifold and called an (f, g, u, v, λ)-structure has been studied in [1], [2], [3], [4]. The submanifolds of codimension 2 in an even-dimensional Euclidean space in terms of this structure have been studied by Ki [4], [5], Okumura [7], Pak [4], Yano [5], [6], and the others.

In the present paper, we study submanifolds of codimension 2 of the even-dimensional Euclidean space under the assumptions such that the linear transformations $h_j^{\ i}$ and $k_j^{\ i}$ which are defined by the second fundamental tensors anti-commute with $f_j^{\ i}$.

In §2, we consider a submanifold of codimension 2 of a Kaehlerian manifold and find several equations which the induced (f, g, u, v, λ)-structure satisfies.

In §3, we study submanifolds of codimension 2 of the even dimensional Euclidean space under the our assumptions stated above. In the last §4, we study submanifolds under the same assumptions in a locally Fubinian manifold.

2. Certain submanifolds of codimension 2 of a Kählerian manifold ([4], [6]).

Let M be a 2n-dimensional differentiable manifold which is covered by a system of coordinate neighborhoods $\{U;\ x^h\}$ and which is differentiably immersed in a (2n+2)-dimensional Kählerian manifold M covered by a system of coordinate neighorhoods $\{\tilde{U};\ y^\kappa\}$ as a submanifold of codimension 2 by the equations

$$y^\kappa = y^\kappa(x^h),$$

where, hear and in the sequel the indices $\kappa,\ \lambda,\ \mu,\ \nu,\ \cdots$ run over the range $\{1, 2, \cdots, 2n+2\}$ and $h,\ i,\ j,\ \cdots$ over the range $\{1, 2, \cdots, 2n\}$ respectively.

We put $(F_\mu^{\ \kappa},\ G_{\mu\lambda})$ be the Kählerian structure, that is,

$$F_\mu^{\ \kappa} F_\lambda^{\ \mu} = -\delta_\lambda^\kappa,$$

and $G_{\mu\lambda}$ a Riemannian metric such that

$$G_{\beta\alpha} F_\mu^{\ \beta} F_\lambda^{\ \alpha} = G_{\mu\lambda},$$

$$\tilde{\nabla}_\mu F_\lambda^{\ \kappa} = 0,$$

where $\tilde{\nabla}$ denotes by the operator of covariant differentiation with respect to the Christoffel symbols $\left\{ {\kappa \atop \mu\lambda} \right\}$ formed with $G_{\mu\lambda}$ and put

$$B_i^{\kappa}=\partial_i y^{\kappa},\quad (\partial_i=\partial/\partial x^i).$$

Then we find B_i^{κ} is, for fixed i, a local vector field of M tangent to M and the vectors B_i^{κ} are linearly independent in each coordinate neighborhood. B_i^{κ} is also, for fixed κ, a local 1-form of M and then the transforms $F_{\lambda}^{\kappa}B_i^{\lambda}$, $F_{\lambda}^{\kappa}C^{\lambda}$ and $F_{\lambda}^{\kappa}D^{\lambda}$ may be respectively expressed as linear combinations of B_i^{κ}, C^{κ} and D^{κ}, that is,

$$\begin{aligned} &F_{\lambda}^{\kappa}B_i^{\lambda}=f_i^{\ h}\ B_h^{\ \kappa}\ +u_i\ C^{\kappa}+v_i\ D^{\kappa},\\ (2.1)\quad &F_{\lambda}^{\kappa}C^{\lambda}=-u^i\ B_i^{\kappa}+\lambda D^{\kappa},\\ &F_{\lambda}^{\kappa}D^{\lambda}=-v^i\ B_i^{\kappa}-\lambda C^{\kappa},\end{aligned}$$

where C^{κ} and D^{κ} are two mutually orthogonal unit vectors of M normal to M and chosen in such a way that $2n+2$ vectors B_i^{κ}, C^{κ}, D^{κ} give the positive orientaion of M, g_{ji} being the Riemannian metric on M induced from that of $\tilde{M}$, λ is a function on M and

$$u^i=u_t g^{ti},\quad v^i=v_t g^{ti}$$

We can easily verify that λ is a function globally defined on M. From (2,1) and taking account of itself, we find

$$\begin{aligned} &f_j^{\ t}\ f_t^{\ h}=-\delta_j^{\ h}+u^h u_j+v^h v_j,\\ (2.2)\quad &f_i^{\ h}u^i=-\lambda v^h,\quad f_i^{\ h}v^i=\lambda u^h,\\ &f_h^{\ i}u_i=\lambda v^h,\quad f_h^{\ i}v_i=-\lambda u_h,\\ &u^i u_i=1-\lambda^2=v^i v_i,\\ &u_i v^i=0,\quad v_i u^i,\ =0,\end{aligned}$$

that is, M admits an (f,g,u,v,λ)-structure [6].

Moreover, f_{it} is skew-symmetric with respect to i and t, where

$$f_{it}=f_i^{\ s}g_{ts}$$

We denote by $\left\{ {h \atop ji} \right\}$ and ∇_i the Christoffel symbols formed with g_{ji} and by the operator of covariant differentiation with respect to $\left\{ {h \atop ji} \right\}$ respectively.

Then the equations of Gauss and Weingarten of M are

$$\nabla_j B_i^\kappa = h_{ji} C^\kappa + k_{ji} D^\kappa,$$

(2.3) $$\nabla_j C^\kappa = -h_j^i B_i^\kappa + j_j D^\kappa$$

and

$$\nabla_j D^\kappa = -k_j^i B_i^\kappa - l_j C^\gamma$$

respectively, where h_{ji} and k_{ji} are the second fundamental tensors with respect to C^κ and D^κ respectively, and h_j^i, k_j^i are Weingarten maps corresponding the normals defined by

$$h_j^i = h_{jt} g^{ti}, \quad k_j^i = k_{jt} g^{ti},$$

and l_j is the third fundamental tensor.

From (2.1) and (2.3), we have [6]

(2.4)
$$\nabla_j f_i^s = -h_{ji} u^s + h_j^s u_i - k_{ji} v^s + k_j^s v_i,$$
$$\nabla_j u^i = -h_{jt} f_i^t - \lambda k_j^i + l_j v_i,$$
$$\nabla_j v_i = -k_{jt} f_{it} + \lambda h_{ji} - l_j u_i$$
$$\nabla_j \lambda = k_{jt} u^t - h_{jt} v^t.$$

From now and in the sequel we suppose that in the submanifold M h_j^i and k_j^i anti-commute with f_j^i, that is,

(2.5) $$f_j^t h_t^i = -h_j^t f_t^i, \quad f_j^t k_t^i = -k_j^t f_t^i,$$

or equivalently $f_j^t h_{ti}$ and $f_j^t k_{ti}$ are symmetric with respect to j and i and that the globally defined function λ is constant different from 0 and 1 on the submanifold M.

Transvecting (2.5) with f_j^i and using of (2.2), we get

$$h_t^t = (1-\lambda^2)(\alpha+\gamma),$$

where we have put

$$h_{st} u^s u^t = (1-\lambda^2)\alpha, \quad h_{st} v^s v^t = (1-\lambda^2)\gamma.$$

Transvecting again (2.5) by u^i and taking account of (2.2), we also get

$$0 = h_s^t u^s f_{ti} - \lambda h_{ti} v^t ,$$

and then by transvecting the above equation with f^{ij} we obtain

$$h_j^t u_t = \alpha u_j + \beta v_j,$$

where

$$h_{st} u^s v^t = (1-\lambda^2)\beta$$

On the other hand, transvecting (2.5) by v^j, we can also find

$$h_s^t v^s f_{ti} + \lambda h_{ti} u^t = 0.$$

Transvecting the above equation with f^{ij} and taking account of (2,2), we have

$$h_j^t v_t = \beta u_j + \gamma v_j.$$

From these relations we can see

$$\lambda(\alpha+\gamma)=0.$$

By the similar method we can also verify that

$$k_j^t u_t = \bar{\alpha} u_j + \bar{\beta} v_j, \quad k_j^t v_t = \bar{\beta} u_j + \bar{\gamma} v_j,$$

$$k_s^s = 0,$$

where we have put

$$k_{st} u^s u^t = (1-\lambda^2)\bar{\alpha}, \quad k_{st} u^s v^t = (1-\lambda^2)\bar{\beta},$$

$$k_{st} v^s v^t = (1-\lambda^2)\bar{\gamma}.$$

Moreover, from (2.4) we have

$$h_{ji} v^i = k_{ij} u^i.$$

Thus, summing up, we find

$$(2.6)\qquad \begin{aligned} h_{ji} u^i &= \alpha u_j + \beta v_j, \\ h_{ji} v^i &= \beta u_j - \alpha v_j, \\ k_{ji} u^i &= \beta u_j - \alpha v_j, \\ k_{ji} v^i &= -\alpha u_j - \beta v_j, \\ h_s^s &= 0, \ k_s^s = 0. \end{aligned}$$

§ 3. Anti-submanifold of codimension 2 in a Euclidean space.

In this section we consider the submanifold M of codimension 2 under the assumptions stated in the previous section in a $(2n+2)$-dimensional Euclidean space.

In this submanifold M, it is well known that the equations of Gauss, Codazzi and Ricci are

$$(3.1)\qquad R_{kji}{}^s = h_k{}^s h_{ji} - h_j{}^s h_{ki} + k_k{}^s k_{ji} - k_j{}^s k_{ki}$$

$$(3.2)\qquad \begin{aligned} &\nabla_k h_{ji} - \nabla_j h_{ki} - l_k k_{ji} + l_j k_{ki} = 0, \\ &\nabla_k k_{ji} - \nabla_j k_{ki} + l_k h_{ji} - l_j h_{ki} = 0, \end{aligned}$$

and

(3.3) $\nabla_j l_i - \nabla_i l_j + h_j^t k_{it} - h_i^t k_{jt} = 0,$

respectively, where $R_{kji}{}^s$ are components of the curvature tensor of M.

Now, covariantly differentiating the first equation of (2,6), we have

$$(\nabla_k h_{ji}) u^i + h_{ji} \nabla_k u^i$$
$$= (\nabla_k \alpha) u_j + (\nabla_k \beta) v_j + \alpha \nabla_k u_j + \beta \nabla_k v_j \ .$$

Taking the skew-symmetric part of this equation with respect to k and j, and then substituting (2.6) and (3.2) we can see

(3,4) $2h_{ks} h_{jt} f^{st} + \lambda (h_{kt} k_j^{\ t} - h_{jt} k_k^{\ t})$
$$= (\nabla_k \alpha - 3\beta l_k) u_j - (\nabla_j \alpha - 3\beta l_j) u_k$$
$$+ (\nabla_k \beta + 3\alpha l_k) v_j - (\nabla_j \beta + 3\alpha l_j) v_k$$

by virtue of (2.2), (2,4) and (2.5).

Tranvecting (3.4) with u^j and taking account of (2.6), we have

$$0 = (1-\lambda^2)(\nabla_k \alpha - 3\beta l_k) - u^t (\nabla_t \alpha - 3\beta l_t) u_k$$
$$- u^t (\nabla_t \beta + 3\alpha l_t) v_k$$

or

$$\nabla_k \alpha - 3\beta l_k = \frac{1}{1-\lambda^2} \{u^t (\nabla_t \alpha - 3\beta l_t) u_k + u^t (\nabla_t \beta + 3\alpha l_t) v_k\}$$

Substituting this equation into (3,4), and transvecting again with v^j, we get

$$0 = (1-\lambda^2)(\nabla_k \beta + 3\alpha l_k) - u^t (\nabla_t \beta + 3\alpha l_t) u_k - v^t (\nabla_t \beta + 3\alpha l_t) v_k$$

by virtue of (2.6).

Substituting again this relation into (3.4), we fine

(3.5) $2h_{kt} h_{js} f^{ts} + \lambda (h_{kt} k_j^{\ t} - h_{jt} k_k^{\ t}) = 0.$

On the other hand, covariantly differentiating the second equation of (2.6), we obtain

$$(\nabla_k h_{ji}) v^i + h_{ji} \nabla_k v^i$$
$$= (\nabla_k \beta) u_j - (\nabla_k \alpha) v_j + \beta \nabla_k u_j - \alpha \nabla_k v_j$$

Taking the skew-symmetric part of this equation with respect to k and j, and substituting again (2.6) and (3.2), we also find

$$(h_{ks}k_{jt}-h_{js}k_{kt})f^{st}=(\nabla_k\beta+3\alpha l_k)u-(\nabla_j\beta+3\alpha l_j)u_k$$
$$-(\nabla_k\alpha-3\beta l_k)v_j+(\nabla_j\alpha-3\beta l_j)v_k.$$

by virtue of (2.2), (2.4), (2.5) and (2.6).

Since

$$(h_{ks}k_{jt}-h_{js}k_{kt})f^{st}u^j=0$$

and

$$(h_{ks}k_{jt}-h_{js}k_{kt})f^{st}v^j=0,$$

from the above relations, we can verify

$$(h_{ks}k_{jt}-h_{js}k_{kt})f^{st}=0.$$

Transvecting this equation with f_i^j, we have

(3.6) $h_{kt}k_j^{\ t}+h_{jt}k_k^{\ t}=0$

by using of (2.5) and (2.6).

Comparing (3.6) with (3.5), we find

(3.7) $h_{ks}h_{jt}f^{st}+\lambda h_{kt}k_j^{\ t}=0.$

Similarly, taking the covariant differentiation of the last equation of (2.6), we obtain

$$(\nabla_k k_{ji})v^i+k_{ji}\nabla_k v^i$$
$$=-(\nabla_k\alpha)u_j-(\nabla_k\beta)v_j-\alpha\nabla_k u_j-\beta\nabla_k v_j$$

Taking the skew-symmetric part of this relation with respect to k and j, and substituting (2.6) and (3.2), we get

$$2k_{kt}k_{js}f^{ts}+\lambda(h_k^{\ t}k_{tj}-h_j^{\ t}k_{tk})$$
$$=-(\nabla_k\alpha-3\beta l_k)u_j+(\nabla_j\alpha-3\beta l_j)u_k$$
$$-(\nabla_k\beta+3\alpha l_k)v_j+(\nabla_j\beta+3\alpha l_j)v_k$$

by virtue of (2.2), (2.4), (2.5) and (2.6).

Comparing this equation with (3.4) and taking account of (3.5) and (3.6), we also get

(3.8) $k_{ks}k_{jt}f^{st}+\lambda h_k^{\ t}k_{tj}=0.$

From (3.7) and (3.8), we can easily see that

(3.9) $h_{kt}h_j^{\ t}=k_{kt}k_j^{\ t}.$

On the other hand, taking the covariant differentiation of (2.5) and taking account of (2.2), (2.4), (2.5) and (2.6), we have

(3.10) $R_t^t = R = -4(\alpha^2+\beta^2)$

and

(3.11) $R_{ji}u^i = \frac{R}{2}u^j, \quad R_{ji}v^i = \frac{R}{2}v_j$

by virtue of (3.1).

Moreover, transvecting (3.7) with f_i^j and using of (2.2), (2,4), (2.5) and (2.6), we get

(3.12) $R_{ji} = \frac{R}{2}(u_j u_i + v_j v_i) + R_{st} f_j^s f_i^t.$

Thus we have

PROPOSITION 3.1. *Let the submanifold M of codimension 2 of a $(2n+2)$-dimensional Euclidean space be such that H and K anti-commute with f, where H and K are Weingarten maps with respect to the normals C and D respectively. If λ is constant different from 0 and 1, then the relation*

$$R_j^t f_t^i + f_j^t R_t^i = 0,$$

that is, Ricci tensor R of M anti-commute with f on M.

From (3.12), we can see that

$$R_{ks} R^s{}_t R^t{}_j = \frac{R}{2} R_{kt} R^t{}_j$$

by virtue of (3.9), (3.10) and (3.11).

Thus the only eigenvalue of the tensor R_j^i is $\frac{R}{2}$ or 0. We denote the eigenspaces corresponding to the eigenvalues $\frac{R}{2}$ and 0 by $V_{\frac{R}{2}}$ and V_0 respectively. Since the multiplicity of $\frac{R}{2}$ is 2, $V_{\frac{R}{2}}(X)$ at x and $V_0(X)$ at x, $X \in M$, define respectively 2-and $(2n-2)$-dimensional distributions $V_{\frac{R}{2}}$ and V_0 over M. They are mutually orthogonal and their Whiteney sum is $T(M)$.

Now, we assume that

(3.13) $\nabla_k R_{ji} = 0$,

(that is, Ricci tensor is parallel)

on M.

Then R is constant on M.

Let p^h and q^h be two arbitrary eigenvectors of R_j^i with constant eigenvalue $\frac{R}{2} \neq 0$, then we have

(3.14) $R_j^i p^j = \frac{R}{2} p_i, \quad R_j^i q^j = \frac{R}{2} q^i,$

from which

$$R_i^h \nabla_j p^i = \frac{R}{2} \nabla_j p^i,$$
$$R_i^h \nabla_j q^i = \frac{R}{2} \nabla_j q^i,$$

Thus

$$R_i^h (p^j \nabla_j q^i - q^j \nabla_j p^i) = \frac{R}{2} (p^j \nabla_j q^i - q^j \nabla_j p^i)$$

that is, if p^h and q^h belong to $V_{\frac{R}{2}}$, then $[p, q]^h$ also belong to $V_{\frac{R}{2}}$. Consequenly the distribution $V_{\frac{R}{2}}$ is integrable.

Similarly we can prove that the distribution V_0 is also integrable.

Differentiating the first eauation of (3.14) covariantly, we get

$$R_i^h \nabla_j p_h = \frac{R}{2} \nabla_j p_i,$$

from which

$$R_i^h \nabla^j p_h - R_j^h \nabla_i p_h = \frac{R}{2} (\nabla_j p_i - \nabla_i p_j).$$

Transvecting this equation with q^j and using of (3.14), we obtain

$$R_i^h (q^t \nabla_t p_h) - \frac{R}{2} q^t \nabla^i p_t = \frac{R}{2} q^t (\nabla_t p_i - \nabla_i p_t),$$

from which

$$R_i^t (q^s \nabla_s p_t) = \frac{R}{2} (q^t \nabla_t p_i),$$

or

$$R_i^h (q^s \nabla_s p^i) = \frac{R}{2} (q^s \nabla_s p^h),$$

which shows that if p^h and q^h are two arbitrary vectors belonging to the distribution $V_{\frac{R}{2}}$, then $q^t \nabla_t p^h$ also belongs to the distribution $V_{\frac{R}{2}}$. Thus each integral manifold of $V_{\frac{R}{2}}$ is totally geodesic in M.

Similarly we can verify that each integral manifold of V_0 is totally geodesic in M.

Moreover, if p^i and w^i belong respectively to $V_{\frac{R}{2}}$ and V_0, we have

$$0 = (w^t \nabla_t R_i^h) P^i = w^t \nabla_t (R_i^h P^i) - R_i^h w^t \nabla_t P^i$$
$$= -R_i^h w^t \nabla_t P^i + \frac{R}{2} w^t \nabla_t P^h$$

and

$$0=(P^t\nabla_t R_i^{\ h})w^i=P^t\nabla_t(R_i^{\ h}w^i)-R_i^{\ h}P^t\nabla_t w^i$$
$$=-R_i^{\ h}P^t w^i,$$

that is,

$$0=\frac{R}{2}(w^t\nabla_t P^h)-\frac{R}{2}(w^j\nabla_j P^h)_{\frac{R}{2}}$$
$$=\frac{R}{2}(w^t\nabla_t P^h)_0,$$

and

$$0=\frac{R}{2}(P^t\nabla_t w^i)_{\frac{R}{2}},$$

vector of the form q^h being written as $(q^h)_{\frac{R}{2}}+(q^h)_0$, where $(q^h)_{\frac{R}{2}}$ and $(q^h)_0$ respectively denote the $V_{\frac{R}{2}}$ and V_0 components of q^h.

Consequently we have

$$(w^t\nabla_t P^h)_0=0, \text{ that is, } w^t\nabla_t P^h\in V_{\frac{R}{2}}$$

and

$$(P^t\nabla_t w^h)_{\frac{R}{2}}=0, \text{ that is, } P^t\nabla_t w^h\in V_0.$$

Thus the distributions $V_{\frac{R}{2}}$ and V_0 are parallel. So, using de Rham's decomposition theorem, we have

THEOREM 3.2. *Let M be a complete submanifold of codimension* 2 *in a* $(2n+2)$-*dimensional Euclidean space such that H and K anti-commute with f, where H and K are Weingarten maps with respect to the normals C and D respectively. If* λ *is constant different from* 0 *and* 1 *and*

$$\nabla_k R_{ji}=0,$$

(that is, Ricci tensor is parallel)

on M, then M is the product of $M^2\times E^{2n-2}$ *of a two-dimensional manifold* M^2 *and a* $(2n-2)$-*dimensional Euclidean space* E^{2n-2}.

§4. Sumanifolds of codimension 2 in a locally Fubinian manifold.

A Kählerian manifold is called a locally Fubinian manifold if the holomorphic sectional curvature at every point is independent of the holomorphic section at the point. In this case, its curvature tensor is given by

$$\bar{R}_{\nu\mu\lambda\kappa}=\kappa(G_{\nu\kappa}G_{\mu\lambda}-G_{\mu\kappa}G_{\nu\lambda}+F_{\nu\kappa}F_{\mu\lambda}-F_{\mu\kappa}F_{\nu\lambda}-2F_{\nu\mu}F_{\lambda\kappa}),$$

κ being a constant [1].

Substituting this equation into the equations of Gauss, Codazzi, Ricci respectively;

$$\bar{R}_{\nu\mu\lambda\kappa}B_k^{\ \nu}B_j^{\ \mu}B_i^{\ \lambda}B_h^{\ \kappa}=R_{kjih}-h_{kh}h_{ji}+h_{jh}h_{ki}-k_{kh}k_{ji}+k_{jh}k_{ki},$$

$$\bar{R}_{\nu\mu\lambda\kappa}B_k^{\ \nu}B_j^{\ \mu}B_i^{\ \lambda}C^{\kappa}=\nabla_k h_{ji}-\nabla_j h_{ki}-l_k k_{ji}+l_j k_{ki}$$

$$\bar{R}_{\nu\mu\lambda\kappa}B_k^{\ \nu}B_j^{\ \mu}B_i^{\ \lambda}D^{\kappa}=\nabla_k k_{ji}-\nabla_j k_{ki}+l_k h_{ji}-l_j h_{ki},$$

$$\bar{R}_{\nu\mu\lambda\kappa}B_k^{\ \nu}B_j^{\ \mu}C^{\lambda}D^{\kappa}=\nabla_k l_j-\nabla_j l_k+h_{kt}k_j^{\ t}-h_{jt}k_k^{\ t},$$

we find [3]

$$\text{(4.1)}\quad \nabla_k h_{ji}-\nabla_j h_{ki}-l_k k_{ji}+l_j k_{ki}=\kappa(u_k f_{ji}-u_j f_{kj}),$$

$$\text{(4.2)}\quad \nabla_k k_{ji}-\nabla_j k_{ki}+l_k h_{ji}-l_j h_{ki}=\kappa(v_k f_{ji}-v_j f_{ki}-2v_i f_{kj}),$$

$$\text{(4.3)}\quad \nabla_k l_j-\nabla_j l_k+h_{kt}k_j^{\ t}-h_{jt}k_k^{\ t}=\kappa(v_k u_j-v_j u_k-2\lambda f_{kj}).$$

Taking the similar method to the first equation of (2.6) as in the previous section and using of (2.2), (2.4), (2.5) and (4.1), we find

$$\text{(4.4)}\quad 2h_{ki}h_{jt}f^{it}+\lambda(h_{ki}k_j^{\ i}-h_{ji}k_k^{\ i})+\kappa\{\lambda(u_k v_j-u_j v_k)-2(1-\lambda^2)f_{kj}\}$$
$$=(\nabla_k\alpha-3\beta l_k)u_j-(\nabla_j\alpha-3\beta l_j)u_k+(\nabla_k\beta+3\alpha l_k)v_j-(\nabla_j\beta+3\alpha l_j)v_k$$

Transvecting (4.4) with u^j and taking account of (2.6), we have

$$\text{(4.5)}\quad \nabla_k\alpha-3\beta l_k=\frac{1}{1-\lambda^2}\{u^t(\nabla_t\alpha-3\beta l_t)u_k+u^t(\nabla_t\beta+3\alpha l_t)u_k\}-3\lambda\kappa v_k.$$

Substituting (4.5) into (4.4) and transvecting with v^j, we get

$$\text{(4.6)}\quad \nabla_k\beta+3\alpha l_k=\frac{1}{1-\lambda^2}\{u^t(\nabla_t\beta+3\alpha l_t)u_k+v^t(\nabla_t\beta+3\alpha l_t)v_k\}.$$

From (4.5) and (4.6), we can see

$$\text{(4.7)}\quad 2h_{ki}h_{jt}f^{it}+\lambda(h_{kt}k_j^{\ t}-h_{jt}k_k^{\ t})+\kappa\{\lambda(u_k v_j-u_j v_k)-2(1-\lambda^2)f_{kj}\}$$
$$=3\lambda\kappa(u_k v_j-v_k u_j)=(\nabla_k\alpha-3\beta l_k)u_j-(\nabla_j\alpha-3\beta l_j)u_k$$
$$+(\nabla_k\beta+3\alpha l_k)v_j-(\nabla_j\beta+3\alpha l_j)v_k.$$

Taking also the similar way to the last equation of (2.6) as in the previous section and taking account of (2.2), (2.4), (2.5) and (4.2), we obtain

$$\text{(4.8)}\quad 2k_{ks}k_{jt}f^{st}+\lambda(h_{kt}k_j^{\ t}-h_{jt}k_k^{\ t})+\kappa\{\lambda(u_k v_j-u_j v_k)-2(1-\lambda^2)f_{kj}\}$$
$$=-3\lambda\kappa(u_k v_j-u_j v_k)$$

by virtue of (4.7).

From (4.8), we can see

$$0=6\lambda(1-\lambda^2)\kappa v^k$$

by virtue of (2.6). It means that $\kappa=0$ on M.

Thus we have

THEOREM 4.1. *Let a submanifold M of codimension 2 of a locally Fubinian manifold $\tilde{M}$ be such that H and K anti-commute with f, respect to the normals C and D respectively. If λ is constant different from 0 and 1, then there is no such a M unless $\tilde{M}$ is locally Euclidean.*

Kyung-pook University

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