Analytical Methods of Leakage Rate Estimation from a Containment under a LOCA

Moon-Hyun Chun
Korea Advanced Institute of Science and Technology
(Received April 6, 1981)

Abstract

Three most outstanding maximum flow rate formulas are identified from many existing models. Outlines of the three limiting mass flow rate models are given along with computational procedures to estimate approximate amount of fission products released from a containment to environment for a given characteristic hole size for containment-isolation failure and containment pressure and temperature under a loss of coolant accident. Sample calculations are performed using the critical ideal gas flow rate model and the Moody's graphs for the maximum two-phase flow rates, and the results are compared with the values obtained from the mass leakage rate formula of CONTEMPT-LT code for converging nozzle and sonic flow. It is shown that the critical ideal gas flow rate formula gives almost comparable results as one can obtain from the Moody's model. It is also found that a more conservative approach to estimate leakage rate from a containment under a LOCA is to use the maximum ideal gas flow rate equation rather than the mass leakage rate formula of CONTEMPT-LT.

요 약

많은 기존 공식중 세개의 가장 우수한 극한 유량공식을 찾아, 그 세개의 한계유량공식의 개요와, 
방계심사고사 격납용기로부터 유출하는 것중 특성상의 폭의 크기와 격납용기의 압력 및 온도 등이 주어진 상태에서 격납용기로부터 외부기로 방출되는 개략적인 흡발적심사기물의 압을 추정하기 위한 계산법을 제시하였다. 이성기계의 일체유량균직과 이상유(two-phase flow) 
의 최대유량을 산출하기 위한 모드(Moody)의 도표를 이용하여 계산법을 제시하였으며, 그 정 
과를 콘버게노즐- 알아래 (CONTEMPT-LT) 자장 코드의 형량유량공식을 가용음 노즐(converging 
nozzle)을 통과하는 음속류(sonic flow)의 경우에 적용하여 산출한 값과 비교하여 보았다. 이하 
여 이성 기계의 일체유량공식은 모드(Moody)의 공식이 주는 값과 거의 비슷한 결과를 중등 입증하 
하였. 또한 방계심사고사 격납용기로부터 유출로 문출을 추정하기 위해서는 콘버게노즐- 알아래 (CONTEMPT-LT)의 형량유량공식을 사용하는 것보다 이성 기계의 한계유량공식을 사용하는 것으 
며 보수적인 방법임을 보여 주었다.

--- 121 ---
1. Introduction

Estimates of the leakage rate from a nuclear reactor containment atmosphere following a loss-of-coolant accident (LOCA) are important to the prediction of the amount of fission products released to the environment. The first step to ascertain radiological consequences of reactor accident is to calculate released doses to the public by estimating the leakage rate from the containment.

In the event of a LOCA in a pressurized water reactor (PWR), cooling water is expelled from the primary coolant system to the dry containment building. Flashing of this liquid to steam and a rapid rise in pressure and temperature occur in the containment. Continued addition of mass and energy to the containment necessitate operation of containment safety systems to control the pressure and temperature of the containment atmosphere.

Several different types of leakage from reactor containment to the atmosphere can occur during accident sequences. These can be classified as follows\(^1\): (1) Low (design) leakage from one or more small undefined paths, (2) isolation loss leakage from a single and relatively large open path, and (3) massive leakage (a puff) from a failed containment vessel.

The driving force for fission product leakage from a containment consists of steam and noncondensable gases that are generated during the course of an accident. In a reactor accident, radionuclides may be released into the containment, which may then leak at one or more points to the outside environment. Thus, the release of fission products to the environment is controlled by the leak rate from the containment. The leak rate of the containment, on the other hand, depends on such factors as (1) the containment leak hole size and (2) the containment pressure and temperature.

Once the containment leak hole size and the containment pressure and temperature are known, the next step to determine the amount of fission products released to the environment under a LOCA is to identify a proper flow rate formula for computation. A conservative method of leak rate estimation is to use a limiting leakage flow rate model for the given conditions.

A number of models have been proposed for maximum two-phase flow at low pressures by Linning\(^2\), Faletti\(^3\), Da Cruz\(^4\), Isbin et al.\(^5\), and Massena\(^6\). These models rely on some degree of empirical correlation for the extra degree of freedom encountered in two-phase flows, in which the liquid and vapor travel at different average velocities. Fauske\(^7\) considers annular flow with both phases in equilibrium and he assumed the slip ratio at maximum flow could be determined by maximizing the pressure gradient just upstream from discharge. Also, based on the concept of critical mass flow in a single-phase homogeneous fluid, Moody\(^8\) obtained the maximum flow rate in a two-phase mixture model. However, the most simple formula is the critical ideal gas flow rate model which can be readily obtained from the relations developed for isentropic flow in a converging nozzle for compressible fluid\(^9\).

Currently there are numerous computer codes for analysis during a LOCA. Among these are various versions of RELAP\(^10\) computer programs for transient thermal-hydraulic analysis during postulated acci-
dents or power transients, BEACON/MOD2\textsuperscript{12} and CONTEMPT-LT\textsuperscript{23} computer codes for the pressure and temperature response of a containment building to a LOCA. BEACON/MOD2\textsuperscript{12} is the current version of the BEACON\textsuperscript{12} code and is suitable for the simulation of the short term transient behavior of a containment system where heat transfer effects may be neglected. CONTEMPT-LT\textsuperscript{23}, on the other hand, is a digital computer program developed to describe the long-term behavior of water-cooled nuclear reactor containment systems subjected to postulated LOCA conditions. The program calculates the time variation of compartment pressure, temperatures, mass and energy inventories, heat structure temperature distributions, and energy exchange with adjacent compartments. This program is capable of describing the effects of leakage on containment response.

This paper presents an outline of three limiting mass flow rate models and computational procedures to estimate approximate amount of fission products released for a given hole size, containment pressure and temperature under a LOCA. Sample calculations are given and compared with the results obtained from the equation of the mass leakage rate of CONTEMPT-LT\textsuperscript{23} for converging nozzle and sonic flow.

2. Maximum Flow Rate Models

For a conservative estimation of fission products released from the containment under a LOCA, one of the limiting flow rate formulas developed for an ideal gas flow, or single-phase and homogeneous flow, and/or two-phase flow may be employed. For the present purpose, only three limiting mass flow rate models, which are the most applicable and outstanding among many existing models, are selected and briefly outlined here along with the equation of the mass leakage rate of CONTEMPT-LT\textsuperscript{23} for converging nozzle and sonic flow.

2.1. Ideal Gas Flow Rate Model

Ideal gas flow properties entering and leaving an ideal nozzle are shown in Fig. 1. Employing the relations developed for "isentropic flow in a converging nozzle for compressible fluid", one can obtain the following expression for mass flow rate ($W$) at a flow cross sectional area $A$ in terms of stagnation pressure $p_o$ and stagnation temperature $T_o$:

$$W = p_o A M \sqrt{\frac{r R}{RT_o}} \left(1 + \frac{r-1}{2} \frac{M^2}{(r+1)/(2-2r)} \right)$$

where $r$ is the specific heat ratio, $M$ is the mach number, $g_c$ is the acceleration due to gravity, and $R$ is the gas constant. This equation is applicable for isentropic flow of an ideal gas.

2.2. Maximum Single-Phase, Homogeneous Flow Rate

Local static and stagnation properties may be related by postulating an ideal nozzle in which flow is described by the

Fig. 1. Ideal Nozzle and Flow Properties
following general assumptions: (1) Entrance velocities are zero; (2) total entropy flow rates at entrance and exit are equal (i.e., isentropic flow); (3) total energy flow rates at entrance and exit are equal; and (4) the flow is steady. Velocity \( V \) is assumed to be constant over the exit plane. Continuity and energy equations are given by

\[ G = \frac{W}{A} = \frac{V}{v} \]  \hspace{1cm} (2)

\[ h_s = h + \frac{V^2}{2g_f} \]  \hspace{1cm} (3)

For isentropic flow, \( s \), is constant. Therefore \( h \) and \( v \) are functions of \( p \) only:

\[ h = h(s, \ p) \]  \hspace{1cm} (4)

\[ v = v(s, \ p) \]  \hspace{1cm} (5)

In the above equation, \( G \) is the mass flow rate per unit area, \( V \) is the fluid velocity, \( v \) is the specific volume, \( h \) is the specific enthalpy, \( J \) is the mechanical equivalent of heat, and a subscript zero denotes the properties at stagnation state, respectively.

Solving for \( G \) from Eqs. (2) and (3)

\[ G = \sqrt{\frac{2g_f(J_s - h)}{v'}} \]  \hspace{1cm} (6)

Equations (4), (5) and (6) show that \( G \) is a function of \( p \) alone when \( h_s \) and \( s_s \) are known. A maximum value of \( G \) then must satisfy \( \frac{dG}{dp} = 0 \) which leads to the following expression.

\[ G_M = \sqrt{\frac{2g_f d_s}{J d_s + 2d_s d_s}} \]  \hspace{1cm} (7)

where the subscript \( M \) denotes the property at maximum flow rate. The first law of thermodynamics may be written as

\[ T ds = dh - \frac{v'}{J'} dp \]  \hspace{1cm} (8)

where \( J' \) is a conversion factor. For isentropic process, Eq. (8) leads to

\[ \left( \frac{dh}{dv} \right)_s = \frac{v'}{J'} \left( \frac{dp}{dv} \right)_s \]  \hspace{1cm} (9)

Substituting Eq. (9) into Eq. (7), we have

\[ G_M = \sqrt{\frac{2g_f J}{J d_s + 2d_s d_s}} \]  \hspace{1cm} (10)

Equation (10) is the well-known expression for critical mass flow of a single-phase, homogeneous fluid.

2.3. Maximum Two-Phase Flow Rate

Based on the concept of critical mass flow in a single-phase, homogeneous fluid, Moody obtained a theoretical model for predicting the maximum flow rate of a single component, two-phase mixture. It assumes annular flow, uniform linear velocities of each phase, and equilibrium between liquid and vapor. Flow properties for this case are shown in Fig. 2. Flow rate is maximized with respect to local slip ratio and static pressure for known stagnation conditions.

The maximum two-phase flow formula, in terms of local static properties, given by Moody is as follows:

\[ G_M = \sqrt{\frac{2g_f J}{J d_s + 2d_s d_s}} \]  \hspace{1cm} (11)

where \( d_1, d_2, d_3, d_4, \) and \( d_5 \) are specified functions of \( p \) and the quality \( x \) (vapor mass flow fraction).

Also, Moody presented his solution in the form of five graphs giving maximum steam/water flow rates for: local static pressures between 1.70 and 204.14 atm.
with local qualities from 0.01 to 1.00: local stagnation pressures and enthalpies which cover the range of saturation states.

2.4. Mass Leakage Rate Formula of CONTEMPT-LT

CONTEMPT-LT\textsuperscript{123} allows two types of leakage calculations: normal compartment leakage determined from tabular input and program calculations and penetration leakage determined from program calculations. For penetration leakage, the flow is assumed to be through one of three types of nozzles: a converging nozzle, a diverging nozzle, or a converging-diverging nozzle. In each case, the equation of the mass leakage rate is

\[ W = C_o A Z \rho_0 \sqrt{\frac{2 \tau g}{(\gamma - 1) R_\alpha T_\alpha}} \sqrt{1 - Z^{-1}} \]  \hspace{1cm} (12)

where \( C_o \) is an input constant, usually 1.0, \( A \) is the nozzle throat area, \( \rho_0 \) is the pressure at inlet side, \( T_\alpha \) is the absolute temperature of flowing mixture, \( R_\alpha \) is the gas constant of the the air and water vapor mixture, and \( Z \) is the function of pressure ratio which depends on the type of nozzle and the pressure difference across the nozzle.

For converging nozzle and sonic flow, in particular, Eq. (12) becomes

\[ W = C_o A \left( \frac{\rho_1}{\rho_0} \right)^\frac{1}{\gamma} \rho_0 \sqrt{\frac{2 \tau g}{(\gamma - 1) R_\alpha T_\alpha}} \sqrt{1 - \left( \frac{\rho_1}{\rho_0} \right)^\frac{\gamma - 1}{\gamma}} \] \hspace{1cm} (13)

where the subscript \( e \) stands for exit conditions.

3. Leakage Rate Analysis and Discussion

3.1 Assumptions and Sample Calculations

The PWR has two small (5.08cm diameter) vacuum pump line penetrations each having two isolation valves outside containment that are called upon to close when a LOCA occurs. However, both of these penetrations ultimately connect to a common vacuum pump exhaust line. In this line there is a normally open valve which is designed to trip closed on a high radiation signal. Thus failure to isolate each of those penetrations could occur by the failure of these three valves to close. Thus, were "consequence limiting control system" of the PWR to fail, the leakage rate for the containment atmosphere to external environment could be equivalent to that due to about a 7.62 cm diameter hole. Also, it may be noted here that the design leak rate for a PWR containment is 0.1 volume percent (v/o) per day, with temporary increases up to 1 v/o per day considered permissible\textsuperscript{113}.

During a design basis accident the containment is pressurized rapidly to a peak of 3.67 atm as a result of primary system blowdown\textsuperscript{117}. Design pressure of a typical PWR containment is about 4.08 atm (60 psig).

For a conservative estimate of the containment leakage rate under a LOCA following assumptions may be made:

(1) The reference containment building is a dry, single-volume structure with internal masses.

(2) The pressure-temperature response of a dry containment to a LOCA is known from the use of code such as CONTEMPT-LT\textsuperscript{123} or BEACON/MOD2\textsuperscript{123} along with the RELAP\textsuperscript{109} codes.

(3) Containment atmosphere filled with a mixture of air, steam, and noncondensible gases behaves as a perfect gas with a constant specific heat. This assumption is required only when Eq. (1) is used.
(4) Containment leakage can be represented by an equivalent diameter (i.e., a characteristic hole size) of a converging nozzle as shown in Fig. 3.

(5) Containment free volume is large enough so that negligible changes in pressure and temperature occur as the fluid is exhausted through the leak hole.

With these assumptions sample calculations are made to examine the validity and the applicability of the maximum flow rate models to the present case.

Ideal gas flow properties entering and leaving an ideal nozzle are shown in Fig. 1, whereas the ideal nozzle and two-phase flow properties are shown in Fig. 2. Initial conditions and numerical data are selected for direct comparison with WASH-1400\textsuperscript{9} results. They are summarized and shown in Table 1.

Of the three flow rate equations, i.e., Eqs. (1), (10) and (11), the Eq. (10) is not readily applicable to the present sample calculations, because it requires the knowledge of \(\frac{\partial p}{\partial r} s\), which is not available. Therefore, only two different methods are used in the present sample calculations for comparison: For the maximum perfect gas flow rate Eq. (1) is used, whereas graphs presented by Moody\textsuperscript{3} are used for the computation of the maximum two-phase flow rates. Flow rates \(W\) obtained from both methods and the mass leakage rate of Eq. (13) are shown in Table 1.

In addition, maximum flow rate calculations are performed for typical containment pressure ranges under a LOCA using both models, and the results are compared with the mass leakage rate obtained from Eq. (13) as shown in Fig. 4.

3.2. Discussion of the Results

Result of WASH-1400\textsuperscript{9} shows that if the internal pressure exceeds about 1.7 atm., the leakage flow through an assumed orifice will be choked, and a leak rate of 200 v/o per day would require a hole of 9.14 cm in diameter or its equivalent.

![Fig. 3. Reference Containment Building and Initial Conditions](image)

![Fig. 4. Stagnation Pressure versus Flow Rate](image)
Table 1. Parameters and Comparison of the Results

<table>
<thead>
<tr>
<th>Parameters and Results</th>
<th>Max. Perfect Gas Flow Rate Model (Eq. (1))</th>
<th>Max. Two-phase Flow Rate Model (Ref. 8)</th>
<th>CONTEMP-LT (Ref. 12 Eq. (13))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>X=1.0</td>
<td>X=0.8</td>
</tr>
<tr>
<td>$P_e$ (atm.)</td>
<td>3.67 (54 psia)</td>
<td>3.67</td>
<td>3.67</td>
</tr>
<tr>
<td>$T_e$ (°K)</td>
<td>414 (75.92  °R)</td>
<td>414</td>
<td>414</td>
</tr>
<tr>
<td>Diameter (m)</td>
<td>$9.14 \times 10^{-2}$ (0.36 in)</td>
<td>$9.14 \times 10^{-2}$</td>
<td>$9.14 \times 10^{-2}$</td>
</tr>
<tr>
<td>$A$ (m²)</td>
<td>$6.57 \times 10^{-2}$ (10.18 in²)</td>
<td>$6.57 \times 10^{-2}$</td>
<td>$6.57 \times 10^{-2}$</td>
</tr>
<tr>
<td>$M$</td>
<td>1</td>
<td>Not Applicable</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>$r$</td>
<td>1.4</td>
<td>Not Applicable</td>
<td>Not Applicable</td>
</tr>
<tr>
<td>$R$ (erg/K) or $R_e$</td>
<td>$2.8709 \times 10^6$ (53.31 ft-lbf/1bm·°R)</td>
<td>$2.8709 \times 10^6$</td>
<td>$2.8709 \times 10^6$</td>
</tr>
<tr>
<td>$G_{c}$ (Kg-m/Newton·sec⁴)</td>
<td>1 (32.2 lbm·ft/lbf·sec⁴)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$G_s$ (Kcal/Kg)</td>
<td>Not Applicable</td>
<td>655.27</td>
<td>550.94</td>
</tr>
<tr>
<td>$G_m$ (Kg/sec·m²)</td>
<td>Not Applicable</td>
<td>1220.61</td>
<td>976.48</td>
</tr>
<tr>
<td>$W$ (Kg/sec)</td>
<td>8.01 (10.71 1bm/sec)</td>
<td>5.41</td>
<td>4.12</td>
</tr>
</tbody>
</table>

As shown in Table 1, Eq. (1) gives the maximum flow rate of 4.86 kg/sec for the leak hole of 9.14 cm in diameter, and this is equivalent to 263 v/o per day, which is in general agreement with WASH-140013 results. The maximum two-phase flow rate model gives a larger value than Eq. (1), whereas Eq. (13) of CONTEMP-LT12 for converging nozzle and sonic flow gives smaller value than Eq. (1). It may be noted here that part of the reason for this deviation may be due to the difference in the temperature of the containment atmosphere.

Temperature of the containment atmosphere used in the WASH-140013 is not specified, whereas it is assumed to be 414 °k for Eq. (1), Moody's89 graph, and Eq. (13).

Also, inspection of Fig. 4 shows that Moody's89 maximum two-phase flow rate model gives larger flow rate than the critical ideal gas flow rate formula Eq.(1) for pressures below 10 atm., and beyond this pressure both results tend to approach the same value. The mass leakage rate formula, Eq. (13), of CONTEMP-LT12, on the other hand, gives smaller flow rate than the critical ideal gas flow rate formula for pressures above 2 atm., and beyond this pressure the magnitude of the difference between the two flow rates becomes larger.

4. Application to Computer Program

From the foregoing discussions it may be deduced that the maximum ideal gas flow rate model is the most convenient tool to be used for a subroutine program of a main computer codes (or for an entirely in dependent program). That is, in spite of its simplicity, Eq.(1) gives almost comparable results as one can obtain from the maximum two-phase flow rate model. Also, since Eq. (1) predicts larger flow rate than Eq. (13) of the CONTEMP-LT12 it would be more conservative to employ Eq.(1) rather than Eq.(13) for choked flow condition to estimate leakage rate from a containment under a LOCA. The maximum two-phase flow model89, on the other hand, has an inherent drawback for applications to computer programs because of its extreme complexity, and it would require an
enormous amount of data, such as steam tables. In addition, graphs presented by Moody\(^6\) are not directly applicable for computer programming.

Therefore, in order to estimate leakage rates from a containment and its effect on the containment pressure and temperature response under an initiating event, such as a double-ended cold- leg break, one may proceed as shown in the flow diagram for computer program (Fig. 5):

(1) The pressure-temperature response of a dry containment building to a LOCA may be obtained from the computer programs such as CONCEPT\(^{14}\) and BEACON\(^{15}\). For these codes, the mass and energy input rates to the containment may be obtained as functions of time from the RELAP\(^{30}\) computer codes.

(2) Results of the pressure and temperature response from one time interval are used as initial conditions for the leakage rate estimations. In addition, the effect of the leakage on the containment pressure and temperature are estimated and these results are used as initial conditions for the next time interval \((t = t + \Delta t)\).

(3) Continuation of this process would give the containment leak rate as a function of time under an initiating event such as a LOCA.

5. Conclusions

This work is initiated in an effort to improve and upgrade current computer codes. Outlines of three limiting mass flow rate models are given along with computational procedures to estimate approximate amount of fission products released from a containment to environment for a given characteristic hole size for containment-isolation failure and containment pressure and temperature under a LOCA.

Sample calculations are also performed using the maximum ideal gas flow rate model and the Moody's\(^6\) graphs for the maximum two-phase flow rates, and the results are compared with the values obtained from the mass leakage rate formula of CONCEPT-LT\(^{12}\) for converging nozzle and choked flow. This comparison shows that the critical ideal gas flow rate formula, Eq. (1), gives almost comparable results as one can obtain from the Moody's\(^6\) model. It is also found that a more conservative approach to estimate leakage rate from a containment under a LOCA is to employ Eq. (1) rather than Eq. (13) of CONCEPT-LT\(^{12}\). Based on this result and because of its simplicity and
immediate applicability, a recommendation is made here to select the maximum ideal gas flow rate model for the "computer program of an approximate containment leakage rate estimation and its effect on containment transient under a LOCA." For this purpose, a flow diagram for the computer program is also presented.

References