Several Detection Criteria and Fuzzy Data

여러가지 검파 기준과 퍼지 자료

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Abstract

In this paper, fuzzy statistical techniques are introduced into the optimum signal detection problems. We show that various conventional decision criteria can be applied in fuzzy signal detection area. The likelihood ratio for the fuzzy detection of known signals is obtained. Some properties of the optimum fuzzy nonlinearities of detectors are also described. As a special case of the fuzzy signal detectors, a fuzzy sign detector is considered and its performance characteristics are investigated.

요 약

이 논문에서는 고전적인 결정 기준들이 퍼지 신호 검파에서도 쓰일 수 있음을 보였다. 알려진 신호퍼지 검파에 쓰이는 우도비률(likelihood ratio) 얻어보았고 퍼지신호 검파기를 이루는 퍼지 비선형성의 몇가지 성질을 기술하였다. 퍼지 신호 검파기의 한 보기로 퍼지 부호 검파를 얻어서 그 성능 특성을 살펴보았다.

I. Introduction

In signal processing areas, it is certain that imprecise information is often unavoidable.

Based on this observation, applications of fuzzy set theory have been considered in several signal processing problems including digital signal restoration, image segmentation, and biomedical signal processing [*e. g.*, 3]. Although the fuzzy set theory has already been introduced into a signal detection area based on the concept of interval-valued fuzzy sets [4], it is also a reasonable approach to construct a detection system based on the fuzzy testing of statistical hypotheses.

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The possibility to connect the fuzzy theory and the signal detection theory may partially be explained as follows. Although we can not exactly estimate the statistical characteristics, say, the mean and the variacne, of the noise process and it is sometimes hard to assume that the noise process is stationary in practice, we usually neglect both facts in modeling the physical situations. This leads us to a conclusion that when a certain value is observed under the assumptions of incorrect estimation and nonstationarity of noise, the only sentence we can say with confidence is 'the value just around the observed one is received'.

In this paper, we reconsider the signal detection theory based on the fuzzy set theory and fuzzy tests. We consider several possible decision criteria including the fuzzy maximum likelihood criterion, the fuzzy minimax criterion, and fuzzy Bayes criterion, etc. Finally we discuss a fuzzy sign detector as a particular example of the fuzzy decision problems applied to signal detection.

II. Fuzzy Decision Criteria

2.1. Preliminaries

Let $X=(X, Bx, Fx)$ be an *experiment*, where X is a set in the real line, Bx is the smallest Borel σ-field on X, and Fx belongs to a family of probability distributions Λ on the measurable space (X, Bx).

Suppose that only fuzzy information is available. In such cases, one of the ways to handle such fuzzy observations is to employ a *fuzzy information system* (FIS) x [1].

For the case of decision-making with observations of the sample size n, sample fuzzy information, which is an element of a *fuzzy random sample* $x^{(n)}$ from an FIS, has been generally used. The probability distribution of $x^{(n)}$ is given by

$$P(\kappa_1, \ldots, \kappa_n) = \int_{X^n} \mu_{\kappa_1}(x_1) \cdots \mu_{\kappa_n}(x_n)\, dF_{X^n}(x_1, \ldots, x_n),$$

where $F_{X^n}(x_1, x_2, \cdots, x_n)$ is the probability distribution on (X^n, B_{X^n}) determined by Fx on (X, B_X), μ_{κ_i} is called the *membership function* of κ_i, and $X^n = \underbrace{X \times \cdots \times X}_{n \text{ times}}$. From now on we will abbreviate $(\kappa_1, \kappa_2, \cdots, \kappa_n)$ to κ for notational convenience.

2.2. Various Decision Criteria

In classical detection theory, there are a number of decision criteria [*e.g.*, 6]. Although the classical signal detection problem and the fuzzy signal detection problem seem to be somewhat different, it is interesting that all of the conventional classical decision criteria can also be used in fuzzy signal detection problems.

Some detection criteria for fuzzy signal detection problems are shown below :

Maximum Likelihood (ML) Criterion : choose H_1 if

$$\frac{P(\kappa \mid H_1)}{P(\kappa \mid H_0)} > 1.$$

Bayes Criterion : choose H_1 if

$$\frac{P(\kappa \mid H_1)}{P(\kappa \mid H_0)} > \frac{P(H_0)(C_{10}-C_{00})}{P(H_1)(C_{01}-C_{11})}.$$

where C_{ij} is the cost associated with choosing hypothesis H_i when in fact hypothesis H_j is true and $P(H_i)$ is the *a priori* probaility of H_i

Minimax Criterion : choose H_1 if

$$\frac{P(\kappa \mid H_1)}{P(\kappa \mid H_0)} > \gamma_M.$$

In(4), γ_M should be chosen so that

$$C_{10}P(D_1 \mid H_0) + C_{00}P(D_0 \mid H_0) = C_{01}P(D_0 \mid H_1) + C_{11}P(D_1 \mid H_1),$$

where $P(D_i|H_j)$ is the probability that we choose H_i given that H_j is true.

Neyman-Pearson Criterion : choose H_1 if

$$\frac{P(\kappa \mid H_1)}{P(\kappa \mid H_0)} > \gamma_{NP}.$$

In(6), γ_{NP} should be chosen to satisfy the constraint,

$$\sum_{\Psi} P(\mathbf{\kappa} \mid H_0) = \alpha,$$

where

$$\Psi = \{ \mathbf{\kappa} : P(\mathbf{\kappa} \mid H_1) > \gamma_{NP} \cdot P(\mathbf{\kappa} \mid H_0) \}.$$

Proofs of the Bayes and Neyman-Pearson criteria can be found in [1]. There are more criteria including the maximum *a posteriori* probability criterion, the approximate version of Neyman-Pearson criterion [2], and the fuzzy set theoretic version of the generalized Neyman-Pearson criterion [5]. It is no doubt that these criteria can also be applicable to fuzzy signal detection.

Ⅲ. Fuzzy Known Signal Detection

Let us now consider the following descriptions under the two hypotheses :

$$H_0 : Y_i = W_i$$

versus

$$H_1 : Y_i = \theta e_i + W_i$$

for $i=1,\cdots,n$. In Equations (8) and (9), e_i is the deterministic (known) signal component, Y_i is the observation at the i-th sampling instant, and θ is a signal amplitude parameter which controls the signal strength. The purely-additive noise terms W_i, $i=1,\cdots,n$, are usually assumed to be independent and identically distributed (i.i.d.) with common continuous probability density function (pdf) f.

Since all of the test statistics for the decision criteria described in Section 2 are defined in terms of the likelihood ratio, it is quite natural to deriv-eonly the ratio of the likelihood functions of each hypothesis. The likelihood ratio for the above fuzzy known-signal detection problem can easily be derived to be

$$T(\mathbf{\kappa}) = \sum_{i=1}^{n} e_i g_{opt}(\kappa_i),$$

where

$$g_{opt}(\kappa_i) = \ln \frac{P(\kappa_i \mid H_1)}{P(\kappa_i \mid H_0)}.$$

In (11), the function $g_{opt}(\kappa_i)$ is called the optimum fuzzy nonlinearity and characterizes the detector structure.

As a special case of the fuzzy signal detector, let us consider the situation where the available information is imprecise (because of the reasons described in Section 1) : let τ_1="*approximately less than* 0" and τ_2="*approximately greater than* 0". The membership functions are

$$\mu_{\tau_1}(x) = \begin{cases} 1 & , \quad x \le -\Delta \\ -\dfrac{x}{2\Delta}+\dfrac{1}{2} & , \quad -\Delta < x \le \Delta \\ 0 & , \quad x > \Delta , \end{cases}$$

and

$$\mu_{\tau_2}(x) = \begin{cases} 0 & , \quad x \le -\Delta \\ \dfrac{x}{2\Delta}+\dfrac{1}{2} & , \quad -\Delta < x \le \Delta \\ 1 & , \quad x > \Delta . \end{cases}$$

Since the criteria described in Section 2 are not fundamentally quite different, let us consider the minimum error probability criterion which chooses H_1 when

$$\frac{P(\kappa \mid H_1)}{P(\kappa \mid H_0)} > \frac{P(H_0)}{P(H_1)} .$$

In Table 1, the average error probabilities (AEP) of the crisp and fuzzy sign detectors for various situations are tabulated. We assumed that $\underline{P}(H_0) = \underline{P}(H_1) = 0.5$, $n=60$, $\theta=1$, and f is the standard normal pdf. When the variance of the noise process is exactly estimated, we see that the AEP of the crisp sign detector is smaller than of the fuzzy sign detector. However, if the noise variance is incorrectly estimated to be 1, when the actual variance of the noise process is 1.5, 2, 3, and 4, we see that the fuzzy sign detector provides less AEP.

Table 1. Comparison of average error probabilities of the fuzzy sign detector and the crisp sign detector.

σ	Crisp Detector	Fuzzy Detector	
		$\Delta=0.5$	$\Delta=1.0$
1	0.0016	0.0019	0.0035
1.5	0.0780	0.0450	0.0319
2	0.2476	0.1940	0.1459
3	0.4214	0.3848	0.3413
4	0.4661	0.4451	0.4167

From Table 1, we can also that the AEP of the fuzzy sign detector ia to some degree sensitive the parameter Δ which we call the *incredibility* (or the content of fuzziness) of the received observations. We may conclude that the incredibility is an important paramenter of fuzzy signal detection. Investigation of the properties of the incredibility would be an interesting future research topic.

IV. Concluding Remarks

In this paper, we reconsidered the signal detection theory based on the fuzzy set theory. Several decision criteria were discussed and the optimum fuzzy detector structure of known signals was investigated. As an example, a fuzzy sign detector was introduced. We showed that the fuzzy sign detector provides less average error probabilites than the crisp sign detector when fuzziness is involved in the observations.

The same procedure can applicable to the random signal detection problem which is now under investigation. It is believed that the performance characteristics of the fuzzy signal detectors are strongly related to the incredibility Δ of the available information. An adaptive fuzzy signal detection scheme based on this concept seems to be interesting and is also under investigation.

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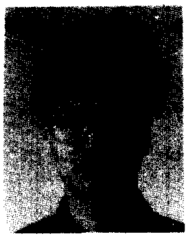

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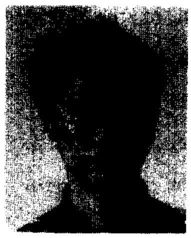

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