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〈연구논문〉

The Instability Due to the Parasitic Modes in the Electron Storage Ring

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전자 저장링에서 고차 모우드에 기인하는 불안정성

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Abstract — The wake potential, total loss and the impedances for the sum of parasitic modes in our designed cavity are computed. We see that the analytical calculations are agree well with a numerical calculations by the computer codes URMEL and TBCI. The instability phenomena for our cavity is also calculated based on the Fokker-Planck equation for synchrotron motion. We see that our calculations satisfy the stability criterion for the higher order modes. It was found that strong Robinson instability does not occur in our case. We find that the threshold with the potential well distortion(PWD) is 1.65×10^{10} by the tracking calculation using the long range wakefield for the longitudinal single bunch instabilities. We have also calculated the coupled bunch instability growth rates for the 15 parasitic modes of the designed RF cavity.

요 약 — 전자 저장링에서 고차 모우드에 대한 웨이크 포텐셜 및 임피던스 등이 계산되었고, 설계된 고주파 가속공동에 대한 불안정성에 관한 현상이 포커-프랑크 방정식에 의해 계산되었다. 이 결과로, 고차 모우드에 대해서 안정영역을 만족함을 보였으며, 강한 로빈슨 불안정성은 일어나지 않음을 알 수 있었다. 웨이크 필드를 사용한 threshold가 tracking에 의해 계산되었고, 15개의 고차모우드에 의한 growth rate가 계산되었다.

1. Introduction

The wake fields give rise to a parasitic energy loss of the bunch, to an induced energy spread of the electrons in the bunch, and to transverse forces that tend to increase the effective beam emittance. A. Chao [1] gives an elegant introduction to the basic physics of wake potentials in cylindrically symmetric systems. K. Bane *et al* [2] give the most detailed presentation of the calculation of the wake potentials in closed cavities. Bane and Sands [3] review wakefield effects in the diffraction limit for short bunches passing through a cavity with beam tubes, and develop useful analytic expressions. In this work, the wake potential, total loss and the impedance for the sum of parasitic modes in our designed cavity are computed by TBCI code [4] and analytical calculations. The instability phenomena for our cavity is also calculated based on the Fokker-Planck equation [5] for synchrotron motion. We have calculated the threshold current with the potential well distortion(PWD) by the simulation method using the wakefield and the coupled bunch instability growth rates for the 15 parasitic modes of the designed RF cavity.

2. Wake Fields and the Loss Factors

A test charge trails the driving charge at a distance s. The longitudinal wake potential is defined by the total voltage loss of the test particle as [6-8]

$$W_{\parallel}(r,r',s) = -\frac{1}{q}\int_{z2}^{z1} dz[E_{\parallel}(r,z,t)]t=(z+s)/c \quad (1)$$

where z is a unit vector in the direction of motion of both the driving and test charges which are taken to be parallel to the z axis.

The driving charge is assumed to enter the cavity structure at z, t=0 and to exit at z=L. The test particle enters and leaves the cavity at z_1 and z_2. If the line density of the charge distribution is λ(s) per unit length, the longitudinal potentials are

$$V_{\parallel}(s) = \int_0^{\infty} ds\ '\lambda(s-s')\ W_{\parallel}(s') \quad (2)$$

The total energy loss to the wake fields is given by

$$\Delta U = \int_0^{\infty} ds\ '\lambda(s-s')\ V_{\parallel}(s) \quad (3)$$

The loss factor is now defined as

$$k_{\parallel} = \frac{\Delta U}{q^2} \quad (4)$$

3. The Higher Order Modes and Loss Factors

The wake potential $W(\tau)$ is the potential seen by a particle crossing the cavity at time τ behind a charge impulse of unit amplitude. The wake potential can be obtained from the cavity modes

$$W(\tau) = 2\sum_n(\omega_n/4)(R/Q)_n \cos\omega_n\tau \quad (5)$$

The total potential at any time t within the bunch due to all charges passing through the cavity or system previous to time t is given by

$$k(t) = \frac{1}{Q}\int_0^{\infty} W(\tau)I(t-\tau)d\tau \quad (6)$$

For a Gaussian bunch, Eq.(6) becomes

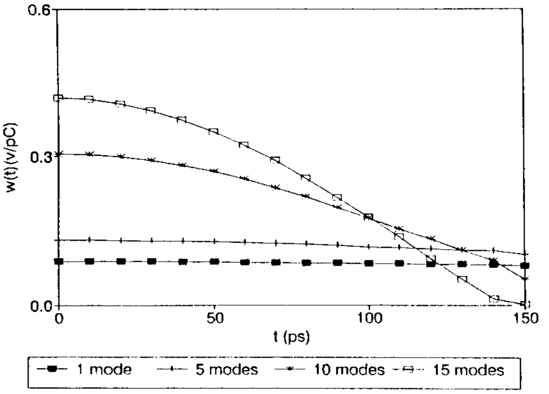

Fig. 1. The wake potentials from the sum over the parasitic modes.

$$k(k) = \frac{1}{\sqrt{2\pi\sigma}}\int_0^{\infty} W(\tau)e^{-(t-\tau)2/2\sigma 2}\ d\tau \quad (7)$$

The total loss per unit charge is obtained from k(t) for the general and Gaussian cases as

$$K_{tot} = \frac{1}{Q}\int_{-\infty}^{\infty} k(t)I_0(t)dt = \frac{1}{\sqrt{2\pi\sigma}}\int_{-\infty}^{\infty} k(t)e^{-t2/2\sigma 2}\ dt \quad (8)$$

Fig. 1 shows the wake potential from the sum over the parasitic modes of 1,5,10 and 15 modes in our cavity. The total loss and impedance for the sum of parasitic modes are shown in Fig. 2 and Fig. 3, respectively. From the results with the relative errors of 1.09% for the total loss and of 1.15% for the total impedances, we see that the analytical calculations are agree well with a numerical analysis by the computer code URMEL [9].

4. The Instability Due to the Cavity

The instability phenomina for our cavity is calculated based on the Fokker-Planck equation [5] for the synchrotron motion. We calculate the instability phenomena when mode coupling is neglected. We take one azimuthal mode m and neglect the coupling between different azimuthal modes

$$\left(\lambda - m + i\frac{|m|+2l}{\tau\omega_s}\right)a_l^{(m)} = i\frac{m\xi_e}{2\pi\sigma_\theta^2}\sum_{k=0}^{\infty} M_{mk}^{ml}a_k^{(m)}, \quad (9)$$

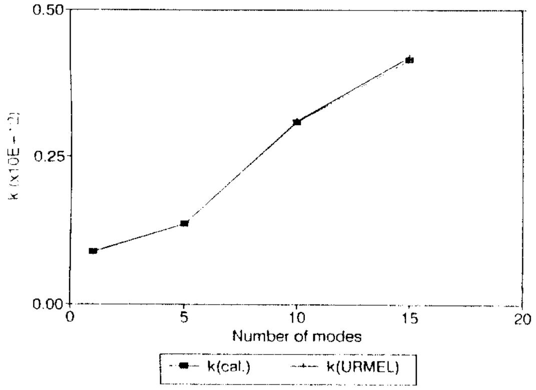

Fig. 2. The loss factor for the sum of the parasitic modes.

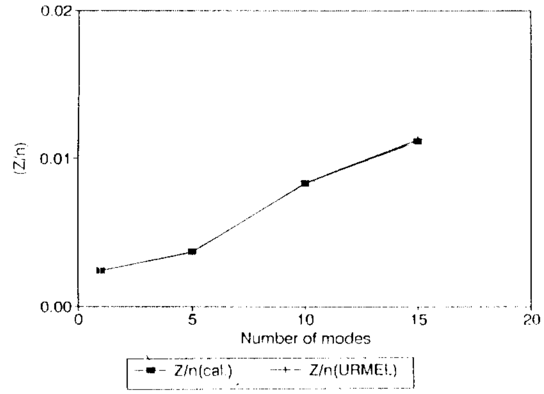

Fig. 3 The impedance for the sum of the parasitic modes.

where ω_s is the incoherent synchrotron angular frequency, ξ scaling parameter. A resonator impedance $Z(\omega)$ is assumed as

$$Z(\omega)=\frac{R}{1-iQ(\omega/\omega_r-\omega_r/\omega)} \tag{10}$$

Where R is the shunt impedance, Q is the quality factor and $\omega_r/2\pi$ is the resonant frequency of a cavity. The matrix element M^{ml} is given by

$$M^{ml}_{mk}=\frac{1}{\sqrt{(m+k)!k!(m+l)!l!}}\sum_{p=-\infty}^{\infty}\frac{Z(p+m\nu_s)}{p+m\nu_s}\left(\frac{\sigma_\theta(p+m\nu_s)}{\sqrt{2}}\right)^{2m+2k+2l}\times\exp[-\sigma_\theta^2(p+m\nu_s)^2] \tag{11}$$

with

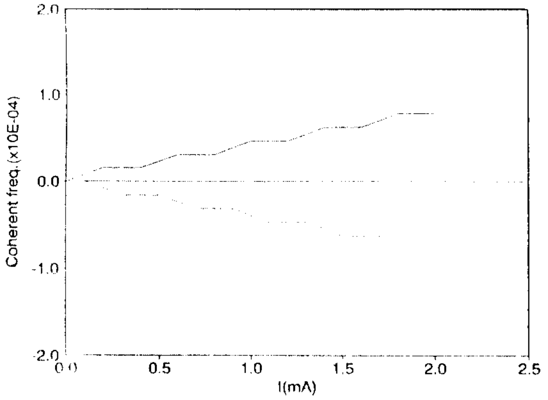

Fig. 4. Coherent frequency vs. current for the dipole mode.

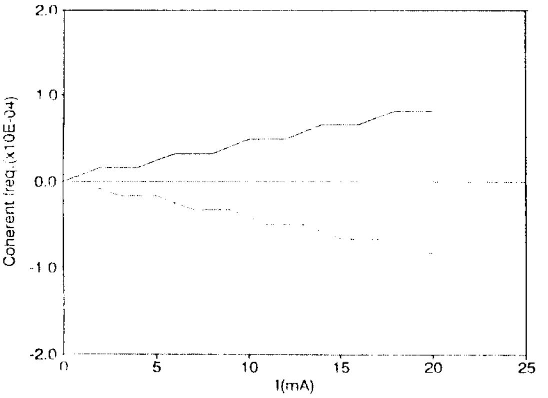

Fig. 5. Coherent frequency vs. current for the quad rupole mode.

$$\frac{Z(p+m\nu_s)}{p+m\nu_s}=\frac{iR}{2Q\sqrt{1-1/4Q^2}}\left[\frac{1}{p-p_1}-\frac{1}{p-p_2}\right] \tag{12}$$

where

$$p_1=p_r\sqrt{1-1/4Q^2}-m\nu_s-ip_r/2Q$$
$$p_2=-p_r\sqrt{1-1/4Q^2}-m\nu_s-ip_r/2Q \tag{13}$$
$$p_r=\omega_r/\omega_0$$

The coherent oscillation frequencies in unit of synchrotron frequency for higher order mode impedance of our rf cavity and storage ring parameters are calculated by this formalism. The results are shown in Fig. 4-6 for the dipole, qua drupole and sextupole modes, respectively.

The solid curve and the dashed curve show the real part and imaginary part of the coherent os-

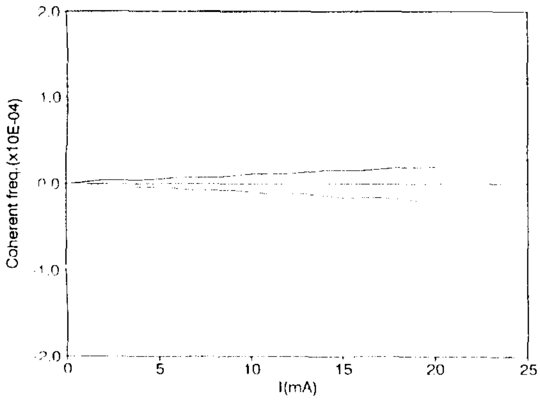

Fig. 6. Coherent frequency vs. current for the sextupole mode.

cillation frequency, respectively. Three radial mo des(k,l=0-2) are taken into account but the results did not changed. In this case, strong Robinson instability does not occurs. The stability criterion when we use the Vlasov equation is given by

$$\text{growth rate} < |m|/\tau \tag{14}$$

where τ is the radiation damping time. We see that our calculations satisfy the stability criterion for the higher order modes. If we assume that the coupling of [m=0, k=0] and [m=−1, k=0] modes occurs when λ equals −1 and then the threshold current is determined by the stability criterion for the monopole mode.

$$I_{th} = \frac{8\,\nu_s E/e}{\beta P_r (R/Q)} \tag{15}$$

We calculated the threshold current of $I_{th}=6.7$ mA, which is less close to the value of SPEAR scaling law [10].

The longitudinal single bunch collective motion of an electron beam in a storage ring is described by the Vlasov equation [11]

$$-\frac{\partial \Psi}{\partial \theta} = p\frac{\partial \Psi}{\partial q} + (-q + V(q,\theta))\frac{\partial \Psi}{\partial p} \tag{16}$$

where $\Psi = \Psi(p,q,\theta)$ is the distribution function in the longitudinal phase space, $p \equiv (E_0 - E)/E_0\sigma_E$ the relative energy deviation, $q \equiv z/\sigma_z$ the longitudinal position, and $\theta \equiv \omega_s t$ the phase of the synchrotron motion. The charge of the bunch induces the longitudinal field V(q) with the longitudinal wake function W(q) as

$$V(q,\theta) = I\int_{-\infty}^{+\infty} f(q',\theta)W(q'-q)dq' \tag{17}$$

where $f(q,\theta) = \int_{-\infty}^{+\infty}\Psi(p,q,\theta)dq$ and normalized as $\int f(q,\theta)dq = 1$,

and the parameter I represents the beam intensity as

$$I = \frac{Ne}{2\pi\nu_s\sigma_E}\left(\frac{e}{E_0}\right) \tag{18}$$

with N the number of the particles in the bunch, ν_s the synchrotron tune and E_0 the nominal beam energy. To solve this problem as exactly as possible, the mode-coupling method and action angle variables (J,Φ) are introduced. Then Eq. (16) can be rewritten by expansion of ψ around the stationary distribution, $\psi = \psi_0 + \psi_1$ as

$$-\frac{\partial \Psi_1}{\partial \theta} = \omega(J)\frac{\partial \Psi_1}{\partial \phi} + V_1(q,\theta)\frac{\partial \Psi_0}{\partial p} \tag{19}$$

where $\omega(J) = d\phi/d\theta = \partial H/\partial J$ is the angular fre quency of the single-particle motion in the potential well and Ψ_1 can be expanded in terms of the orthogonal modes as

$$\Psi_1 = \sum_{nm}(C_{nm}\cos m\phi + S_{nm}\sin m\phi)\Delta_n(J)\exp(-i\mu\theta) \tag{20}$$

where the function $\Delta_n(J)$ taken the value $1/\Delta J_n$ in the strip around the n-th mesh point $J=J_n$ with the thickness ΔJ_n, and zero outside. After substituting (20) to (19), we multiply $\Delta_n(J)\Delta J_n\cos m\phi$ or $\Delta_n(J)\Delta J_n \sin m\phi$ on the both side, and integrate them over J and ϕ, then we obtain a linear equation for C_{nm}: $-\mu^2 C_{nm} = -\sum_{n'm'} M_{nmn'm'} C_{n'm'}$ (21)

with

$$M_{nmn'm'} = m^2\omega_n^2\delta_{nn'}\delta_{mm'} + I\frac{m^2\omega_n^2\Psi_0(J_n)\Delta J_n}{\pi}\int_0^{2\pi}\int_0^{2\pi}$$
$$\cos m\phi \cos m'\phi' \times F(q(J_{n'},\phi') - q(J_n,\phi))d\phi d\phi' \tag{22}$$

where $\delta_{nn'}$ is Kronecker's delta and $F'(q) = W(q)$. The system becomes unstable when the matrix M has a negative or a complex eigenvalue.

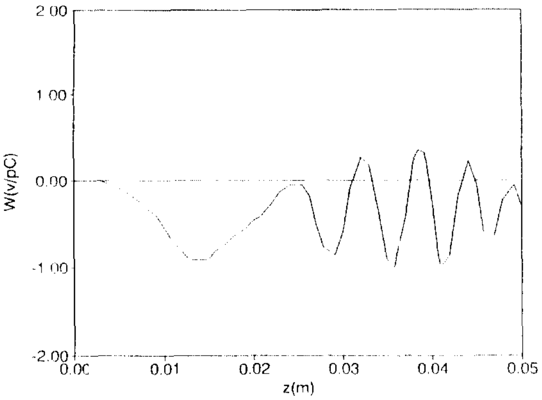

Fig. 7. The Green function wake potential as a function of the bunch coordinates z in [m].

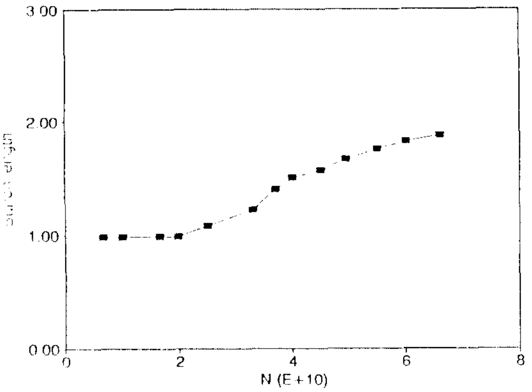

Fig. 8. The bunch length due to the PWD by the simulation method.

We used a tracking method [11] for simulating the effect of the wakefield on the longitudinal phase space of the beam. The deformation of the equilibrium distribution in a bunch due to the wakefield induced by itself is included in this method. The long range wakefield as a function of the bunch coordinates z for the ring with cavities was calculated for the gaussian driving bunch with $\sigma_z = 1$ mm as shown in Fig. 7.

For the simulations we take the synchrotron tune of 0.0059, momentum spread of 7.15E-04, energy 2 GeV, bunch length of 0.0078 m, 5 azimuthal space harmonics and 60 mesh points. Fig. 8 shows the bunch length(σ_z/σ_{z0}) due to the potential well distortions (PWD) by the simulation method using the wakefield of Fig. 7. From this

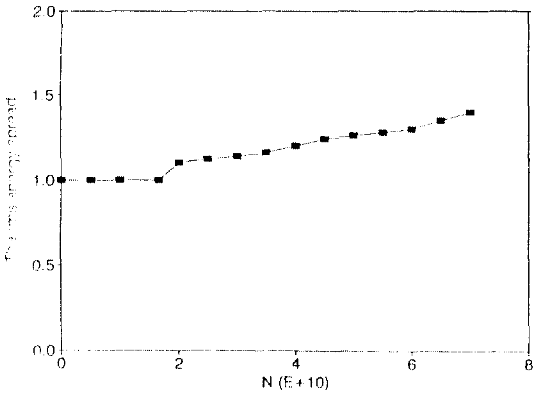

Fig. 9. The rms energy spread due to the PWD by the simulation method.

method applied to the current storage ring we find that the threshold to be 1.65×10^{10} which is 4.699 mA.

In Fig. 9, we plot the average rms energy spread$(\sigma_\varepsilon/\sigma_{\varepsilon 0})$ due to the PWD as function of current by the simulation method using the long range wakefield of Fig. 7.

We find that the threshold with the potential well distortion (PWD) is 1.65×10^{10} which is the value between 0.88 mA for without SPEAR and 6.24 mA for with SPEAR scaling. The first unstable mode appears at this current by the tracking calculation.

The results of the coupled bunch calculations is given in Table 1 for the first few fastest-growing longitudinal modes. Calculations were done for all the higher order modes of a single rf cell, with the resonant shunt impedances enhanced by a factor of three because of the presence of three cell. The dipole and quadrupole synchrotron mode oscillations are unstable and not Landau damped by the small synchrotron frequency spread of the bunch for the longitudinal modes. Modes with synchrotron mode numbers higher than a=2 longitudinary are all effectively Landau damped by the synchrotron frequency spread in the beam.

The bunch lengthening and momentum spread due to the microwave instability are shown in Fig.

Table 1. Growth time and tune shifts for the fastest growing longitudinal coupled bunch modes

Energy(GeV)	Synchrotron mode no. (a)	Coherent growth time(msec)	Tune shift	Landau damping
2	1	0.25	1.27E-3	undamped
	2	3.60	5.3E-3	〃
	3	88.0	1.39E-3	damped
	4	509	8.15E-4	〃

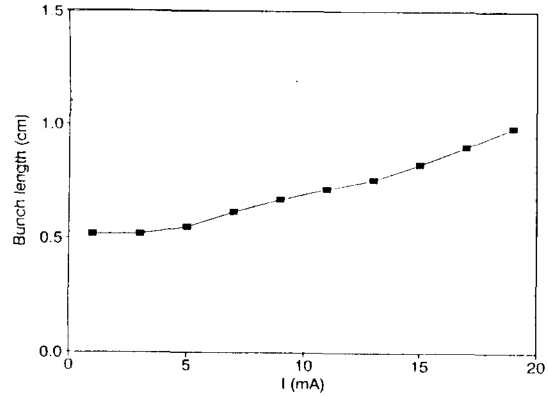

Fig. 10. Bunch lengthening due to the microwave instability.

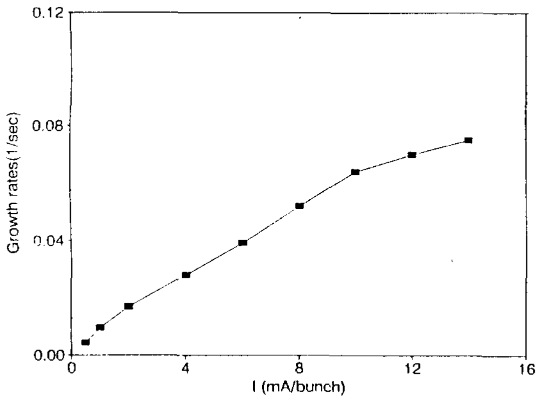

Fig. 12. Longitudinal coupled bunch instability for single bunch.

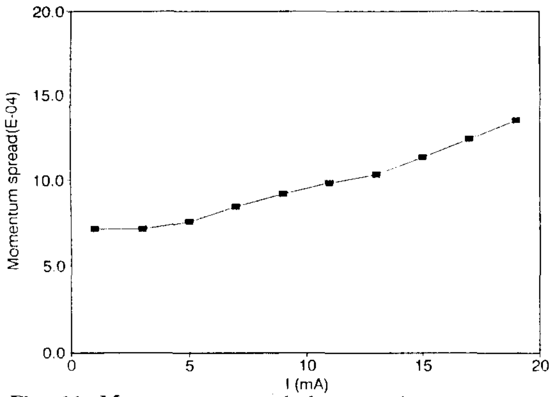

Fig. 11. Momentum spread due to microwave instability.

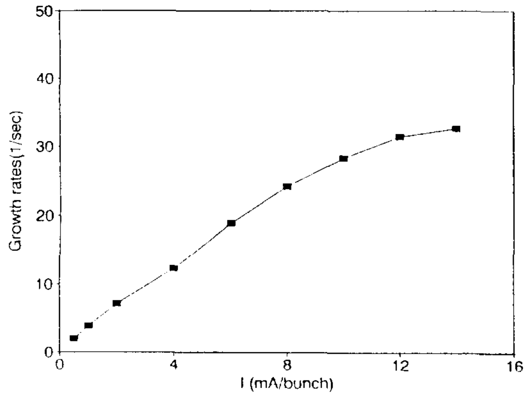

Fig. 13. Longitudinal coupled bunch instability for multi bunch.

10 and Fig. 11. The influence of the longitudinal microwave instability is determined by the effective broadband impedance(Z/n) assumed for the ring. The broadband impedance is made up of contributions from the vacuum chamber and the rf cavities. We take the momentum spread of 7.15E-04, rf voltage of 1.5 MV and Z/n of 2.023 ohms as longitudinal bunch parameters.

The calculated maximum growth rates due to the longitudinal coupled bunch instability for single and multibunch are shown in Fig. 12 and Fig. 13, respectively.

In calculations of the growth rate by the computer code ZAP [12], we have used total number of bunches of 282, the average beam current of 100 mA, bunch length of 0.0078 m at 2 GeV and ring radius 26.853. We calculated total 15 modes

Table 2. Higher order modes causing beam instabilities

Freq.(MHz)	Mode type	R/Q(ohm)	Q-value
499.113	TM010	58.899	35716
843.999	TM011	5.565	30279
1114.140	TM020	0.014	34027
1463.590	TM021	0.193	32396

including fundamental one in the longitudinal case of our designed RF cavity with the length of 40 cm, diameter 46.035 cm, peak voltage 1.5 MV and frequency 499.113 MHz. After the calculation of the coupled bunch oscillation growth rate for each modes, we have choose the worst cases of TM011 (843.99 MHz) mode for longitudinal case. Table 2 summerizes the higher order modes to be troublesome except for the fundamental mode among 15 modes. We have calculated the coupled bunch instability growth rates for the parasitic modes of the designed RF cavity. The growth rates are small enough compared to the radiation damping times of 4.5 ms longitudinally.

5. Conclusions

We have calculated the total loss, wake potential and the impedances for the sum of higher order modes in our designed model cavity. We see that the analytical calculations are agree well with a numerical calculation by the computer code URMEL and TBCI. The instability phenomena is also calculated based on the Fokker-Planck equation for synchrotron motion. It was found that strong Robinson instability does not occurs in our case. We calculated the threshold current of 6.7 mA for the modes coupling, which is less but close to the value of SPEAR scaling law and the design value.

We find that the threshold with the potential well distortion (PWD) by the simulation method using the long range wakefield is 1.65×10^{10} which is the value between 0.88 mA for without SPEAR and 6.24 mA for with SPEAR scaling. The first unstable mode appears at this current by the tracking calculation for the longitudinal single bunch instabilities.

We have calculated the coupled bunch instability growth rates for the 15 parasitic modes of the designed RF cavity. The growth rates are small enough compared to the radiation dampling times of 4.5 ms longitudinally. Since the radiation damping times are of the order of 10 ms, synchrotron radiation times are much longer than the instability growth times.

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