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Current Carrying Iron Whiskers: A New Magnetic Configuration

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The magnetization pattern of the central cross section deduced from the ac susceptibility measurement is described with an analytical function. The function is based on a charge-free configuration. The thickness of the 90° wall lying in a (100) plane and the wall energy density are calculated analytically. Total energy of the domain structure has been minimized with Ritz's method. As the result of the minimization, the energy density of the 90° wall lying in a (100) plane is 0.58 erg/cm^2 and the one for a (110) plane is 1.18 erg/cm^2. Thicknesses of these walls are calculated numerically. Also, the calculation indicates there is a small central domain at the cross section without applied current. With the ac susceptibility measurement the existence of the domain without current can be identified.

I. INTRODUCTION

The magnetic response of an iron whisker with a current along its axis is a challenging problem. The domain structure depends on the current I, on an external field applied along the axis H, and the past history of each. The structure can be altered further by applying fields perpendicular to the axis. The material parameters include the saturation magnetization M_s, the cubic anisotropy constants K_1 and K_2, the exchange interaction constant A_{ex}, and the magnetostriction constants B_m. In addition the dimensions of the whisker must be considered. The simplest case is the {100} whisker with all sides parallel to the set of {100} planes. Typically the {100} whiskers have almost square cross sections with widths ranging from 10 to 500 μm and lengths from 5 to 50 mm.

We study the magnetic response by measuring the ac susceptibility as a function of the fields, the current and past history. We also use techniques for deducing the domain structure from observation of surface magnetization and external magnetic fields. The latter techniques are of little value in deducing the basic structure for which the principal domain walls do not intersect the surface. The basic structure is presumed to be similar to that shown in Fig. 1[1]. It consists of five domains, four of which form a sheath about the central core. The evidence is compelling that each cross section is as illustrated in Fig. 2. The illustration of the variation with distance along the whisker of the cross section of the central core is schematic, for this remains one of the unsolved parts of the problem.

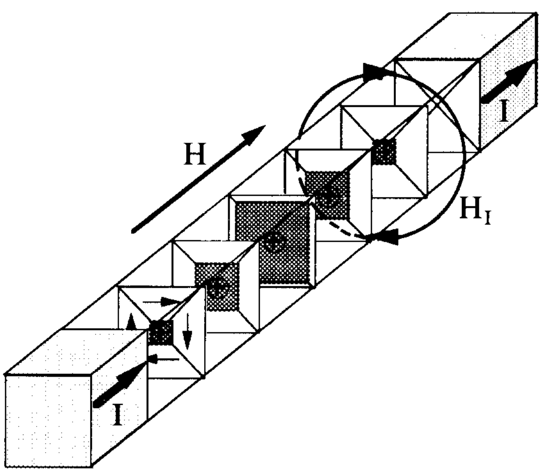

Fig. 1 Basic domain structure in a {100} whisker with external field and current along the principal axes. Lead wires are connected to patterned edges of the whisker.

The purpose of this note is to discuss the micromagnetics of the cross section shown in Fig. 2. This includes calculating the exchange and anisotropy energy associated primarily with the domain walls. A mathematical description of the magnetization configuration would be most helpful for treating this part of the problem. An approximate description is given below.

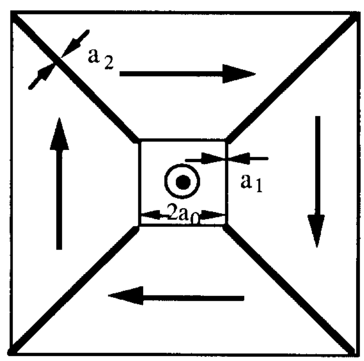

Fig. 2 Domain structure in the central cross section of the {100} iron whisker with current applied opposite to the direction of the central domain.

II. MICROMAGNETIC CALCULATION BASED ON A SIMPLE DOMAIN CONFIGURATION

The micromagnetic treatment also includes the magneto-elastic energy that arises because the core is elongated in the direction of its magnetization while the sheath is expanded circumferentially. The most important energy terms are the magnetostatic interactions from the magnetization of the central cross section with the applied field, the field from the current, and the demagnetizing field arising from magnetic charge at cross sections away from the central cross section, which is charge free.

An ansatz used for an approximate treatment of the ac susceptibility is to assume that a calculation valid for the central cross section can be extended to describe the fields from the rest of the material. The main effect of the magnetization in cross sections away from the central cross section is incorporated in the assumption that the demagnetizing field is proportional to the net magnetization in the central cross section. The calculation of that proportionality constant is part of the problem.

The essential feature of the ac susceptibility is that the out-of-phase response is dominated by eddy currents. The eddy currents are particularly sensitive to the movement of the four walls separating the inner core from the outer sheath.

The mathematics of the response is approximately given in terms of the dimension s of the inner core[2]. The net magnetic moment per unit length along the whisker is proportional to s^2. There is an effective field obtained from the derivative of the wall energy with respect to the net magnetic moment which is inversely proportional to s. The derivative of the magneto-elastic energy with respect to the magnetic moment contributes an effective field that is rather complicated with many terms. Its most important terms are one proportional to log s and another proportional to s^2. These effective fields are balanced by the real field from the current, the applied field H and the demagnetizing field. The demagnetizing field is proportional to s^2. The effect of the field from the current is expressed as an effective field by taking the derivative with respect to the magnetic moment of the integrated energy of the magnetization in the field of the current. This is proportional to Is the product of the current and the width of the central cross section. The relation between the applied field H and the magnetic moment per unit length m is given parametrically in terms of s, which is positive definite:

$H = CIs + D_o s^2 - \sigma / s - b(\log s - cs^2);\quad m = M_s s^2.$

The corresponding dc susceptibility is given by

$$\chi_0 = 2M_s s / \{CI + 2D_o s + \sigma / s^2 - b(1/s - 2cs)\}. \qquad (1)$$

The ac susceptibility is approximately given by

$1/\chi = 1/\chi_0 + i\omega / \omega_1 \log(s/d)$

where d is the width of the whisker.

The constants appearing in the ac susceptibility are C, D_o, σ, b, c, and ω_1. Once the challenging problem of calculating these parameters has been achieved, it is then interesting to note the richness of the response to changes in the independent variables H and I.

The first thing to note is that the magnetization of the central core does not reverse. If the central core is magnetized along the axis in the positive z direction, a field in the positive z direction will increase the size of the central core. If the field is in the negative z direction it only serves to drive the core to a smaller and smaller cross section as the field energy competes with the wall energy.

The second thing to note is that the magnetization of the central core maintains a very small value for small positive values of H. This is because of the log s term in the effective field from the magneto-elastic energy.

The third thing to note is that m can be a multi-valued function of H. This gives rise to hysteresis in positive fields. For particular choices of I and H there are two configurations of equal energy corresponding to different values of s. The barrier between these two states can be made arbitrarily small by choosing the critical I and the critical H. The hysteretic behavior can be exhibited by changes in either I or H.

III. CHARGE-FREE DOMAIN CONFIGURATION

All of these, and many other, aspects of the phenomenological relation given by Eq. 1 are seen in the magnetic response of {100} iron whiskers with current along the axis. This is sufficient motivation for an attempt to describe the magnetization pattern of the central cross section by an analytical function. $A_z(x,y)$ is the z component of a vector potential used to obtain the M_x and M_y components of the magnetization by taking its curl. The M_z component is then found by completing the square. The resulting magnetization pattern will be divergentless because the x and y components are the curl of a vector and the z component does not depend on z. The following function $A_z(x,y)$ is given for this purpose:

$$A_z(x,y) = A_0 + A_1 + A_2 + A_c \qquad (2)$$

where

$$A_0 = \frac{a_1}{\sqrt{2}} \log\left[\tan\left(\frac{\pi}{4} + \frac{\tan^{-1} e^{\frac{|x|+|y|-a_0}{a_1}}}{2}\right)\right],$$

$$A_1 = \frac{a_2}{\sqrt{2}}\left[\log\left(2\cosh\frac{|x|}{a_2}\right) - |x|\right]\sin\left(\tan^{-1} e^{\frac{|y|-a_0}{a_1}}\right),$$

$$A_c = \frac{a_2}{\sqrt{2}} \log\left[\cosh\frac{x-\frac{d}{2}}{a_2}\cosh\frac{x+\frac{d}{2}}{a_2}\cosh\frac{y-\frac{d}{2}}{a_2}\cosh\frac{y+\frac{d}{2}}{a_2}\right],$$

and A_2 is the same as A_1 with y and x interchanged.

$A_Z(x,y)$ is a pyramid with its top cut off and its edges rounded. The term A_0 generates a pyramid with its top cut off and the top edges rounded. The terms A_1 and A_2 round the edges of the sloping sides. The term A_c puts flats in the four corners of the base of the pyramid. The coordinate system is rotated by 45 degrees from that coinciding with the sides of the whisker and the anisotropy axes.

$A_z(x,y)$ has three parameters a_0, a_1 and a_2. The width of the central core will depend on a_0 which determines the size of the top of the pyramid. The width of the walls surrounding the central core will depend on a_1 which determines the rounding of the top edges. The width of the walls separating the four domains of the sheath will depend on a_2 which determines the rounding of the sloping edges. The parameter a_2 also characterizes the magnetization rotation in the four corners of the cross section where it is assumed, arbitrarily and without consequences, that the magnetization is parallel to the magnetization of the central core.

The M_y component of the magnetization is obtained by taking the derivative of $-A_z(x,y)$ with respect to x. The M_x component is found from the derivative of $A_z(x,y)$ with respect to y. The M_y component is given by:

$$M_y / M_s = -\frac{sgn(x)}{\sqrt{2}}\left[\sin\theta + (\tanh\frac{|x|}{a_2} - 1)\sin\psi\right] - \frac{sgn(x)}{\sqrt{2}a_1}\left[\left(a_2 \log(2\cosh\frac{|y|}{a_2} - |y|)\right)\cos^2\phi\sin\phi\right] + \frac{1}{\sqrt{2}}\left[\tanh\frac{x+\frac{d}{\sqrt{2}}}{a_2} + \tanh\frac{x-\frac{d}{\sqrt{2}}}{a_2}\right] \quad (3)$$

where

$\theta = \tan^{-1} e^{\frac{|x|+|y|-a_0}{a_1}}$, $\psi = \tan^{-1} e^{\frac{|y|-a_0}{a_1}}$, and $\phi = \tan^{-1} e^{\frac{|x|-a_0}{a_1}}$.

The M_x component has the same functional form with x and y interchanged and with the inverse sign of the M_y component. The in-plane component of the magnetization near the central domain is shown in a contour plot in Fig. 3.

The anisotropy energy for a cubic crystal is given by:

$$E_K = \frac{K_1}{M_s^4}\int_V \left(M_1^2 M_2^2 + M_3^2\left(M_1^2 + M_2^2\right)\right)d\tau \quad (4)$$

where

$M_1 = (M_x + M_y)/\sqrt{2}$, $M_2 = (M_x - M_y)/\sqrt{2}$,

and $M_3^2 = M_s^2 - M_x^2 - M_y^2$, and $d\tau$ is the volume element integrating over the volume V of a magnetic material.

The exchange energy is given by:

$$E_{ex} = \frac{A_{ex}}{M_s^2}\int_V \left(\nabla \times \vec{M}\right)^2 d\tau = \frac{4A_{ex}L}{M_s^2}\int_0^{\frac{d}{2}}\int_0^{\frac{d}{2}}\left(\nabla \times \vec{M}\right)^2 dxdy \quad (5)$$

where L is the length of the material.

The width of the walls surrounding the central core can be determined with the exchange and anisotropy energy. The energy density of the 90° wall lying in a (100) plane can be obtained analytically

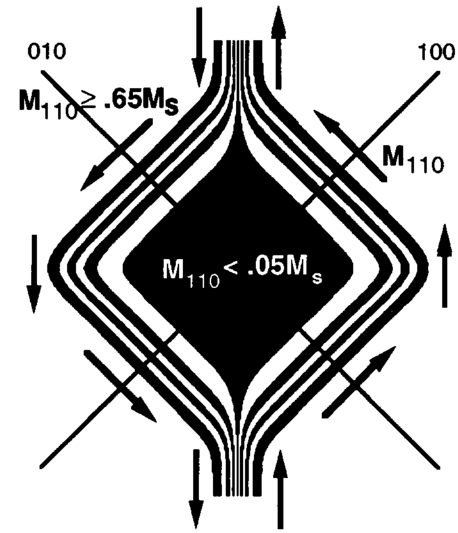

Fig. 3 Contours for the in-plane component of the magnetization $M_{110} = \sqrt{M_x^2 + M_y^2}$ with a_0=0.2, a_1=1/40, and a_2=1/60. Lines of equal values of M_{110} are shown as the edges of strips across which M_{110} decreases by 20% of the saturation magnetization.

with Eqs. 4 and 5. The energy density of the wall is given by:

$$e_{wall} = e_K + e_{ex} = \frac{K_1}{M_s^4}\left[M_1^2 M_2^2 + M_3^2(M_1^2 + M_2^2)\right] + \frac{A_{ex}}{M_s^2}\left(\nabla \times \vec{M}\right)^2 = \frac{K_1 \sin^2 2\theta}{4} + A_{ex}\frac{\sin^2 2\theta}{2a_1^2} = \left(\frac{K_1}{4} + A_{ex}\frac{1}{2a_1^2}\right)\sin^2 2\theta. \quad (6)$$

It seems that the anisotropy energy does not depend on a_1; while the exchange energy wants a_1 as large as possible. We need to look at the volume integration to see how the total energy depends on a_1, because the spatial dependence of the angles depends on a_1. A double integration of the exchange energy is linear in $1/a_1$. With Eq. 6, the thickness of the 90° wall is calculated as:

$$\frac{\partial e_{wall}}{\partial a_1} \approx \frac{\partial}{\partial a_1}\left(\frac{A_{ex}}{2a_1} + \frac{K_1 a_1}{4}\right) = 0; \quad a_1 = \sqrt{\frac{2A_{ex}}{K_1}}.$$

To obtain a better theoretical value for the wall energy, a micromagnetic calculation is carried out. The total energy consists of the exchange energy, the anisotropy energy, the field energy due to the applied current, the magnetostatic energy, and the energy from an applied field. To simplify the situation, the magneto-elastic energy is not considered with the assumption of a rigid body. For the magnetic field from the applied current, Küpfmüller's expression is used[3]. Bloomberg's formula is used for the demagnetizing factor D of a rectangular bar[4]. The energy due to the demagnetizing field H_D in the mid plane is approximated as:

$$E_D = -\int_V \tfrac{1}{2}\vec{M}\cdot\vec{H}_D d\tau = 2\pi D\int_V M_z^2 d\tau = 2\pi D < M_z >^2 d^2 L$$

where $<M_z>$ is the z-component of the magnetization averaged over the cross section. The ratio of the length and the width of the whisker is 50. The total energy is minimized while varying Ritz parameters, a_0, a_1, and a_2 for fixed currents and fields. The external field is assumed to be along the direction of the magnetization of the central domain. The energy from the field is approximated by the energy at the central cross section as:

$$E_H = -\int_V \vec{H}\cdot\vec{M} d\tau = - < M_z > Hd^2 L.$$

The result of the minimization shows our basic domain structure is stable for each applied external field and current. As the demagnetizing field and the applied current increase, the central domain becomes smaller. The increase of the external field makes the central domain larger. These results agree with our previous ones. The numerical calculation predicts the existence of a small domain near the center of the cross section without applied field and current. The central domain generates the demagnetizing energy, which keeps walls surrounding the central core small.

The energy densities of 90° walls are calculated for a (110) plane and a (100) plane numerically. As the result of the minimization, the exchange energy is equal to the anisotropy energy at the minimum. The energy density of the 90° wall lying in a (110) plane is 1.19 erg/cm^2 and the one for a (100) plane is 0.58 erg/cm^2. Our model has the same energy density for a (110) plane calculated by Aharoni and Jakubovics[5]. But the energy density of a 90° wall lying in a (100) plane is less than half of the energy density of a 180° wall lying in a (100) plane. The reason for the low energy is due to the magnetostriction which makes the 90° domain wall narrower than half of the 180° wall.

The thickness of the 90° domain wall lying in a (110) plane is defined as $6a_1 d/\sqrt{2}$(511 Å) which is the distance over which M_x or M_y change from $0.9M_s/\sqrt{2}$ to $-0.9M_s/\sqrt{2}$. The thickness of the wall for a (100) plane, 487Å, is calculated as M_z varies from $0.9M_s$ to $0.1M_s$ while crossing the 90° wall. These wall thicknesses can not be compared with the previous result because the thickness is defined differently. But these values are within the range of Jakubovics' wall thickness[6].

IV. CONCLUSION

Fig. 4 is the experimental data measured without applied current. Before taking the data, current which was large enough to make the basic structure shown in Fig. 1 was applied along a {100} whisker without any external field. After current was turned off, the external field has been changed only within a few oersted. The data show there is no out-of-phase signal near zero field. It indicates that the domain structure has only 90° walls lying in {110} planes. These walls are not moved easily with a small driving field. To move the walls, the driving field has

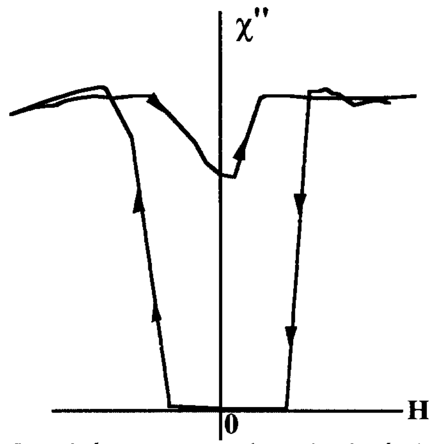

Fig. 4 Out-of-phase response for a {100} whisker while varying external field without applied current. The depth near zero field will disappear after several oscillations of the external field.

to be large enough to compute the exchange and anisotropy field. On the other hand, the wall surrounding the central domain can be easily oscillated even with the small driving field. If a small domain is at the center of the cross section, that would be too small to induce any signal. The existence of the basic structure without applied current can be explained with this data. The basic structure is very stable when small external field is applied along the whisker. With a large external field (about 3 Oe), the basic structure is transformed to the Landau structure. Once the basic structure is changed, the Landau structure does not go back to the old structure unless large current is applied along the whisker.

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