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FINITE ELEMENT ANALYSIS OF ROTATIONAL HYSTERESIS LOSS USING TWO DIMENSIONAL PERMEABILITY TENSOR

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Abstract - Finite element analysis using two dimensional magnetic permeability tensor that can represents phase lag between magnetic field intensity and flux density under rotational flux is examined. Considered problem is confined to two dimensional magnetostatic case. And we applied proposed method to calculate the core loss of the test model and compare the result with that of experiment.

I. INTRODUCTION

Currently, there are increasing demands for energy efficient electrical machine. For the design of high efficient electrical machine, accurate loss analysis is very important. In the various kinds of loss in electrical machines, iron loss - hysteresis loss & eddy current loss - analysis is one of the most difficult problem because of the nonlinear, anisotropic, and hysteresis characteristic of magnetic materials.

Existing hysteresis loss analysis is based on the magnetic characteristics which is measured under the alternating magnetic fields. But, in the electrical machines such as induction machines and transformers there exist rotating magnetic fields and high power loss is associated with its presence.[1][2]

Scalar permeability is measured under the assumption that magnetic field intensity $\vec{H}$, magnetization $\vec{M}$, and so the flux density $\vec{B}$ vectors are of the same direction. But, if magnetic field varies two dimensionally, for example, fields are rotating the three vectors are not of the same direction in general.

If rotational flux is occurred, magnetic field analysis using scalar permeability is not good enough, so we need alternative analysis method.

In this paper, finite element analysis using two dimensional permeability tensor and calculation of power loss under rotating flux are examined. Next, we applied this analysis method for the computation of the iron loss in the test model.

II. FINITE ELEMENT FORMULATION

A. Two Dimensional Permeability Tensor

In the case of rotational flux, there is a phase lag between $\vec{H}$ and $\vec{B}$. And this phase lag is well expressed by the two dimensional magnetic permeability tensor.[3]

$$\begin{pmatrix} B_x \\ B_y \end{pmatrix} = \begin{pmatrix} \mu_{xx} & \mu_{xy} \\ \mu_{yx} & \mu_{yy} \end{pmatrix} \begin{pmatrix} H_x \\ H_y \end{pmatrix} \tag{1}$$

Where H_x and H_y are x and y component of $\vec{H}$.

B. Formulation

There are some kinds of method for the finite element formulation. One of them is using energy functional. The energy functional used for the static magnetic field is as follows.

$$I = \int_\Omega \left(\int \vec{H} \cdot d\vec{B} \right) d\Omega - \int_\Omega \vec{J} \cdot \vec{A}\, d\Omega \tag{2}$$

Where Ω is analysis domain, $\vec{J}$ is free current density, and $\nabla \times \vec{A} = \vec{B}$. But this energy functional is not uniquely defined unless the symmetry of the tensor is not guaranteed. So, this energy functional is of no use for the asymmetric tensor. As a result, Galerkin method is used for the formulation.

We confine our problem to two dimensional magnetoquasistatic problem. That is, analysis

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domain is uniform in one direction, say z-axis and displacement current is negligible. So, we use Maxwell' equations in a static magnetic field and equation(1).

$$\nabla \times \vec{H} = \vec{J}_f \qquad \nabla \cdot \vec{B} = 0 \quad (\nabla \times \vec{A} = \vec{B}) \tag{3}$$

Where $\vec{J}_f$ is free current density and $\vec{A}$ is magnetic vector potential. Taking equation(1) into consideration, equation(3) becomes equation(4) for the two dimensional problem.

$$\frac{\partial}{\partial x}\left(\nu_{yy}\frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_{xx}\frac{\partial A}{\partial y}\right) - \frac{\partial}{\partial y}\left(\nu_{xy}\frac{\partial A}{\partial x}\right) - \frac{\partial}{\partial x}\left(\nu_{yx}\frac{\partial A}{\partial y}\right) = -J \tag{4}$$

Where $\vec{A} = A\hat{k}$, $\vec{J}_f = J\hat{k}$, and ν_{ij} is the element of inverse of permeability tensor. This equation is formulated by the Galerkin approximation.

III.ROTATIONAL POWER LOSS

The total magnetic power loss under arbitrary flux condition can be obtained by using the properties of Poynting vector.[4]

The power transferred by electromagnetic field into a volume can be calculated from integrating Poynting vector over the surface enclosing the volume. Let p_t be the total power transferred into a volume V from its environment and the surface enclosing V be S, then p_t is given by

$$p_t = -\int_S \vec{S} \cdot \hat{n}\, ds \tag{5}$$

,where $\vec{S}$ is the Poynting vector and $\hat{n}$ is outward unit vector normal to the surface. Using divergence theorem equation(5) becomes equation(6).

$$p_t = -\int_V \nabla \cdot \vec{S}\, dv = -\int_V \nabla \cdot (\vec{E} \times \vec{H})\, dv \tag{6}$$

Vector identity,

$$\nabla \cdot (\vec{C} \times \vec{D}) = \vec{D} \cdot (\nabla \times \vec{C}) - \vec{C} \cdot (\nabla \times \vec{D}) \tag{7}$$

Maxwell's equation,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{H} = \vec{J}_f \tag{8}$$

and the assumption that there's no free current in the volume V yield equation(9).

$$p_t = \int_V \vec{H} \cdot \frac{\partial \vec{B}}{\partial t}\, dv \tag{9}$$

The volume V above is usually a part of magnetic material, so there is no incident current and if eddy current is negligible, little is lost by the above assumption.

If the time average of the spatial average of this power(p_t) is positive, this will be a power loss per unit volume. So the average P_t is expressed as equation(10).

$$P_t = \frac{1}{T}\int_T \left(H_x \frac{dB_x}{dt} + H_y \frac{dB_y}{dt}\right) dt \tag{10}$$

Here, T is the period of excitation.

Expressing this power loss by polar coordinates yields two terms.

$$P_t = \frac{1}{T}\int_T \frac{d\theta}{dt}(\vec{B} \times \vec{H})_z\, dt + \frac{1}{T}\int_T |\vec{H}| \frac{d|\vec{B}|}{dt} \cos\alpha\, dt \tag{11}$$

Where θ is the phase of $\vec{B}$, and α the lag angle between $\vec{H}$ and $\vec{B}$.

For the pure rotating flux, the second term on the right of equation(11) vanishes and the first term gives total power loss per unit area. This term is easily discretized as equation(12).

$$P_r = \frac{1}{T}\int_T \frac{d\theta}{dt}(\vec{H} \times \vec{B})_z\, dt \approx \frac{1}{T}\sum_{i=1}^{N} \Delta\theta_i (\vec{H}_i \times \vec{B}_i)_z \tag{12}$$

Where pure rotating flux exists, by simply applying equation(12) rotational power loss can be computed. Another term represents hysteresis loss due to alternating flux component.

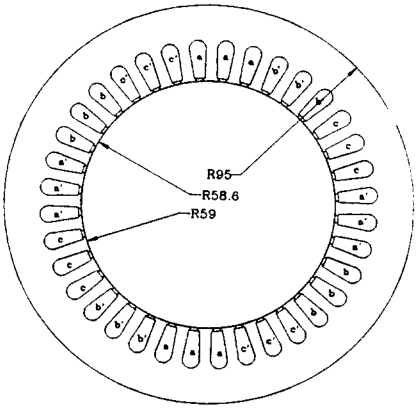

Fig.1 Simulation & Experiment Model

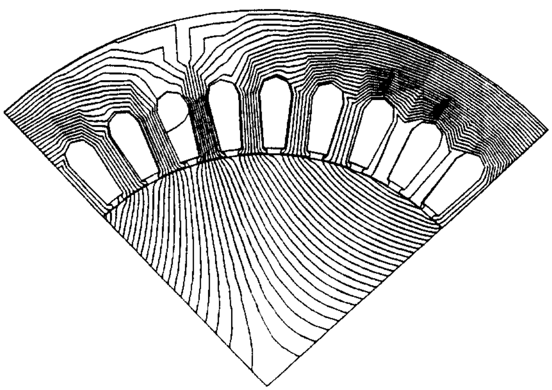

Fig.2 Analyzed Result (Flux Distribution)

IV.SIMULATION & EXPERIMENT RESULTS

A. Simulation

Fig 1. shows simulation and experiment model. outer frame is a stator of 5 [Hp], 220 [V], 4 pole induction motor. Inner core is made of silicon steel plates S60. Applying 3 phase current to the stator winding makes nearly pure rotating flux in the circular core. The proposed method is applied to the calculation of rotational hystersis loss in the circular core. One-quater of the model is analyzed using periodic boundary condition. And the elements of the tensor is assumed to be constant. Analyzed flux distribution is shown in Fig.2.

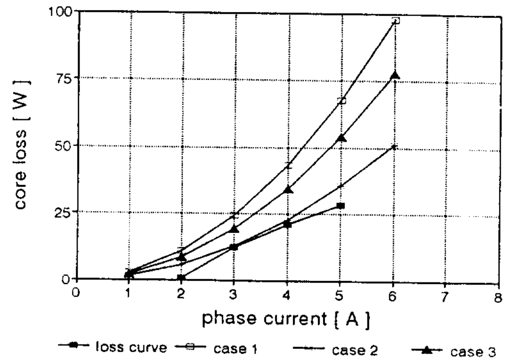

Fig.3 Proposed vs. Conventional

Comparison with conventional loss analysis is shown in Fig.3. Where case 1 : μ_{xx}=1000, μ_{xy}=-800, μ_{yx}=800, μ_{yy}=1000, case 2 : μ_{xx}=2000, μ_{xy}=-1600, μ_{yx}=1600, μ_{yy}=2000, case 3 : μ_{xx}=1000, μ_{xy}=-500, μ_{yx}=500, μ_{yy}=1000. Conventional loss analysis is using loss curve provided by the steel manufacturer. Calculated power loss is twice or three times that of conventional loss analysis

B. Experiment

Power loss measuring apparatus is shown in Fig.4. An induction motor in the left hand side is connected to the test model. If 3 phase current is applied to the stator winding of the test model, separation of core loss in the circular core from that in the stator is difficult. Upon this, instead of rotating flux system, static flux & rotating core type is used. That means, stator winding is connected to a DC power supply and static flux is generated and then rotation of circular core makes rotating flux in view of the core. Then, the power loss can be calculated from measuring power input of the motor. Fig.5 shows the comparison between experiment & simulation results. Case1,2,&3 are as described above.

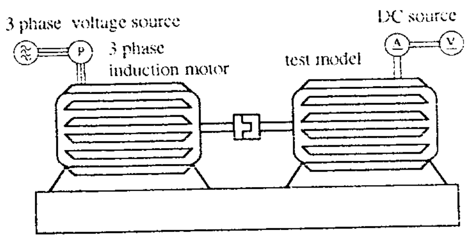

Fig.4 Power Loss Measuring Apparatus

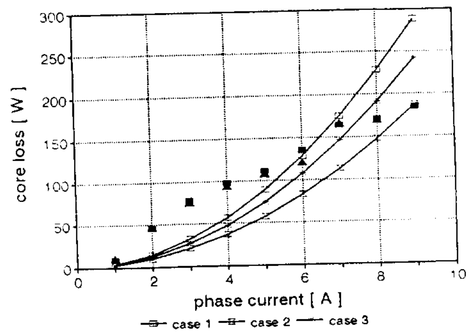

Fig.5 Experiment & Simulation Results

The error comes from the rough assumptions that eddy current is negligible, pure rotating flux occur in the circular core and that the elements of permeability tensor is constant. In fact, the elements are function of some factors.

V.CONCLUSION

Loss analysis using two dimensional permeability tensor is more accurate than the existing method. More research in considering nonlinearity of the elements of permeability tensor will yield more accurate loss analysis method.

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