Radiation Power Estimation for the Planar Array Acoustic Sensor Considering Mutual Coupling Effects

상호간섭영향을 고려한 평면배열형 음향센서의 방사출력 예측

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요 약 : 평면 배열형 소나 센서에서는 트랜스듀서 상호간의 간섭효과들이 음을 방사하는 각각의 트랜스듀서 및 평면 배열의 빔패턴에 영향을 주게된다. 따라서 음향 방사출력의 계산은 소나용 트랜스듀서의 성능 및 효율을 평가하는데 필수적이다. 음향 방사출력을 예측하기 위하여 무한 강성 배플에 고정된 수개의 트랜스듀서를 이론해석의 대상으로 설정하였다. 각 트랜스듀서는 자기방사 임피던스 및 상호방사 임피던스로 구성되어 있으며 이것의 총 방사 임피던스 및 음향방사 출력의 추출은 등가 전기회로 모델을 이용하였다. 이론 및 수치해석의 결과에 근거하여 음향방사 출력은 각 트랜스듀서 상호간의 간섭의 양에 의존함을 보였으며 상호간섭에 의한 음향출력 손실은 25.05%에서 최고 51.52%정도임을 확인하였다.

Keywords: planar array acoustic transducer, radiation power, self-radiation impedance, mutual radiation impedance, equivalent circuit, impedance matrix

I. Introduction

Rectangular shape planar array acoustic transducers are usually used in a sonar system. To obtain the radiation characteristics of the transducers in the water, performance estimation by using the beam pattern is general approach. The main source of this beam pattern is coming from the radiation power of the transducer itself. The intensity of the radiation power in the transducer is proportional to the product of current and voltage of the transducer. This radiation power affects to decide the magnitude and shape of main lobe of the beam pattern, directly. Therefore, to extract the radiation power of the transducer, equivalent electric circuit method is reasonable. To predict the radiation power calculations of the self-radiation impedance due to transducer itself and the mutual radiation impedance due to mutual coupling effects are necessary.

The estimation of radiation power for the planar array acoustic transducer considering mutual coupling phenomena seems less attention. Lee et al[1]. suggested quadruple integral equations for the self- and mutual radiation impedances of the planar array acoustic transducer by using geometric relations between square pistons. And the quadruple integral equations simulated numerically. Gilbert[2] measured the radiation impedance of sound projectors working in array and also investigated that the mutual radiation impedances are dominant in close-packed arrays containing projectors which have diaphragms that are small in terms of a wave length. Porter[3] calculated near field sound pressure of the flexural disk and obtained the series expansion of self- and mutual radiation. To calculate the transmitting characteristics of sonar arrays, Audoly[4] developed a computer model for a densely packed array of circular pistons and showed the acoustic interactions may have large effects on the pressure and velocity distributions on the surface of the array. However, other researchers [5-8] have been studied the mutual coupling effects only in the array transducers.

In this paper some reasonable results on the planar array with 37 transducers are shown. It is assumed that the transducer is mounted on the rigid infinite baffle and each transducer is identical. Radiation impedance of each transducer can be obtained as a combination of self- and mutual radiation impedances. By using an equivalent circuit, radiation power of the transducers is calculated considering the mutual coupling effects. Therefore, radiation power calculation can be applied to the performance and efficiency estimation of the sonar arrays with densely packed transducers.

II. Extraction of Radiation Power Considering Mutual Coupling Effects

Underwater acoustic transducer is main parts of sonar system. The transducer consists of radiating front mass, piezoelectric element, and inertia tail mass. Generally, sound propagates to the water due to the vibration of radiating front mass and the transducer called piston simply. In this paper 37 square pistons which are mounted in the rigid infinite baffle are selected. Fig. 1 shows relative positions and the planar array of 37 acoustic transducers which are used to a sonar system. According to the required power, size limitation, and beam pattern the several different combinations are possible. In Fig. 1, a and d are width and distance of the square piston. Fig. 2 shows relative coordinate of the 37 transducers. There are eight types(A-H) of radiation impedance pattern which include self- and mutual impedances. Those types are catagolized by geometric relations between pistons.

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Total radiation impedance, Z_{Ri} of square piston can be expressed as the summation of the self-and mutual radiation impedances,

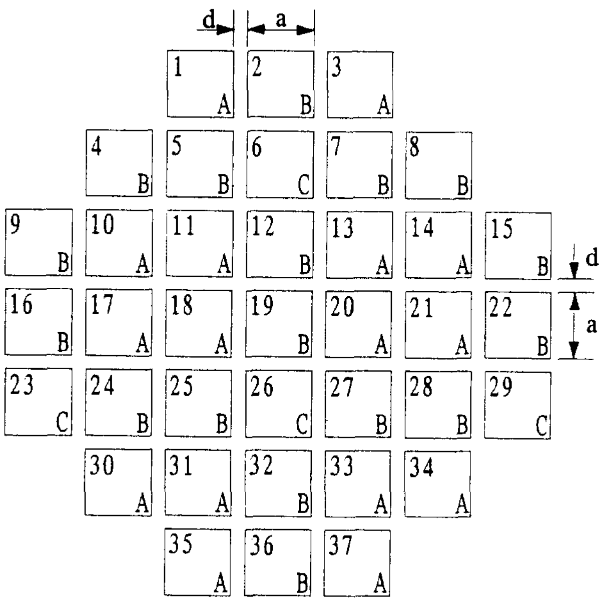
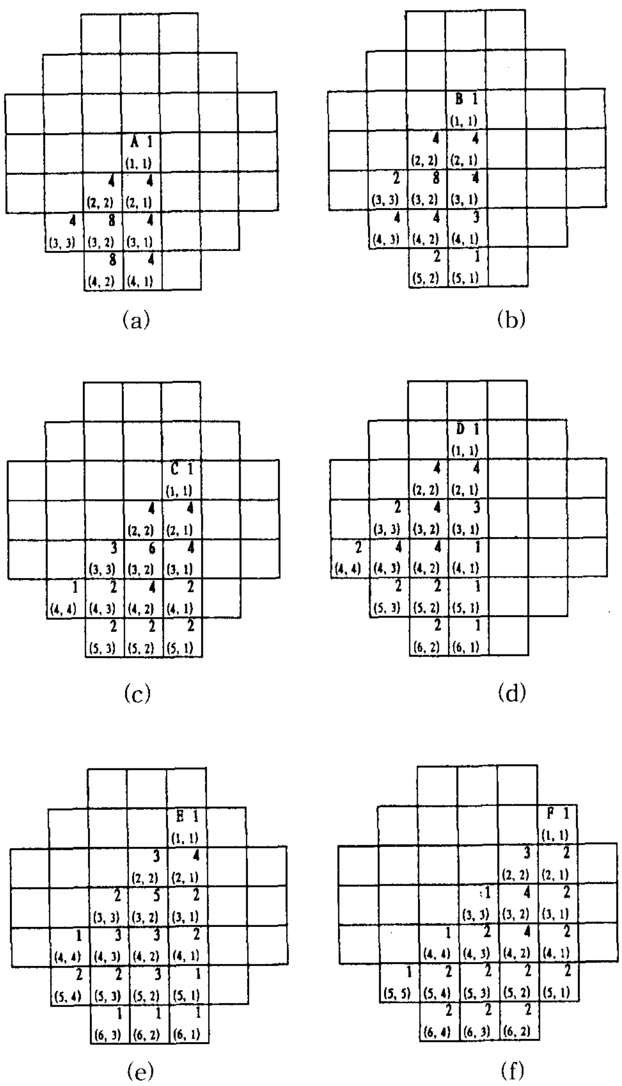


Fig. 1. Planar array acoustic transducer.



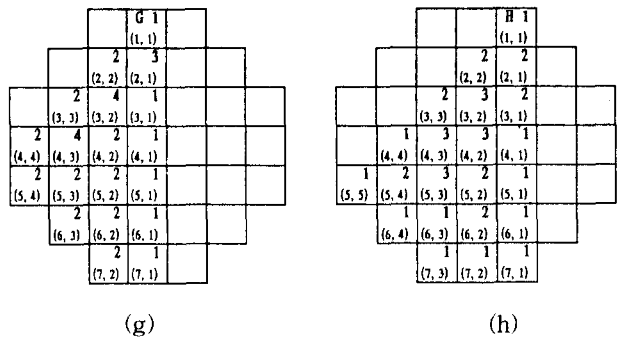


Fig. 2. Coordinate of the pistons.

$$Z_{Ri} = \sum_{j=1}^{N} Z_{ij}\frac{I_j}{I_i} = Z_{i1}\frac{I_1}{I_i} + Z_{i2}\frac{I_2}{I_i} + \cdots + Z_{iN}\frac{I_N}{I_i} \quad (1)$$

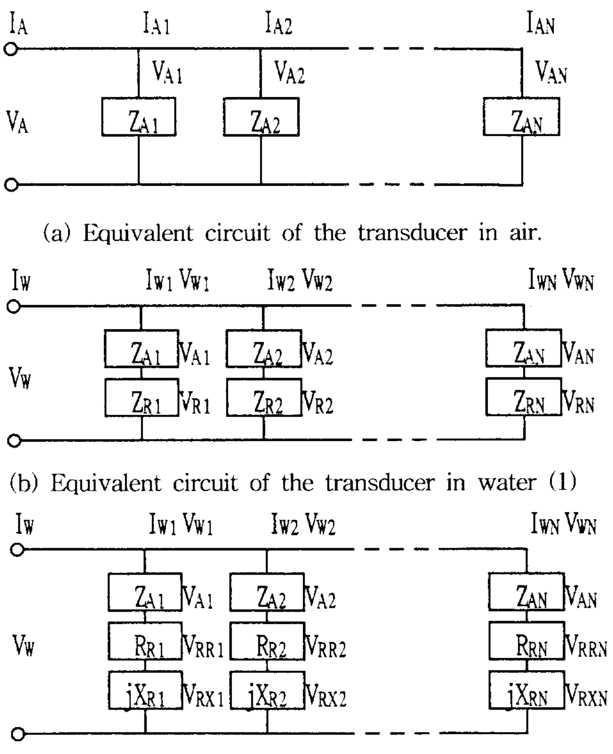
where, N is number of piston, I_i is current of the ith piston. Radiation impedance term Z_{ij} is self-radiation impedance when $i=j$ and mutual radiation impedance when $i \neq j$.

Table 1. Radiation impedance based on the mutual positions.

Nsec/m

Position	Z(x,y)	Resistance	Reactance	Impedance
(1,1)	Z(1,1)	0.76849E+03	0.50269E+03	0.91830E+03
(2,1)	Z(2,1)	0.44699E+02	-017408.E+03	0.17973E+03
(2,2)	Z(2,2)	-0.11104E+03	-0.32813E+02	0.11579E+03
(3,1)	Z(3,1)	0.65946E+01	0.83942E+02	0.84201E+02
(3,2)	Z(3,2)	0.58934E+02	0.46541E+02	0.75095E+02
(3,3)	Z(3,3)	0.15243E+02	-0.57838E+02	0.59813E+02
(4,1)	Z(4,1)	-0.14580E+02	-0.53233E+02	0.55193E+02
(4,2)	Z(4,2)	-0.38533E+02	-0.36037E+02	0.52759E+02
(4,3)	Z(4,3)	-0.35005E+02	0.31177E+02	0.46876E+02
(4,4)	Z(4,4)	0.38084E+02	0.12409E+02	0.40054E+02
(5,1)	Z(5,1)	0.17017E+02	0.37467E+02	0.41151E+02
(5,2)	Z(5,2)	0.29899E+02	0.26866E+02	0.40196E+02
(5,3)	Z(5,3)	0.34175E+02	-0.15595E+02	0.37565E+02
(5,4)	Z(5,4)	-0.19096E+02	-0.28049E+02	0.33932E+02
(5,5)	Z(5,5)	-0.11385E+02	0.27850E+02	0.30087E+02
(6,1)	Z(6,1)	-0.17808E+02	-0.27581E+02	0.32831E+02
(6,2)	Z(6,2)	-0.25401E+02	-0.20054E+02	0.32363E+02
(6,3)	Z(6,3)	-0.29807E+02	0.85460E+01	0.31008E+02
(6,4)	Z(6,4)	0.51147E+01	0.28505E+02	0.28960E+02
(7,1)	Z(7,1)	0.17896E+02	0.20640E+02	0.27318E+02
(7,2)	Z(7,2)	0.22592E+02	0.14887E+02	0.27055E+02
(7,3)	Z(7,3)	0.25667E+02	-0.56255E+01	0.26276E+02

To formulate the array equations, equivalent circuits are used as shown in Fig. 3. Fig. 3(a) is the



(a) Equivalent circuit of the transducer in air.

(b) Equivalent circuit of the transducer in water (1)

(c) Equivalent circuit of the transducer in water (2)

Fig. 3. Equivalent electric circuit for the planar array acoustic transducer.

Table 2. Total radiation impedances of the transducers.

Nsec/m

No.	P_{Ri}	X_{Ri}	No.	P_{Ri}	X_{Ri}
1	0.7654E+3	0.2398E+3	20	-0.8149E+3	-0.1874E+3
2	0.8533E+3	0.7942E+2	21	0.6269E+3	-0.2506E+3
3	0.7645E+3	0.2398E+3	22	0.8533E+3	0.7942E+2
4	0.6223E+3	0.3047E+3	23	0.7645E+3	0.2398E+3
5	0.7750E+3	-0.1336E+3	24	0.7750E+3	-0.1336E+2
6	0.6269E+3	-0.2506E+2	25	0.8681E+3	0.5310E+1
7	0.7750E+3	-0.1336E+3	26	0.8149E+3	0.1874E+3
8	0.6223E+3	0.3047E+3	27	0.8681E+3	0.5310E+1
9	0.7645E+3	0.2398E+3	28	0.7750E+3	-0.1336E+3
10	0.7750E+3	-0.1336E+3	29	0.7645E+3	0.2398E+3
11	0.8681E+3	0.5310E+1	30	0.6223E+3	0.3047E+3
12	0.8149E+3	0.1874E+3	31	0.7750E+3	-0.1336E+3
13	0.8681E+3	0.5310E+1	32	0.6269E+3	-0.2506E+2
14	0.7750E+3	-0.1336E+3	33	0.7750E+3	-0.1336E+3
15	0.7645E+3	0.2398E+3	34	0.6223E+3	0.3047E+3
16	0.8533E+3	0.7942E+2	35	0.7645E+3	0.2398E+3
17	0.6269E+3	-0.2506E+2	36	0.8533E+3	0.7942E+3
18	0.8149E+3	0.1874E+3	37	0.7645E+3	0.2398E+3
19	0.5831E+3	-0.3092E+3			

equivalent circuit of the parallel pistons in air. Z_{Ai} is impedance in air. When the N pistons are identical, supplied voltage in the circuit, V_A is same at each piston ($V_A = V_{A1} = V_{A2} = \cdots = V_{AN}$) and current is $I_A = I_{A1} + I_{A2} + \cdots + I_{AN} = NI_{Ai}$. Fig. 3(b) is an equivalent circuit of the parallel pistons in water. In this case it is easily seen that the supplied voltage and current are expressed as $V_W = V_{W1} = V_{W2} = \cdots = V_{WN}$ and $I_W = I_{W1} + I_{W2} + \cdots + I_{WN}$. Radiation impedance can be separated to radiation resistance and radiation reactance as shown in Fig. 3(c) and voltage is expressed as $V_{Wi} = (Z_{Ai} + R_{Ri} + jX_{Ri}) I_{Wi}$. It is a similar circuit and equation for every transducer in the array, because it is assumed that every transducers are identical. This set of N equations can be written in the form,

$$V_{Wi} = \left(Z_{Ai} + \sum_{j=1}^{N} Z_{ij} \frac{I_{Wj}}{I_{Wi}} \right) I_{Wi}$$

$$\text{(for i=1, 2, 3, ... , N)} \tag{2}$$

when all the transducers in the array are identical. (2) can also be expressed as

$$\begin{Bmatrix} V_{W1} \\ V_{W2} \\ \vdots \\ V_{WN} \end{Bmatrix} = \begin{bmatrix} Z_{A1} + Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{A2} + Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{AN} + Z_{NN} \end{bmatrix} \begin{Bmatrix} I_{W1} \\ I_{W2} \\ \vdots \\ I_{WN} \end{Bmatrix} \tag{3}$$

using matrix form. Using the admittance matrix $[Y_{ij}]$, the current of the ith piston is obtained as

$$\begin{Bmatrix} I_{W1} \\ I_{W2} \\ \vdots \\ I_{WN} \end{Bmatrix} = \begin{bmatrix} Y'_{11} & Y_{12} & \cdots & Y_{1N} \\ Y_{21} & Y'_{22} & \cdots & Y_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{N1} & Y_{N2} & \cdots & Y'_{NN} \end{bmatrix} \begin{Bmatrix} V_{W1} \\ V_{W2} \\ \vdots \\ V_{WN} \end{Bmatrix} = \begin{bmatrix} Y'_{11} + Y_{12} + \cdots + Y_{1N} \\ Y_{21} + Y'_{22} + \cdots + Y_{2N} \\ \vdots \\ Y_{N1} + Y_{N2} + \cdots + Y'_{NN} \end{bmatrix} V_W \tag{4}$$

and the current I_{Wi} which generated at each piston is

$$I_{Wi} = \left(Y_{ii}' + \sum_{j=1}^{N} Y_{ij} \right) V_W, \quad j \neq i \tag{5}$$

Supplied voltage V_{Wi} to the ith piston is $V_{Wi} = V_{Ai} + V_{Ri} = Z_{Ai} I_{Wi} + Z_{Ri} I_{Wi}$. From Fig. 3(b) voltage V_{Ri} at the radiation impedance Z_{Ri} is obtained as

$$V_{Ri} = Z_{Ri} I_{Wi} = \sum_{j=1}^{N} Z_{ij} I_{Wj} \text{ or}$$

$$\begin{Bmatrix} V_{R1} \\ V_{R2} \\ \vdots \\ V_{RN} \end{Bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{Bmatrix} I_{W1} \\ I_{W2} \\ \vdots \\ I_{WN} \end{Bmatrix} \tag{6}$$

Voltage and current which generated at each piston can be calculated using the (5) and (6). Total radiation impedance of the ith piston is $Z_{Ri} = V_{Ri} / I_{Wi}$. Total radiation resistance R_{Ri} is the real part of Z_{Ri}. Therefore acoustic radiation voltage V_{RRi} is obtained as

$$V_{RRi} = Re(Z_{Ri}) I_{Wi} = R_{Ri} I_{Wi} \tag{7}$$

Acoustic radiation power P_{Ri} of the ith piston is the multiplication of voltage and current, hence

$$P_{Ri} = V_{RRi} I_{Wi} = R_{Ri} I^2_{Wi}. \quad (8)$$

The radiation power P_{Ri} is complex number. However, the amount of actual acoustic radiation power is real part of P_{Ri} and imaginary part of P_{Ri} is pseudo radiation power. Therefore, the real part of P_{Ri} have to be considered as an actual radiation power.

Table 3. Radiation power of the transducers.

Watt

No.	Re(P_{Ri})	Im(R_{Ri})	No.	Re(P_{Ri})	Im(R_{Ri})
1	0.2361E-4	0.1123E-4	20	0.2444E-4	0.1208E-4
2	0.2456E-4	0.1319E-4	21	0.1873E-4	0.1152E-4
3	0.2361E-4	0.1123E-4	22	0.2456E-4	0.1319E-4
4	0.2047E-4	0.9440E-5	23	0.2361E-4	0.1123E-4
5	0.2141E-4	0.1389E-4	24	0.2141E-4	0.1389E-4
6	0.1873E-4	0.1152E-4	25	0.2438E-4	0.1388E-4
7	0.2141E-4	0.1389E-4	26	0.2444E-4	0.1208E-4
8	0.2047E-4	0.9440E-5	27	0.2438E-4	0.1388E-4
9	0.2361E-4	0.1123E-4	28	0.2141E-4	0.1389E-4
10	0.2141E-4	0.1389E-4	29	0.2361E-4	0.1123E-4
11	0.2438E-4	0.1388E-4	30	0.2047E-4	0.9440E-5
12	0.2444E-4	0.1208E-4	31	0.2141E-4	0.1389E-4
13	0.2438E-4	0.1388E-4	32	0.1873E-4	0.1152E-4
14	0.2141E-4	0.1389E-4	33	0.2141E-4	0.1389E-4
15	0.2361E-4	0.1123E-4	34	0.2047E-4	0.9440E-5
16	0.2456E-4	0.1319E-4	35	0.2361E-4	0.1123E-4
17	0.1873E-4	0.1152E-4	36	0.2456E-4	0.1319E-4
18	0.2444E-4	0.1208E-4	37	0.2361E-4	0.1123E-4
19	0.1592E-4	0.1232E-4			

Ⅲ. Application of the radiation power extraction method

To predict the quantitative radiation power, extraction method is applied to the planar array which have 37 transducers as shown in Fig. 2. It is assumed that dynamic characteristics of the 37 transducers are identical. The planar array is characterized as nondimensional size ka(=3.137), nondimensional distance kd (=3.272), and nondimensional ratio d/a(=1.043). The transducer in the array is supplied by 1V in underwater(say, $V_W = V_{Wi}$). To obtain the self- and mutual radiation impedances, results of Lee et al.[1] are cited here. Fig. 4 shows radiation resistance and reactance of self-radiation impedance as a function of ka. In this case when ka is 3.137, self-radiation impedance is 918.3 Nsec/m(=768.5+j502.7 Nsec/m). Fig. 5 shows the mutual radiation impedance between transducers as a function of ka in the case of d/a=1.043. Fig. 6 shows mutual radiation impedance as a function of kd. By using the plots self-(Z(i,j), $i=j$ case) and mutual radiation impedances(Z(i,j), $i \neq j$ case) are calculated as shown in Table 1. By using an impedance analyzer (HP4194A) the impedance in air is measured as Z_{Ai} =4510-j1430 Nsec/m. Table 2 shows total radiation impedance of each transducer. Table 3 and Fig. 7 show the quantitative radiation power of each transducer taking real part of (8). Radiation power of the 19th transducer is relatively small. Because the 19th transducer is placed at the center of the array. When the transducers are identical and ignoring mutual coupling effects, the pistons have self-radiation impedance only. In this case, the radiation power of the each piston is calculated as $Re(P_{Ri})$=0.32766E-4 by using the Eqs.

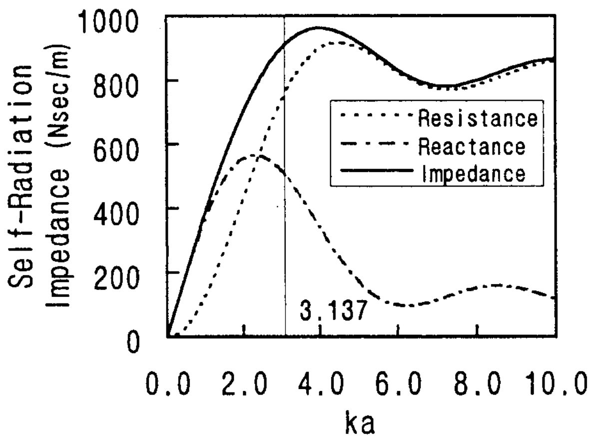


Fig. 4. Self-radiation impedance.

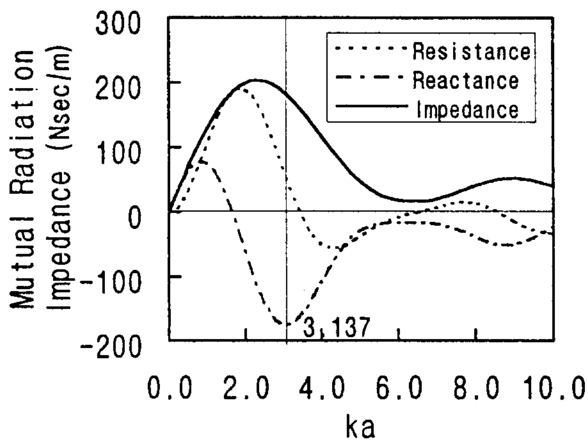


Fig. 5. Mutual radiation impedance as a function of ka (d/a=1.043 case).

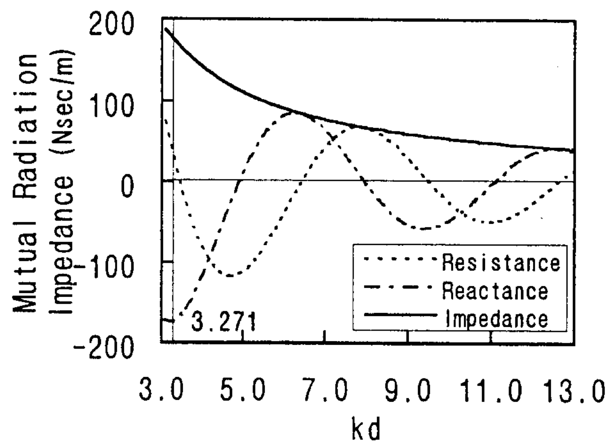


Fig. 6. Mutual radiation impedance as a function of kd (ka=3.137 case).

(2), (5), (7), and (8). To compare the quantitative radiation power of each transducer between with and without mutual coupling effect, acoustic power loss is calculates as maximum of 51.42 %(piston no.: 2, 16, 22, and 36) and minimum of 25.05 %(piston no.: 19).

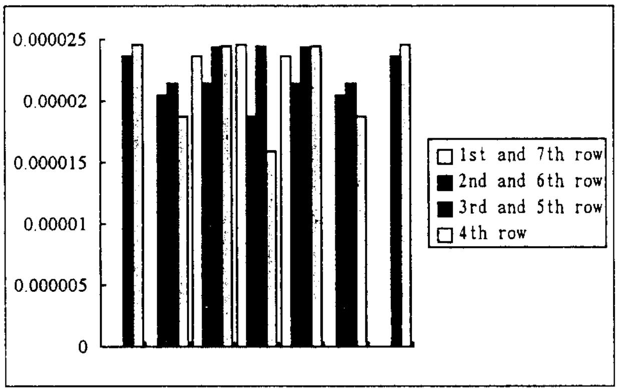


Fig. 7. Radiation power of the transducer from 1st to 7th row(unit: Watt)(Horizontal axis is row configurations in the planar array).

Ⅳ. Conclusions

Planar array acoustic transducer is normally used in sonar system. The main source of the beam pattern is acoustic radiation power. Therefore, acoustic radiation power is necessary to predict the performance and efficiency of the planar array. To predict the acoustic radiation power, 37 acoustic transducers which are mounted on a rigid infinite baffle are considered as a theoretical model. Each piston's acoustic radiation consists of self- and mutual radiation impedances. Total radiation impedances and acoustic radiation power of the transducers are extracted using by an equivalent electric circuit. Based on the theoretical results acoustic radiation power of the transducer depends on the mutual coupling effects. To compare the quantitative radiation power of each transducer between with and without mutual coupling effect, acoustic power loss is calculates as maximum of 51.42 % and minimum of 25.05 %.

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Appendix : nomenclature

a=square piston width
d=distancd between pistons
I_A=transducer current in air
I_{Ai}=ith piston current in air
I_W=transducer current in water
I_{Wi}=ith piston current in water
$j=\sqrt{-1}$ or matrix element number
k=wave number
ka=nondimensional width
kd=nondimensional distance
N=number of pistons
P_{Ri}=radiation power of ith piston
$R_{Ri}(=\mathrm{Re}(Z_{Ri}))$=radiation resistance of ith piston
V_A=voltage supplied to the transducer in air
V_{Ai}=impedance voltage of ith piston in air
V_{Ri}=radiation impedance voltage of ith piston
V_{RRi}=radiation resistance voltage of ith piston
V_{RXi}=radiation reactance voltage of ith piston
$V_W(=V_{Wi})$=supplied voltage of the transducers in water
X_{Ri}=radiation reactance of ith piston
Y_{ij}=element of admittance matrix
Z_{Ai}=ith piston's impedance in air
Z_{ii}=self-radiation impedance of ith piston
$Z_{ij}(=Z_{ji})$=mutual radiation impedance between ith and jth piston
Z_{Ri}=ith piston's total radiation impedance

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