Design of Nonlinear Robust Observer for Robots with Joint Elasticity

유연 조인트 로봇의 견실한 비선형 관측기 설계

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요 약 : 복잡한 비선형성과 불확실한 변수를 가지는 유연 죠인트 로봇의 견실한 관측기 설계에 관한 연구이다. 시간불변 또는 시간 가변성의 불확실한 변수들을 가지는 시스템에 적용한 경우이며 로봇 링크의 각도와 각속도들을 출력으로 하였다. 본 견실 관측기는 리아프노프 방법에 에 기초를 두었으며 불확실한 변수들은 그값은 모르나 그 값들은 지정된 집합내에 존재한다. 제안된 견실 관측기는 실용적인 안정성을 보장한다. 관측기설계의 알고리즘을 2링크 유연 죠인트 로봇에 적용하여 시뮬레이션을 수행하여 우수한 성능을 가짐을 확인할수 있었다.

Keywords: robust observer, practical stability, uncertainty, flexible joint manipulators

I. Introduction

We consider the observer design problem for flexible joint manipulators which have nonlinearity and contain uncertainty. As shown by the experimental work of [1], the joint flexibility must be taken into account in analysis. Gear elasticity, chain, and shaft wind up are common sources of joint flexibility. From modeling point of view internal deflection between the actuator and the driven link can be approximated by inputting a torsional spring at each joint. One of the models of flexible joint manipulator was presented in Spong [2] and we adopt this model in this paper. The observer design problem is one of the open problems in robot control area, specially in flexible joint robot control area. As matter of fact, the robot control with state feedback requires the knowledge of state variables for each link and joint, which may be either positions and velocities of joints and of the links [3] or positions, velocities, accelerations and jerks of links [4]. At this point, it is necessary to consider how these state variables are easily obtainable. However, the added cost of instrumenting both the links and actuators sensors may be high. Therefore, the control design which requires either the link variables or the actuator variables is more desirable. These have motivated the design of *Observers* to reduce the number of sensors needed in implementation. Several algorithms for observer have been proposed in many researchers [5-9]. These algorithms are solved in an approximate way and based on the system with known and constant parameters. Robustness properties of the above observers are not quite analyzed. In this paper, an observer is proposed for flexible joint manipulators, which handles uncertain and constant (time-varying) parameters. The designed observer adopts robust control algorithm based on Lyapunov approach [10-11]. This paper is organized as follows. In section II we introduce uncertain system and practical stability. In section III the flexible joint manipulators model is presented. In section IV, the procedure of observer design is shown. For the system with time-varying uncertainty another observer design is presented in section V. Finally, the simulation results are illustrated in section VI and this report concludes in section VII.

II. Uncertain system and practical stability

We consider the following class of uncertain dynamical systems

$$\dot{\xi}(t)=f(\xi(t),\sigma(t),t) \tag{1}$$

where $t\in R$ is the time, $\xi(t)\in R^n$ is the state, $\sigma(t)\in R^o$ is the *uncertainty*, and $f(\xi(t),\sigma(t),t)$ is the system vector. From now on, unless otherwise stated, the norms in this paper are Euclidean.

Definition [11,12]. The uncertain dynamical system (1) is *practically stable* iff there exists constant $\underline{d}_\xi>0$ such that for any initial time $t_0\in R$ and any initial state, $\xi_0\in R^n$, the following properties hold.

(i) *Existence and continuation of solutions*: Given $(\xi_0,t_0)\in R^n\times R$, system (1) possesses a solution $\xi(\cdot)$: $[t_0,t_1)\to R^n$, $\xi(t_0)=\xi_0$, $t_1>t_0$. Furthermore, every solution $\xi(\cdot):[t_0,t_1)\to R^n$ can be continued over $[t_0,\infty)$.

(ii) *Uniform boundedness*: Given any constant $r_\xi>0$ and any solution $\xi(\cdot):[t_0,\infty)\to R^n$, $\xi(t_0)=\xi_0$ of (1) with $\|\xi_0\|\le r_\xi$ there exists $d_\xi(r_\xi)>0$ such that $\|\xi(t)\|\le d_\xi(r_\xi)$ for *all* $t\in[t_0,\infty)$.

(iii) *Uniform ultimate boundedness*: Given any constant $\bar{d}_\xi>\underline{d}_\xi$ and any $r_\xi\in[0,\infty)$, there exists a finite time $T_\xi(\bar{d}_\xi,r_\xi)$ such that $\|\xi_0\|\le r_\xi$ implies $\|\xi(t)\|\le\bar{d}_\xi$ for all $t\ge t_0+T_\xi(\bar{d}_\xi,r_\xi)$.

(iv) *Uniform stability*: Given any $\bar{d}_\xi>\underline{d}_\xi$, there exists a $\delta_\xi(\bar{d}_\xi)>0$ such that $\|\xi_0\|\le\delta_\xi(\bar{d}_\xi)$ implies $\|\xi(t)\|\le\bar{d}_\xi$ for all $t\ge t_0$.

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III. Flexible joint manipulators

Consider an n serial link mechanical manipulator. The links are assumed rigid. The joints are however flexible. All joints are revolute or prismatic and are directly actuated by DC-electric motors. For the flexible joint robot define vectors $q_1=[q^2\ q^4 \cdots q^{2n-2}\ q^{2n}]^T$ and $q_2=[q^1\ q^3 \cdots q^{2n-3}\ q^{2n-1}]^T$, where $q^2, q^4 \ldots$ are link angles and $q^1, q^3 \ldots$ are joint angles. We model the joint flexibility by a linear torsional spring at each joint. We assume that the rotors are modeled as uniform cylinders so that the gravitational potential energy of the system is independent of the rotor position and is therefore a function only of link position. The flexible joint manipulator can be expressed in terms of the generalized coordinates [2]:

$$D(q_1)\ddot{q}_1+C(q_1,\dot{q}_1)\dot{q}_1+G(q_1)=K(q_2-q_1), \tag{2}$$

$$J\ddot{q}_2+K(q_2-q_1)=u, \tag{3}$$

where $D(q_1)$ represents the inertia matrix of links, $C(q_1,\dot{q}_1)\dot{q}_1$ represents the Coriolis and centrifugal force, $G(q_1)$ represents the gravitational force, and u denotes the input force from the actuators. K is a constant diagonal matrix representing the torsional stiffness between links and joints (hence K^{-1} exists). J is the inertia matrix of actuators.

IV. Robust observer design

We first consider the system with constant uncertainty. Flexible joint manipulators system (2),(3) is considered. Let $X_1=q_1$, $X_2=\dot{q}_1$, $X_3=q_2$ and $X_4=\dot{q}_2$ also $x_1=[X_1^T\ X_2^T]^T$, $x_2=[X_3^T\ X_4^T]^T$, and $x=[x_1^T\ x_2^T]^T$. Then we construct the state equations as

$$\dot{x}_1(t)=f_1(x_1(t),\sigma_1)+B_1(x_1(t),\sigma_1)x_2(t), \tag{4}$$

$$\dot{x}_2(t)=f_2(x(t),\sigma_2)+B_2(\sigma_2)u(t), \tag{5}$$

where

$$f_1(x_1,\sigma_1)=\begin{bmatrix}\dot{q}_1\\ f_{11}(x_1,\sigma_1)\end{bmatrix}, \tag{6}$$

$$\begin{aligned} &f_{11}(x_1,\sigma_1)\\ &=-D^{-1}(q_1,\sigma_1)C(q_1,\dot{q}_1,\sigma_1)\dot{q}_1\\ &\quad -D^{-1}(q_1,\sigma_1)G(q_1,\sigma_1)-D^{-1}(q_1,\sigma_1)K(\sigma_1)q_1 \end{aligned} \tag{7}$$

$$f_2(x,\sigma_2)=\begin{bmatrix}\dot{q}_2\\ -J^{-1}(\sigma_2)K(\sigma_2)q_2+J^{-1}(\sigma_2)K(\sigma_2)q_1\end{bmatrix}, \tag{8}$$

$$B_1(x_1,\sigma_1)=\begin{bmatrix}0 & 0\\ D^{-1}(q_1,\sigma_1)K(\sigma_1) & 0\end{bmatrix},$$

$$B_2(\sigma_2)=\begin{bmatrix}0\\ J^{-1}(\sigma_2)\end{bmatrix} \tag{9}$$

Here $\sigma_1\in R^{o_1}$ and $\sigma_2\in R^{o_2}$ are uncertain parameter vectors in (4) and (5) respectively.

Assumption 1 : The parameter vectors are such that $\sigma_1\in\Sigma_1\subset R^{o_1}$ and $\sigma_2\in\Sigma_2\subset R^{o_2}$ where Σ_1,Σ_2 are prescribed and compact.

With the dynamics for the flexible joint manipulators systems (4),(5) we omit the arguments if a confusion does not arise. We assume that link angles q_1 and link angular velocities $\dot{q}_1$ are measurable. To design observer define

$$e_1:=q_1-\hat{q}_1,\quad \dot{e}_1=\dot{q}_1-\hat{\dot{q}}_1, \tag{10}$$

$$e_2:=q_2-\hat{q}_2,\quad \dot{e}_2=\dot{q}_2-\hat{\dot{q}}_2. \tag{11}$$

Here, $\hat{q}_1$ and $\hat{\dot{q}}_1$ denote the estimated link angles and link angular velocities respectively. $\hat{q}_2$ and $\hat{\dot{q}}_2$ denote the estimated joint angles and joint angular velocities. We try to design an observer such that $e_1,\dot{e}_1,e_2$ and $\dot{e}_2$ converge to some reasonable value or zero possibly. Let $w_1=[q_1^T\ \hat{q}_1^T\ \dot{q}_1^T\ \hat{\dot{q}}_1^T]^T$. For given $S_1=diag[s_{1i}]_{n\times n}, s_{1i}>0$, we have functions $h_1(\cdot), h_2(\cdot)$ as follows:

$$\begin{aligned} &h_1(w_1,e_1,\dot{e}_1,\hat{q}_2,\sigma_1)\\ &:=\tilde{D}(q_1,\sigma_1)\overline{D}^{-1}(q_1)(\overline{C}(q_1,\dot{q}_1)\hat{\dot{q}}_1+\overline{G}(q_1))\\ &+\overline{D}^{-1}(q_1)(\overline{K}(\hat{q}_1-\hat{q}_2)-K_{p1}e_1-K_{v1}\dot{e}_1)\\ &-\tilde{C}(q_1,\dot{q}_1,\sigma_1)\hat{\dot{q}}_1+\overline{G}(q_1)-\overline{K}(\hat{q}_1-\hat{q}_2)\\ &-\tilde{D}(q_1,\sigma_1)\overline{D}^{-1}(q_1)\beta_1(\dot{e}_1+S_1e_1), \end{aligned} \tag{12}$$

$$\begin{aligned} &h_2(q_1,\dot{q}_1,e_1,\dot{e}_1,\sigma_1)\\ &:=-G(q_1,\sigma_1)-K(\sigma_1)e_1+D(q_1,\sigma_1)S_1\dot{e}_1\\ &\quad +C(q_1,\dot{q}_1,\sigma_1)S_1e_1, \end{aligned} \tag{13}$$

where

$$\begin{aligned} &\tilde{D}(q_1,\sigma_1)=D(q_1,\sigma_1)-\overline{D}(q_1),\\ &\tilde{C}(q_1,\dot{q}_1,\sigma_1)=C(q_1,\dot{q}_1,\sigma_1)-\overline{C}(q_1,\dot{q}_1),\\ &\tilde{G}(q_1,\sigma_1)=G(q_1,\sigma_1)-\overline{G}(q_1),\\ &K_{p1}:=diag[k_{p1i}]_{n\times n},\ k_{p1i}>0,\\ &K_{v1}:=diag[k_{v1i}]_{n\times n},\ k_{v1i}>0. \end{aligned} \tag{14}$$

Here, the "overbar" over parameter represents the nominal (*i.e.*, known) portion and β_1 is constant whose proper value is shown later. We choose a function $\rho_1(\cdot): R^{4n}\times R^n\times R^n\times R^n\to R_+$ such that for all $\sigma_1\in\Sigma_1$,

$$\begin{aligned} &\|h_1(w_1,e_1,\dot{e}_1,\hat{q}_2,\sigma_1)+h_2(q_1,\dot{q}_1,e_1,\dot{e}_1,\sigma_1)\|\\ &\le\rho_1(w_1,e_1,\dot{e}_1,\hat{q}_2). \end{aligned} \tag{15}$$

Assumption 2 : There exists a $\lambda_E(q_1)$ for all $q_1\in R^n$ such that

$$\max_{\sigma_1\in\Sigma_1}\|I-D(q_1,\sigma_1)\overline{D}^{-1}(q_1)\|=:\lambda_E(q_1)<1. \tag{16}$$

This assumption implies that the nominal value $\overline{D}$ is not far from $D(q_1,\sigma_1)$. In special case that $\overline{D}=D$ then $\lambda_E(q_1)=0$ and that assumption holds. Practically, this assumption can be checked since the inertia matrix

$D(q_1,\sigma_1)$ is uniformly positive definite for all q_1 there exist positive $\underline{\sigma}^{-1}$ and $\overline{\sigma}^{-1}$ stemming from Assumption 4, which will be shown later. If we choose $\overline{D}^{-1}=\frac{1}{c}I$, where $c=\frac{1}{2}(\underline{\sigma}^{-1}+\overline{\sigma}^{-1})$. It can be seen that $\lambda_E(q_1)\le\frac{\underline{\sigma}^{-1}-\overline{\sigma}^{-1}}{\underline{\sigma}^{-1}+\overline{\sigma}^{-1}}<1$. Since we can choose $\overline{D}^{-1}$ satisfying Assumption 2 we can see that the assumption is reasonable.

Let the function $\rho_1:R^{4n}\times R^n\times R^n\times R^n\to R_+$ be chosen such that

$$\begin{aligned}&\rho_1(w_1,e_1,\dot{e}_1,\hat{q}_2)\\&\ge(1-\lambda_E(q_1))^{-1}\overline{\rho}_1(w_1,e_1,\dot{e}_1,\hat{q}_2).\end{aligned}\tag{17}$$

Let $z_1=[e_1^T\ \dot{e}_1^T]^T$ and $z_2=[e_2^T\ \dot{e}_2^T]^T$. Now we are ready to design a robust observer. For given $\varepsilon_1>0$, we propose a robust observer as follows:

$$\overline{D}(q_1)\ \ddot{\hat{q}}_1+\overline{C}(q_1,\dot{q}_1)\ \dot{\hat{q}}_1+\overline{G}(q_1)+\overline{K}(\hat{q}_1-\hat{q}_2)=-v_1,\tag{18}$$

$$\overline{J}\ \ddot{\hat{q}}_2+\overline{K}(\hat{q}_2-\hat{q}_1)=u,\tag{19}$$

where

$$v_1=-K_{p1}e_1-K_{v1}\dot{e}_1-\beta_1(\dot{e}_1+S_1e_1)+p_1,\tag{20}$$

$$p_1(w_1,z_1,\hat{q}_2)=\begin{cases}-\dfrac{\mu_1(w_1,z_1,\hat{q}_2)}{\|\mu_1(w_1,z_1,\hat{q}_2)\|}\rho_1(w_1,z_1,\hat{q}_2) & \text{if}\ \|\mu_1(w_1,z_1,\hat{q}_2)\|>\varepsilon_1,\\ -\dfrac{\mu_1(w_1,z_1,\hat{q}_2)}{\varepsilon_1}\rho_1(w_1,z_1,\hat{q}_2) & \text{if}\ \|\mu_1(w_1,z_1,\hat{q}_2)\|\le\varepsilon_1,\end{cases}\tag{21}$$

$$\mu_1(w_1,z_1,\hat{q}_2)=(\dot{e}_1+S_1e_1)\rho_1(w_1,z_1,\hat{q}_2).\tag{22}$$

We see that the proposed observer relies on the information q_1 and $\dot{q}_1$ and those estimated states $\hat{q}_1$, $\dot{\hat{q}}_1$ and $\hat{q}_2$. It does not use any joint information on q_2 and $\dot{q}_2$. And, the selection of β_1 is shown later.

From the proposed observer we have error dynamics for links and joints. First, subtracting (18) from (2) we have:

$$D\ddot{e}_1+C\dot{e}_1+G+K(e_1-e_2)=v_1+g_1,\tag{23}$$

where

$$\begin{aligned}&g_1(w_1,e_1,\dot{e}_1,\hat{q}_2,\sigma_1)\\&=-\tilde{D}(q_1,\sigma_1)\ \ddot{\hat{q}}_1-\tilde{C}(q_1,\dot{q}_1,\sigma_1)\ \dot{\hat{q}}_1\\&\quad+\tilde{G}(q_1)-\tilde{K}(\sigma_1)(\hat{q}_1-\hat{q}_2).\end{aligned}\tag{24}$$

From (18) we get

$$\begin{aligned}g_1=&-\tilde{D}(-\overline{D}^{-1}\overline{C}\ \dot{\hat{q}}_1-\overline{D}^{-1}\overline{G})\\&-\tilde{D}(-\overline{D}^{-1}\overline{K}(\hat{q}_1-\hat{q}_2)-\overline{D}^{-1}v_1)\\&-\tilde{C}\ \dot{\hat{q}}_1+\overline{G}-\tilde{K}(\hat{q}_1-\hat{q}_2).\end{aligned}\tag{25}$$

Substituting v_1 in (20) and $h_1(\cdot)$ in (12) it can be seen that

$$\begin{aligned}&g_1\\&=-\tilde{D}(-\overline{D}^{-1}\overline{C}\ \dot{\hat{q}}_1-\overline{D}^{-1}\overline{G}-\overline{D}^{-1}\overline{K}(\hat{q}_1-\hat{q}_2))\\&+\tilde{D}\overline{D}^{-1}(-K_{p1}e_1-K_{v1}\dot{e}_1-\beta_1(\dot{e}_1+S_1e_1)+p_1)\\&-\tilde{C}\ \dot{\hat{q}}_1+\overline{G}-\tilde{K}(\hat{q}_1-\hat{q}_2)\\&=h_1+\tilde{D}\overline{D}^{-1}p_1\\&=h_1+(D\overline{D}^{-1}-I)p_1,\end{aligned}\tag{26}$$

where I denotes the unity matrix with dimension $n\times n$. Therefore, error dynamics for links follows from (26)

$$\begin{aligned}&D\ddot{e}_1+C\dot{e}_1+G+K(e_1-e_2)\\&=v_1+h_1+(D\overline{D}^{-1}-I)p_1.\end{aligned}\tag{27}$$

Next, subtracting (19) from (2) and substituting (19) we get

$$\begin{aligned}&J\ddot{e}_2+Ke_2\\&=Ke_1-\tilde{J}\ \ddot{\hat{q}}_2-\tilde{K}(\hat{q}_2-\hat{q}_1)\\&=Ke_1-\tilde{J}(-\overline{J}^{-1}\overline{K}(\hat{q}_2-\hat{q}_1)+\overline{J}^{-1}u)\\&\quad-\tilde{K}(\hat{q}_2-\hat{q}_1)\\&=:h_3(e_1,\hat{q}_1,\hat{q}_2,u,\sigma_2).\end{aligned}\tag{28}$$

Based on states z_2 we can show (28) as follows:

$$\begin{aligned}\dot{z}_2&=\begin{bmatrix}0&I\\-J^{-1}K&0\end{bmatrix}z_2+\begin{bmatrix}0\\J^{-1}\end{bmatrix}h_3\\&=\begin{bmatrix}0&I\\-L_1&-L_2\end{bmatrix}z_2\\&\quad+\begin{bmatrix}0&0\\L_1-J^{-1}K&L_2\end{bmatrix}z_2+\begin{bmatrix}0\\J^{-1}\end{bmatrix}h_3\\&=A_2z_2+M_2(\sigma_2)z_2\\&\quad+B_2(\sigma_2)h_3(e_1,\hat{q}_2,u,\sigma_2),\end{aligned}\tag{29}$$

where

$$\begin{aligned}&A_2=\begin{bmatrix}0&I\\-L_1&-L_2\end{bmatrix},\\&M_2=\begin{bmatrix}0&0\\L_1-J^{-1}K&L_2\end{bmatrix},B_2=\begin{bmatrix}0\\J^{-1}\end{bmatrix},\\&L_1,L_2\in R^{n\times n}>0.\end{aligned}\tag{30}$$

Choose P_2 such that it is the solution of

$$A_2^TP_2+P_2A_2=-Q_2,\quad Q_2>0.\tag{31}$$

Now, we are ready to select β_1 in v_1 shown as (20). Let

$$\begin{aligned}&d_1:=\|P_2M_2\|,d_2:=\|P_2B_2\|,\\&d_3:=\|\tilde{J}\overline{J}^{-1}\overline{K}\|+\|\tilde{K}\|,\\&d_4:=\|\tilde{J}\|\ \|\overline{J}^{-1}\|,\\&l_1:=\|K\|+d_3,l_2:=d_3.\end{aligned}\tag{32}$$

Selection of β_1 is chosen as follows:

Define

$$\underline{\lambda}_1:=\min\{\lambda_{\min}(K_{p1}),\lambda_{\min}(S_1K_{p1})\}\tag{33}$$

where $\lambda_{\min}(\cdot)$ represents the minimum eigenvalue of the designated matrix.

Step 1: Select τ_2 to satisfy $\underline{\lambda}_1-\tau_2 d_2 l_1>0$, for $\tau_2>0$.

Step 2: Select τ_1 to satisfy

$$\lambda_{\min}(Q_2)-2d_1-d_2 l_1\tau_2^{-1} -2d_2 l_2-\frac{1}{2}\tau_1^{-1}\|K\|>0, \text{ for } \tau_1>0.$$

Step 3: From appropriate value for τ_1 based on Step 1 and Step 2 choose β_1 such that $\beta_1-\frac{1}{2}\tau_1\|K\|>0$.

Assumption 3 : q_1, q_2 and control input u are bounded by constants c_1, c_2 and c_3:

$$\|q_1(t)\| \le \sup_{t\in[0,\infty)}\|q_1(t)\|=:c_1<\infty,$$

$$\|q_2(t)\| \le \sup_{t\in[0,\infty)}\|q_2(t)\|=:c_2<\infty, \tag{34}$$

$$\|u(t)\| \le \sup_{t\in[0,\infty)}\|u(t)\|=:c_3<\infty.$$

Assumption 4 : The inertia matrix $D(q_1)$ is uniformly positive definite and uniformly bounded from above and below; that is, there exist positive scalar constants $\underline{\sigma}$ and $\bar{\sigma}$ such that

$$\underline{\sigma} I \le D(q_1) \le \bar{\sigma} I \quad \forall q_1\in R^n. \tag{35}$$

Theorem 1 : Subject to Assumptions 1-4, the system (27) and (29) is practically stable under v_1 in (20).

Proof.

Choose Lyapunov function candidates as follows:

$$V(z_1, z_2)=V_1(z_1)+V_2(z_2), \tag{36}$$

where

$$V_1(z_1)=\frac{1}{2}(\dot e_1+S_1 e_1)^T D(\dot e_1+S_1 e_1) +\frac{1}{2}e_1^T(K_{p1}+S_1 K_{v1})e_1, \tag{37}$$

$$V_2(z_2)=z_2^T P_2 z_2, \tag{38}$$

We see that both V_1 and V_2 are legitimate Lyapunov function candidates. We shall prove that both V_1 and V_2 are positive definite and decrescent. Based on Assumption 4,

$$\begin{aligned} V_1 &\ge \frac{1}{2}\underline{\sigma}\|\dot e_1+S_1 e_1\|^2+\frac{1}{2}e_1^T(K_{p1}+S_1K_{v1})Z_1 \\ &= \frac{1}{2}\underline{\sigma}\sum_{i=1}^{n}(\dot e_{1i}^2+2s_{1i}\dot e_{1i}e_{1i}+s_{1i}^2 e_{1i}^2) \\ &+ \frac{1}{2}\sum_{i=1}^{n}(k_{p1i}+s_{1i}k_{v1i})e_{1i}^2 \\ &= \frac{1}{2}\sum_{i=1}^{n}[e_{1i}\ \dot e_{1i}]\ \underline{\Omega}_{1i}\begin{bmatrix} e_{1i}\\ \dot e_{1i}\end{bmatrix}, \end{aligned} \tag{39}$$

where

$$\underline{\Omega}_{1i}:=\begin{bmatrix}\underline{\sigma}s_{1i}^2+k_{p1i}+s_{1i}k_{v1i} & \underline{\sigma}s_{1i}\\ \underline{\sigma}s_{1i} & \underline{\sigma}\end{bmatrix}, \tag{40}$$

where e_{1i} and $\dot e_{1i}$ are the i-th components of e_1 and $\dot e_1$, respectively. Since $\underline{\Omega}_{1i}>0\ \forall i$, V_1 is positive definite:

$$\begin{aligned} V_1 &\ge \frac{1}{2}\sum_{i=1}^{n}\lambda_{\min}(\underline{\Omega}_{1i})(e_{1i}^2+\dot e_{1i}^2) \\ &\ge \gamma_1^1\|z_1\|^2, \end{aligned} \tag{41}$$

where

$$\gamma_1^1:=\frac{1}{2}\min_i\left\{\min_{\sigma_1\in\Sigma_1}\lambda_{\min}(\underline{\Omega}_{1i}), i=1,2,\cdots,n.\right\} \tag{42}$$

Next, with respect to the bounded from the above condition:

$$\begin{aligned} V_1 &\le \frac{1}{2}\bar{\sigma}\|\dot e_1+S_1 e_1\|^2+\frac{1}{2}e_1^T(K_{p1}+S_1K_{v1})e_1 \\ &= \frac{1}{2}\bar{\sigma}\sum_{i=1}^{n}(\dot e_{1i}^2+2s_{1i}\dot e_{1i}e_{1i}+s_{1i}^2e_{1i}^2) \\ &+\frac{1}{2}\sum_{i=1}^{n}(k_{p1i}+s_{1i}k_{v1i})e_{1i}^2 \\ &= \frac{1}{2}\sum_{i=1}^{n}[e_{1i}\ \dot e_{1i}]\ \bar{\Omega}_{1i}\begin{bmatrix} e_{1i}\\ \dot e_{1i}\end{bmatrix}, \end{aligned} \tag{43}$$

where

$$\bar{\Omega}_{1i}:=\begin{bmatrix}\bar{\sigma}s_{1i}^2+k_{p1i}+s_{1i}k_{v1i} & \bar{\sigma}s_{1i}\\ \bar{\sigma}s_{1i} & \bar{\sigma}\end{bmatrix}. \tag{44}$$

Therefore, we have

$$\begin{aligned} V_1 &\le \frac{1}{2}\sum_{i=1}^{n}\lambda_{\max}(\bar{\Omega}_{1i})(e_{1i}^2+\dot e_{1i}^2) \\ &\le \gamma_2^1\|z_1\|^2, \end{aligned} \tag{45}$$

where

$$\gamma_2^1:=\frac{1}{2}\max_i\left\{\max_{\sigma_1\in\Sigma_1}\lambda_{\max}(\bar{\Omega}_{1i}), i=1,2,\cdots,n.\right\} \tag{46}$$

V_2 is also positive definite and decrescent. This is since

$$\lambda_{\min}(P_2)\|z_2\|^2\le z_2^T P_2 z_2\le\lambda_{\max}(P_2)\|z_2\|^2. \tag{47}$$

The derivative of V_1 along the trajectory of system (27) is given by

$$\begin{aligned} \dot V_1 &= (\dot e_1+S_1e_1)^T D(\ddot e_1+S_1\dot e_1) \\ &+\frac{1}{2}(\dot e_1+S_1e_1)^T\dot D(\dot e_1+S_1e_1) \\ &+e_1^T(K_{p1}+S_1K_{v1})\dot e_1. \end{aligned} \tag{48}$$

From the skew-symmetric property in $\dot D-2C$ it can be seen that

$$\begin{aligned} \dot V_1 &= (\dot e_1+S_1e_1)^T(-G-K(e_1-e_2) \\ &+DS_1\dot e_1+CS_1e_1+g_1+v_1) \\ &+e_1^T(K_{p1}+S_1K_{v1})\dot e_1. \end{aligned} \tag{49}$$

According to (13), it can be seen that

$$\begin{aligned} \dot V_1 &= (\dot e_1+S_1e_1)^T(h_2+g_1+v_1) \\ &+(\dot e_1+S_1e_1)^T K e_2 \\ &+e_1^T(K_{p1}+S_1K_{v1})\dot e_1. \end{aligned} \tag{50}$$

It follows from (20) and (26)

$$\begin{aligned} \dot V_1 &= (\dot e_1+S_1e_1)^T(h_2+h_1+(D\bar{D}^{-1}-I)p_1) \\ &+(\dot e_1+S_1e_1)^T(-K_{p1}e_1-K_{v1}\dot e_1) \\ &-(\dot e_1+S_1e_1)^T(\beta_1(\dot e_1+S_1e_1)+p_1) \\ &+e_1^T(K_{p1}+S_1K_{v1})\dot e_1+(\dot e_1+S_1e_1)^T K e_2. \end{aligned} \tag{51}$$

From (16) and (33) it can be seen that

$$\begin{aligned}\dot V_1 \le& \|\dot e_1+S_1e_1\|\ \|h_1+h_2\| \\ &+\|\dot e_1+S_1e_1\|\ \|D\bar D^{-1}-I\|\ \|p_1\| \\ &-\beta_1\|\dot e_1+S_1e_1\|^2-\underline{\lambda}_1\|z_1\|^2 \\ &+\|\dot e_1+S_1e_1\|\ \|K\|\ \|e_2\|+(\dot e_1+S_1e_1)p_1 \\ \le& \|\dot e_1+S_1e_1\|\ \bar\rho_1+\|\dot e_1+S_1e_1\|\lambda_E\rho_1 \\ &+(\dot e_1+S_1e_1)p_1-\underline{\lambda}_1\|z_1\|^2-\beta_1\|\dot e_1+S_1e_1\|^2 \\ &+\|\dot e_1+S_1e_1\|\ \|K\|\ \|e_2\|.\end{aligned} \tag{52}$$

If $\|\mu_1\|>\varepsilon_1$ then the first three terms in (52) become

$$\begin{aligned}&\|\dot e_1+S_1e_1\|\ \bar\rho_1+\|\dot e_1+S_1e_1\|\lambda_E\rho_1+(\dot e_1+S_1e_1)p_1 \\ &\le\|\dot e_1+S_1e_1\|(1-\lambda_E)\rho_1+\|\dot e_1+S_1e_1\|\lambda_E\rho_1 \\ &\quad-\|\dot e_1+S_1e_1\|\rho_1 \\ &=0.\end{aligned} \tag{53}$$

If $\|\mu_1\|\le\varepsilon_1$ then it becomes

$$\begin{aligned}&\|\dot e_1+S_1e_1\|\ \bar\rho_1+\|\dot e_1+S_1e_1\|\lambda_E\rho_1+(\dot e_1+S_1e_1)p_1 \\ &\le\|\dot e_1+S_1e_1\|(1-\lambda_E)\rho_1+\|\dot e_1+S_1e_1\|\lambda_E\rho_1 \\ &\quad-\|\dot e_1+S_1e_1\|^2\frac{\rho_1^2}{\varepsilon_1} \\ &\le\frac{\varepsilon_1}{4}.\end{aligned} \tag{54}$$

Therefore, $\dot V_1$ is bounded:

$$\begin{aligned}\dot V_1 \le& -\underline{\lambda}_1\|z_1\|^2-\beta_1\|\dot e_1+S_1e_1\|^2 \\ &+\|\dot e_1+S_1e_1\|\ \|K\|\ \|e_2\|+\frac{\varepsilon_1}{4}.\end{aligned} \tag{55}$$

Based on inequalities $ab\le\frac{1}{2}(a^2+b^2)$, $a,b\in R$, and $\|e_2\|^2\le\|z_2\|^2$, for any constant $\tau_1>0$, $\dot V_1$ can be seen that

$$\begin{aligned}\dot V_1 \le& -\underline{\lambda}_1\|z_1\|^2-\beta_1\|\dot e_1+S_1e_1\|^2 \\ &+\frac{1}{2}\tau_1\|\dot e_1+S_1e_1\|^2\ \|K\| \\ &+\frac{1}{2}\tau_1^{-1}\|K\|\ \|e_2\|^2+\frac{\varepsilon_1}{4} \\ \le& -\underline{\lambda}_1\|z_1\|^2-(\beta_1-\frac{1}{2}\tau_1\|K\|)\|\dot e_1+S_1e_1\|^2 \\ &+\frac{1}{2}\tau_1^{-1}\|K\|\ \|z_2\|^2+\frac{\varepsilon_1}{4}.\end{aligned} \tag{56}$$

Next, the derivative of V_2 along the trajectory of (29) follows from (32)

$$\begin{aligned}\dot V_2 =& 2z_2^TP_2\dot z_2 \\ =& 2z_2^TP_2(A_2z_2+M_2z_2+B_2h_3) \\ =& -z_2^TQ_2z_2+2z_2^TP_2M_2z_2+2z_2^TP_2B_2h_3 \\ \le& -\lambda_{\min}(Q_2)\|z_2\|^2+2\|z_2\|^2\|P_2M_2\| \\ &+2\|z_2\|\ \|P_2B_2\|\ \|h_3\| \\ =& -\lambda_{\min}(Q_2)\|z_2\|^2 \\ &+2d_1\|z_2\|^2+2d_2\|z_2\|\ \|h_3\|.\end{aligned} \tag{57}$$

Here, from (28) we have the following bounding condition from Assumption 3 and (32)

$$\begin{aligned}&\|h_3\| \\ &\le\|K\|\ \|e_1\|+\|\hat J\bar J^{-1}\bar K\|\ \|\hat q_2-\hat q_1\| \\ &\quad+\|\hat J\bar J^{-1}\|\ \|u\|+\|\tilde K\|\ \|\hat q_2-\hat q_1\| \\ &=\|K\|\ \|e_1\|+d_3\|\hat q_2-\hat q_1\|+d_4\|u\| \\ &\le\|K\|\ \|e_1\|+d_3(\|e_2\|+\|e_1\|+\|q_2\|+\|q_1\|) \\ &\quad+d_4\|u\| \\ &\le\|K\|\ \|e_1\|+d_3(\|e_1\|+\|e_2\|+c_2+c_1)+d_4c_3 \\ &=(\|K\|+d_3)\|e_1\|+d_3\|e_2\|+d_3(c_1+c_2)+d_4c_3 \\ &=l_1\|e_1\|+l_2\|e_2\|+l_3,\end{aligned} \tag{58}$$

where $l_3=d_3(c_1+c_2)+d_4c_3$. Thus, $\dot V_2$ follows from (58)

$$\begin{aligned}\dot V_2 \le& -\lambda_{\min}(Q_2)\|z_2\|^2+2d_1\|z_2\|^2 \\ &+2d_2\|z_2\|(l_1\|e_1\|+l_2\|e_2\|+l_3) \\ \le& -(\lambda_{\min}(Q_2)-2d_1)\|z_2\|^2+2d_2l_1\|z_2\|\ \|z_1\| \\ &+2d_2l_2\|z_2\|^2+2d_2l_3\|z_2\|.\end{aligned} \tag{59}$$

By using the inequality for the second term in (59) with $\tau_2>0$ we have

$$\begin{aligned}\dot V_2 \le& -(\lambda_{\min}(Q_2)-2d_1)\|z_2\|^2 \\ &+d_2l_1(\tau_2^{-1}\|z_2\|^2+\tau_2\|z_1\|^2) \\ &+2d_2l_2\|z_2\|^2+2d_2l_3\|z_2\| \\ =& -(\lambda_{\min}(Q_2)-2d_1-d_2l_1\tau_2^{-1}-2d_2l_2)\|z_2\|^2 \\ &+\tau_2d_2l_1\|z_1\|^2+2d_2l_3\|z_2\|.\end{aligned} \tag{60}$$

Now, using (56) and (60),

$$\begin{aligned}\dot V =& \dot V_1+\dot V_2 \\ \le& -\eta_1\|z_1\|^2-\eta_2\|z_2\|^2 \\ &-(\beta_1-\frac{1}{2}\tau_1\|K\|)\|\dot e_1+S_1e_1\|^2 \\ &+2d_2l_3\|z_2\|+\frac{\varepsilon_1}{4},\end{aligned} \tag{61}$$

where

$$\begin{aligned}\eta_1 &:=\underline{\lambda}_1-\tau_2d_2l_1, \\ \eta_2 &:=\lambda_{\min}(Q_2)-2d_1-d_2l_1\tau_2^{-1}-2d_2l_2 \\ &\quad-\frac{1}{2}\tau_1^{-1}\|K\|.\end{aligned} \tag{62}$$

If we choose τ_1,τ_2 and β_1 such that $\eta_1>0$, $\eta_2>0$, and $\beta_1-\frac{1}{2}\tau_1\|K\|>0$ then we have

$$\begin{aligned}\dot V \le& -\min\{\eta_1,\eta_2\}\|z\|^2+2d_2l_3\|z_2\|+\frac{\varepsilon_1}{4} \\ \le& -\min\{\eta_1,\eta_2\}\|z\|^2+2d_2l_3\|z\|+\frac{\varepsilon_1}{4}.\end{aligned} \tag{63}$$

Therefore, $\dot V<0$ for all $\|z\|>R_z$, where

$$R_z=\frac{d_2l_3+\sqrt{(d_2l_3)^2+\min\{\eta_1,\eta_2\}\frac{\varepsilon_1}{4}}}{\min\{\eta_1,\eta_2\}}. \tag{64}$$

Following (64) for $r_z\ge0$, if $\|z_0\|\le r_z$, we can satisfy the requirements of uniform boundedness, uniform ultimate boundedness and uniform stability by selecting [10]

$$d_z(r_z)=\begin{cases}R_z\sqrt{\dfrac{\gamma_2}{\gamma_1}} & \text{if } r_z\le R_z \\ r_z\sqrt{\dfrac{\gamma_2}{\gamma_1}} & \text{if } r_z>R_z,\end{cases} \tag{65}$$

$$T_z(\bar d_z,r_z)=\begin{cases}0 & \text{if } r_z\le\bar d_z\sqrt{\dfrac{\gamma_1}{\gamma_2}} \\ \dfrac{\gamma_2r_z^2-\gamma_1^2\gamma_2^{-1}\ \bar d_z^{\,2}}{\min\{\eta_1,\eta_2\}\ \bar R_z^{\,2}-\dfrac{\varepsilon_1}{4}} & \textit{otherwise},\end{cases} \tag{66}$$

$$\delta_z(\bar d_z)=R_z, \tag{67}$$

where

$$\gamma_1 = \min\{\gamma_1^1, \lambda_{\min}(P_2)\},\ \ \gamma_2 = \max\{\gamma_2^1, \lambda_{\max}(P_2)\},$$

$$\overline{R}_z = \overline{d}_z\sqrt{\frac{\gamma_1}{\gamma_2}}\ ,\ \ \overline{d} = \gamma_1^{-1}\cdot\gamma_2(R).\ \text{Q.E.D.}$$

Remark 1 : The proposed observer handles the flexible joint manipulator system with uncertainty. The error system in (27,29) satisfies practical stability. Thus, the estimation errors have uniform ultimate boundedness ball $\overline{d}_z$ after time T_z.

Remark 2 : For the proposed observer we need the information on the boundedness of q_1, q_2 and u. This constraint is less strong than the boundedness of $\dot{q}_1$ in [5] since angular velocity may have large value in some cases.

Remark 3 : The design parameters in observer design K_{p1}, K_{v1}, S_1, L_1 and L_2 decide uniform ultimate ball size which is expressed as R_z in (64). In details, K_{p1}, K_{v1}, and S_1 contribute to the magnitude of η_1 shown in (62) and determine the estimation time. Also L_1 and L_2 affect a constant d_2 in (32), which is also affecting ultimate boundedness ball size. The bigger one choose those parameters the bigger the uniform ultimate ball size becomes. Thus, we need to consider a conservativeness in observer design and a careful selection of those parameters gives a nice estimation performance. Fortunately, we have a room to adjust the ball size by a suitable choice of ε_1.

V. System with time-varying uncertainty

We have considered the system with constant uncertainty in section IV. When some parameters are time-varying in case the system has time varying payload, inertia and stiffness in joints, we can not use the skew-symmetric property on $\dot{D}-2C$ in stability analysis [12]. Therefore, in this section we want to show another observer design procedure for the system with time-varying uncertainty by modifying the procedure shown in section IV.

Assumption 5 : The mappings $\sigma_1(\cdot): R\to\Sigma_1\subset R^{o_1}$, $\sigma_2(\cdot): R\to\Sigma_2\subset R^{o_2}$ are Lebesgue measurable with Σ_1, Σ_2 prescribed and compact. Furthermore, the mappings $\dot{\sigma}_1(\cdot): R\to\Sigma_{1t}\subset R^{o_1}$, $\dot{\sigma}_2(\cdot): R\to\Sigma_{2t}\subset R^{o_2}$ are Lebesgue measurable with Σ_{1t}, Σ_{2t} prescribed and compact.

The $h_2(\cdot)$ in (12) is changed to

$$\begin{aligned} & h_2(q_1, \dot{q}_1, e_1, \dot{e}_1, \sigma_1(t), \dot{\sigma}_1(t)) \\ & = -C(q_1, \dot{q}_1, \sigma_1(t))\,\dot{e}_1 - G(q_1, \sigma_1(t)) \\ & \quad -K(\sigma_1(t))e_1 + D(q_1, \sigma_1(t))S_1\dot{e}_1 \\ & \quad +\frac{1}{2}\dot{D}(q_1, \dot{q}_1, \sigma_1(t), \dot{\sigma}_1(t))(\dot{e}_1 + S_1 e_1). \end{aligned} \tag{68}$$

This in turn shows a different bounding function $\overline{\rho}_1(q_1, \dot{q}_1)$ in (15) as following.

$$\begin{aligned} & \|h_1(w_1, e_1, \dot{e}_1, \hat{q}_2, \sigma_1(t)) \\ & \ + h_2(q_1, \dot{q}_1, e_1, \dot{e}_1, \sigma_1(t), \dot{\sigma}_1(t))\| \\ & \le \overline{\rho}_1(w_1, e_1, \dot{e}_1, \hat{q}_2),\ \forall\sigma_1\in\Sigma_1,\ \forall\dot{\sigma}_1\in\Sigma_{1t}. \end{aligned} \tag{69}$$

Based on the different bounding function we use the same procedure derived in Section IV to obtain $v_1(\cdot)$. We can also see the difference in (49) for the system with time-varying uncertainty when proving Theorem 1. As there is no longer skew symmetric property in $\dot{D}-2C$, we need the modified $h_2(\cdot)$ as (68) to prove Theorem 1.

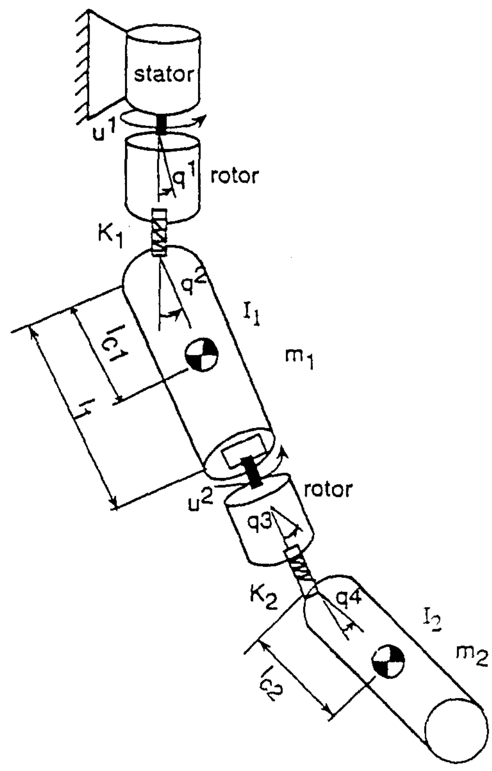

Fig. 1. 2-link flexible joint manipulator mechanism..

VI. Illustrative example

Consider a 2-link flexible revolute joint manipulator (Fig. 1). Let link angle vectors $q_1 = [q^2\ q^4\ ]^T$ and joint angle vectors. $q_2 = [q^1\ q^3\ ]^T$. First, we consider the system with constant uncertainty. Then we have $D(q_1)$, $C(q_1, \dot{q}_1)$, $G(q_1)$, J and K:

$$D(q_1) = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}, \tag{70}$$

$$C(q_1, \dot{q}_1) = \begin{bmatrix} -m_2 l_1 l_{c2}\sin q^4\dot{q}^4 & -m_2 l_1 l_{c2}\sin q^4(\dot{q}^4+\dot{q}^2) \\ m_2 l_1 l_{c2}\sin q^4\dot{q}^2 & 0 \end{bmatrix}, \tag{71}$$

$$G(q_1) = \begin{bmatrix} (m_1 l_{c1} + m_2 l_1)g\sin q^2 + m_2 l_{c2} g\sin(q^2+q^4) \\ m_2 l_{c2} g\sin(q^2+q^4) \end{bmatrix}, \tag{72}$$

$$J = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix},\ K = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix},\ S_1 = \begin{bmatrix} s_{11} & 0 \\ 0 & s_{21} \end{bmatrix}, \tag{73}$$

$$K_{p1} = \begin{bmatrix} k_{p11} & 0 \\ 0 & k_{p21} \end{bmatrix},\ K_{v1} = \begin{bmatrix} k_{v11} & 0 \\ 0 & k_{v21} \end{bmatrix}, \tag{74}$$

where

$$
\begin{aligned}
&d_{11} := 2a_{11}\cos(q^4) + a_{12},\\
&d_{12} := a_{11}\cos(q^4) + a_{22},\ d_{21} = d_{12},\\
&d_{22} := a_{22},\\
&a_{11} = m_2 l_1 l_{c2},\\
&a_{12} = m_2(l_1^2 + l_{c2}^2) + m_1 l_{c1}^2 + I_1 + I_2,\\
&a_{22} = m_2 l_{c2}^2 + I_2.
\end{aligned} \tag{75}
$$

Suppose that D, G, and C are known. Then we have

$$h_1(\cdot) = -C\ \hat{\dot{q}}_1 + G - \overline{K}(\ \hat{q}_1 - \ \hat{q}_2), \tag{76}$$

$$h_2(\cdot) = -G - K\ e_1 + D\ S_1\ \dot{e}_1 + C\ S_1\ e_1. \tag{77}$$

Thus, we obtain a bounding function $\overline{\rho}_1(\cdot)$ as follows:

$$\|h_1 + h_2\| \leq \overline{\rho}_1. \tag{78}$$

Since D is known we have $\lambda_E(q_1) = 0$. This gives a bounding function $\rho_1(\cdot) = \overline{\rho}_1(\cdot)$. We choose

$$L_1 = L_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ Q_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{79}$$

$$
\begin{aligned}
A_2 &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & -1 \end{bmatrix},\\
P_2 &= \begin{bmatrix} 1.5 & 0 & 0.5 & 0 \\ 0 & 1.5 & 0 & 0.5 \\ 0.5 & 0 & 1 & 0 \\ 0 & 0.5 & 0 & 1 \end{bmatrix}.
\end{aligned} \tag{80}
$$

Based on the above values we have the following values:

$$M_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 - \frac{K_1}{J_1} & 0 & 1 & 0 \\ 0 & 1 - \frac{K_2}{J_2} & 0 & 1 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{J_1} & 0 \\ 0 & \frac{1}{J_2} \end{bmatrix}. \tag{81}$$

We choose $\beta_1 = 5$ based on Step 1, 2 and 3 shown in section IV. Now, we design v_1 as followings:

$$
\begin{aligned}
v_1 &= -K_{p1}\ e_1 - K_{v1}\ \dot{e}_1\\
&\quad -\beta_1(\dot{e}_1 + S_1\ e_1) + p_1,
\end{aligned} \tag{82}
$$

where

$$p_1 = \begin{cases} -\dfrac{(\dot{e}_1 + S_1 e_1)}{\|\dot{e}_1 + S_1 e_1\|}\rho_1 & \text{if } \|\dot{e}_1 + S_1 e_1\|\rho_1 > \varepsilon_1, \\[2ex] -\dfrac{(\dot{e}_1 + S_1 e_1)}{\varepsilon_1}\rho_1^2 & \text{if } \|\dot{e}_1 + S_1 e_1\|\rho_1 \leq \varepsilon_1. \end{cases} \tag{83}$$

For simulations, we consider system performance with varying design parameters such as K_{p1}, K_{v1}, S_1 , L_1, L_2 and ε_1. We choose $m_1 = m_2 = 0.16\,kg$, $l_{c1} = l_{c2} = 0.25\,m$, $l_1 = 0.5\,m$, $I_1 = I_2 = 0.031\,kgm^2$, $K_1 = K_2 = 31.0\ Nm/rad$, $J_1 = J_2 = 0.004\ kgm^2$, $\overline{K_1} = \overline{K_2} = 15.5\ Nm/rad$, $\overline{J_1} = \overline{J_2} = 0.002\ kgm^2$.

We change the design parameters and investigate the estimator performance. Simulation results are shown in Figs. 2-4. Fig. 2 shows the history of estimated states by choosing $K_{p1} = K_{v1} = S_1 = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$ and $\varepsilon_1 = 100$. We see that the estimated states track the real states regarding link and joint. After 2 seconds all estimated states converge to the true states. Fig. 3 show the estimation

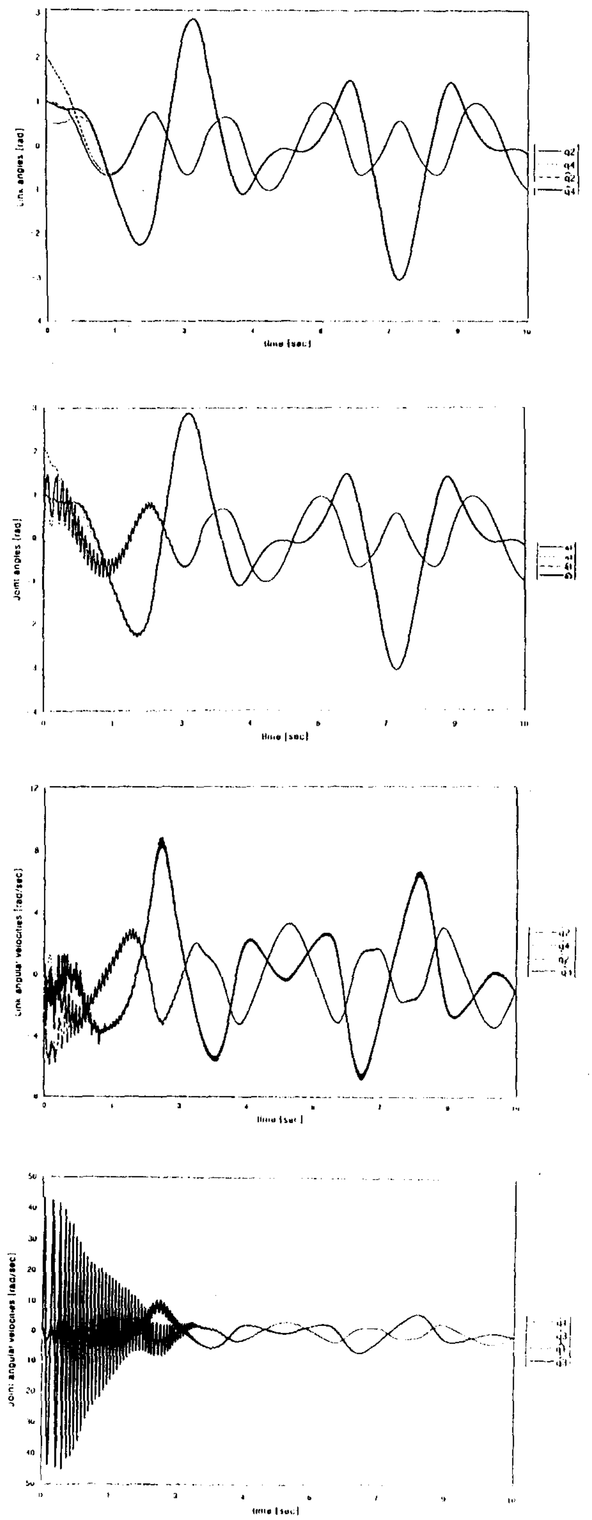

Fig. 2. History of estimation with. ($K_{p1} = K_{v1} = S_1 = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}$, $\varepsilon_1 = 100$)

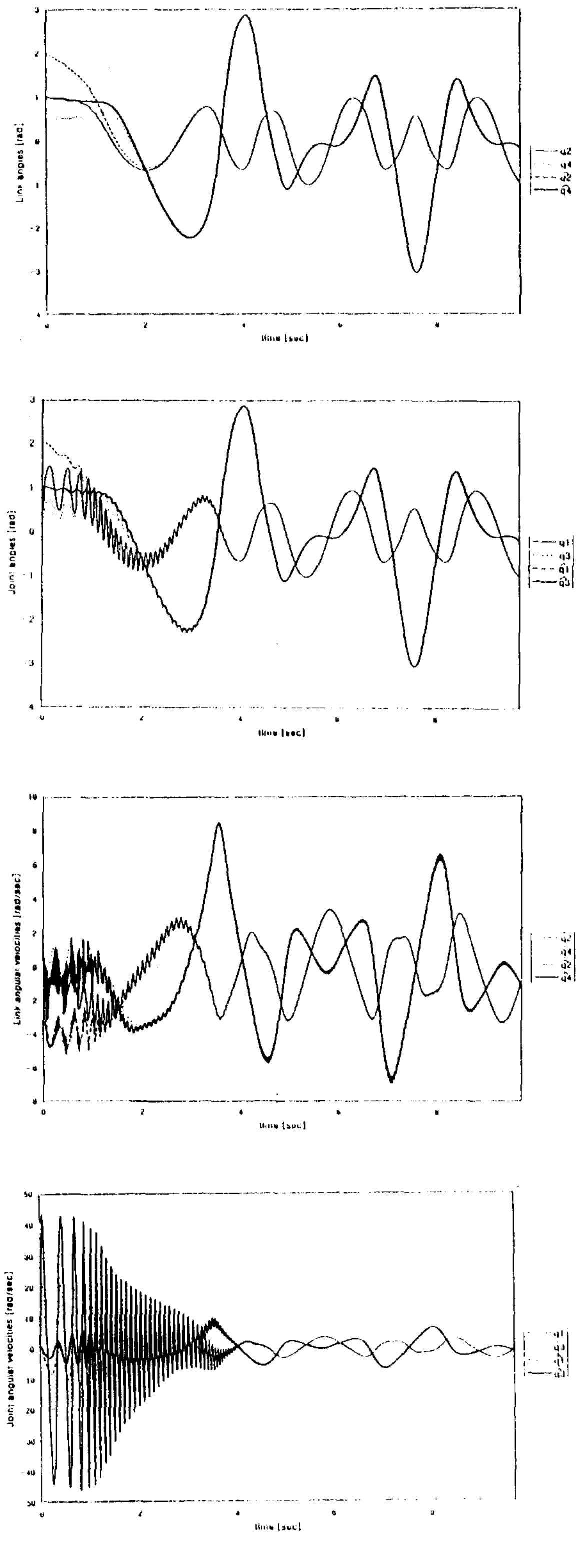

Fig. 3. History of estimation with. ($K_{p1}=K_{v1}=S_1=\begin{bmatrix}5 & 0\\ 0 & 2.5\end{bmatrix}$, $\varepsilon_1=50$)

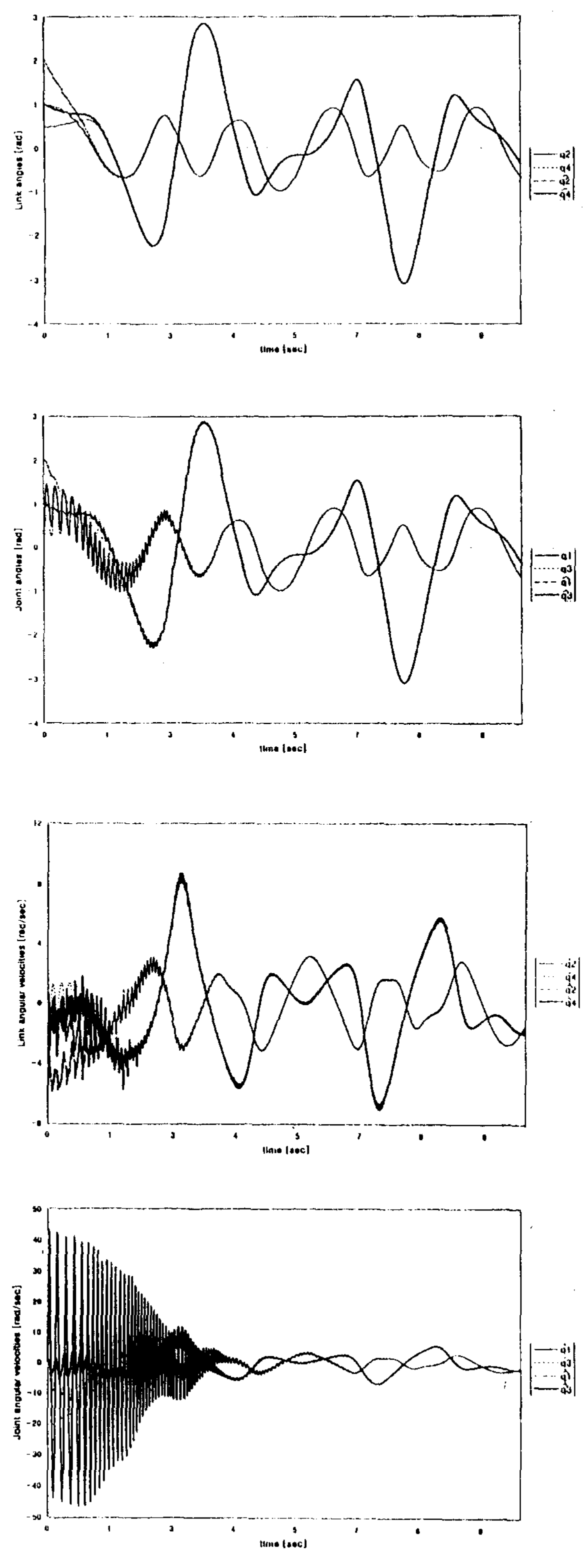

Fig. 4. History of estimation with. ($K_{p1}=K_{v1}=S_1=\begin{bmatrix}10 & 0\\ 0 & 5\end{bmatrix}$, $\varepsilon_1=10$)

performance by choosing $K_{p1}=K_{v1}=S_1=\begin{bmatrix}5 & 0\\ 0 & 2.5\end{bmatrix}$, $\varepsilon_1=50$. It takes somewhat longer time for estimated states to converge than the first case. Since we reduce K_{p1}, and K_{v1} this affects the rising time but arises a smaller uniform ultimate bound ball size than the first case. In Fig. 4 estimation performance shows by choosing design parameters $K_{p1}=K_{v1}=S_1=\begin{bmatrix}10 & 0\\ 0 & 5\end{bmatrix}$, and $\varepsilon_1=10$. Reducing ε_1 gives a small uniform bound and ultimate bound size. However, we see somewhat chattering in transient region. Comparing three cases we see that K_{p1}, K_{v1}, S_1 and ε_1 contribute the system performance. In other words, increasing K_{p1} and K_{v1} results in reducing estimation time to reach within a appropriate estimation range and increases uniform ultimate bound ball size. Decreasing ε_1 results in some chattering while being estimated. Furthermore, the bounding function $\rho_1(\cdot)$ depends on S_1, hence a large S_1 gives a large estimation effort, which is represented by $v_1(\cdot)$ in (82). Considering the estimation performance based on the proposed observer we see that the estimated states are well tracking the real states in good shapes.

Ⅶ. Conclusion

We have developed a robust observer design for flexible joint manipulators, which have uncertainty (*constant or time-varying*). The estimation error dynamical system is practically stable under the designed observation algorithm. Robustness properties of the observer is designed. The observer requires the measurement of the link positions and velocities.

The proposed observer guarantees practical stability as long as link and joint positions and control input are bounded. The further work extends to designing a controller based on the proposed observer design algorithm.

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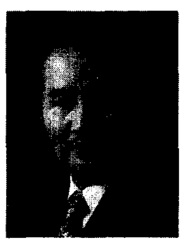

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