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〈**Original**〉

Finite Element Modeling of a Piezoelectric Sensor Embedded in a Fluid-loaded Plate

유체와 접한 판재에 박힌 압전센서의 유한요소 모델링

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Key Words : Hybrid Finite Element Modeling, Piezoelectric Sensor, Fluid-structure Interaction

ABSTRACT

The sensor response of a piezoelectric transducer embedded in a fluid loaded structure is modeled using a hybrid numerical approach. The structure is excited by an obliquely incident acoustic wave. Finite element modeling in the structure and fluid surrounding the transducer region, is used and a plane wave representation is exploited to match the displacement field at the mathematical boundary. On this boundary, continuity of field derivatives is enforced by using a penalty factor and to further achieve transparency at the mathematical boundary, drilling degrees of freedom (d. o. f.) are introduced to ensure continuity of all derivatives. Numerical results are presented for the sensor response and it is found that the sensor at that location is not only non-intrusive but also sensitive to the characteristic of the structure.

요 약

유체와 접한 판재에 박힌 압전센서의 응답을 복합적인 유한요소 해석기법을 이용하여 모델링 하였다. 판재 구조물은 유체영역에서 전파되는 음향파에 의해서 가진된다. 구조물과 압전소자 주위의 유체부분을 유한요소기법을 써서 모델링하였고 임의로 나눈 가상경계에서는 평면파 해를 적용하여 변위를 일치 시켰다. 또한, 이 경계에서 변위의 변분까지도 Penalty factor를 써서 일치 시켰으며 가상경계에서의 투명성을 증가시키기 위해서 유한요소의 각 절점에 회전자유도를 추가시켰다. 압전센서 응답의 수치 결과가 구하여졌고 이것은 센서의 삽입효과가 적을 뿐만 아니라 구조물의 특성에 민감하다는 것이 밝혀졌다.

1. Intorduction

Piezoelectric ceramics have proven to be effective as both sensors and actuators for a wide variety of applications. Although many research studies concerning piezoelectric materials have been performed in the area of ultrasonic transducers[1~3], more interest has been directed towards applications in smart materials or structures which would bring structural revolution[4~7]. Varadan et al.[4] have presented a piezocomposite acturator for active underwater attenuation control of a normally impressed acoustic

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field. An electro-mechanical model of the bilaminate piezocomposite design with an analytical discusstion has been presented. The ability of active cancellation of reflection at normal incidence has been investigated. Barbone and Braga[5,8] have evaluated the sound radiated from an electrically excited laminated piezoelectric plate in an acoustic fluid by employing an invariant imbedding technique. They have investigated the possibility of active cancellation of sound reflection at obilque by prescribing the voltage on the finite size electrodes of the piezoelectric plate. However, in their approach, periodic array of the piezoelectric elements on the structure was assumed; this is not practical since piezoelectric elements can be positioned arbitrarily on the structure in practice.

There are many theoretical and numerical challenges in simulating such a system and its passive(sensor) and active(actuator) functions. In this paper, modeling of piezoelectric sensor is studied. Piezoelectric sensor can be applied in many areas, for examples, damage detection in structures and highways and noise control of automobiles and aircrafts. Figure 1 shows a simple model of sensor function in an acoustical enviromnent; some signals come in and are sensed by the piezoelectric sensor through a change of the displacement field on the structure. In this model, the sensor function can be analyzed by having the model enclose the sensor element only.

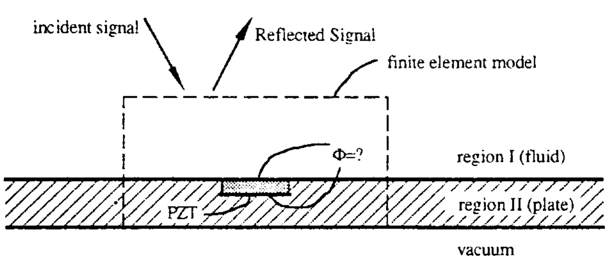


Fig. 1 Sensor problem : voltage induced in PZT sensor by incident acoustic signal

2. A Hybrid Finite Element Modeling

In the sensor problem, there are three regions, the structure, the piezoelectric element and the fluid (Fig. 1). The piezoelectric element is embedded in the structure, and the top surface of the plate is exposed to the infinite fluid medium while the bottom is vacuum. Finite element modeling is used in a region including the piezoelectric element as well as the structure and a portion of the fluid medium. The piezoelectric material is anisotropic and elastodynamic and electric fields are coupled. Therefore, two kinds of variables, displacements and electric potential, are used in the piezoelectric material. Solid region for the structure can be considered as a special case of piezoelectric material which has no electric field. The fluid is considered as inviscid fluid and also described in terms of the displacements and irrotational constraints are included to eliminate zero-energy deformation modes. To model the infinitely layered media, one can put a mathematical boundary surrounding the piezoelectric element. Finite element method can be used to analyze this region (See Fig. 2) and a proper condition must be

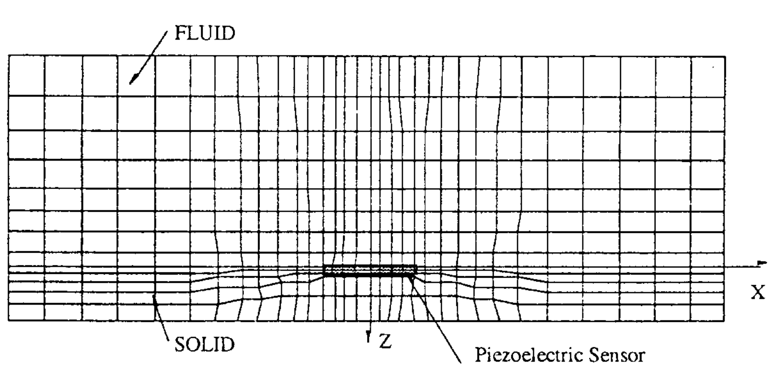


Fig. 2 Finite element model

specified at the mathematical boundary. In the absence of piezoelectric element, the theoretical plane wave solution is already known. If we assume that the model is large enough so that the scattering due to the piezoelectric sensor is small at the mathematical boundary, then a plane wave representation for the infinite plate immersed in the fluid can be used to represent the boundary conditions for the finite element model. Matching of the plane wave solution at the mathematical boundary in terms of displacements and their derivatives is performed by using penalty method to have transparency of the boundary. To further achieve transparency at the boundary, drilling degrees of freedom (d. o. f.) are introduced at each nodes of the finite element model[9].

2.1 Finite Element Formulation

The constitutive equations for the piezoelectric region can be writtern as[10]

$$\boldsymbol{T}=\boldsymbol{C}^{E}\boldsymbol{S}-\boldsymbol{h}^{T}\boldsymbol{E} \text{ and } \boldsymbol{D}=\boldsymbol{h}\boldsymbol{S}+\boldsymbol{b}^{S}\boldsymbol{E} \tag{1}$$

where the superscript T denotes a matrix transpose, $\boldsymbol{T}$ is a stress tensor, $\boldsymbol{S}$ is a strain tensor, $\boldsymbol{D}$ is the electric displacement, $\boldsymbol{E}$ is electric field, $\boldsymbol{C}^{E}$ is the elastic stiffness tensor evaulated at constant $\boldsymbol{E}$ field, $\boldsymbol{h}$ is the piezoelectric coupling constant and $\boldsymbol{b}$ is the dielectric constant at constant strain. The electrical field $\boldsymbol{E}$ is related to the electrical potential ϕ by $\boldsymbol{E}=-\nabla\phi$. $\boldsymbol{u}^{T}=[u_x, u_y, u_z]$ is the displacements in the structure and piezoelectric medium. In the fluid region, displacement is taken as a variable and irrotational constraint is taken into account by using constraint parameter. The slope constraint that has to be imposed on the mathematical boundary is included by using penalty method. The drilling d. o. f. at each nodes of the finite element model is enforced to be same as the physical rotation in a continuum using penalty method.

If we assume that this is the steady state case and after performing discretization, the finite element equation becomes

$$\left\{-\omega^{2}\begin{bmatrix}\boldsymbol{M}_F & 0 & 0\\ 0 & \boldsymbol{M}_{uu} & 0\\ 0 & 0 & 0\end{bmatrix}+\begin{bmatrix}\boldsymbol{K}_F+\alpha\boldsymbol{K}_{\Gamma_F} & 0 & 0\\ 0 & \boldsymbol{K}_{uu}+\gamma\boldsymbol{K}_{\omega}+\alpha\boldsymbol{K}_{\Gamma_S} & \boldsymbol{K}_{u\phi}\\ 0 & \boldsymbol{K}_{\phi u} & \boldsymbol{K}_{\phi\phi}\end{bmatrix}\right\}\begin{Bmatrix}\hat{\boldsymbol{V}}\\ \hat{\boldsymbol{U}}\\ \hat{\boldsymbol{\Phi}}\end{Bmatrix}=\begin{Bmatrix}\alpha\boldsymbol{F}_{\Gamma_F}\\ \hat{\boldsymbol{F}}+\alpha\boldsymbol{F}_{\Gamma_F}\\ \hat{\boldsymbol{Q}}\end{Bmatrix} \tag{2}$$

where $\hat{\boldsymbol{V}}$, $\hat{\boldsymbol{U}}$, $\hat{\boldsymbol{\Phi}}$ are nodal displacements in the fluid, solid and nodal electric field, $\boldsymbol{M}_F$, $\boldsymbol{M}_{uu}$ are mass matrixes in the fluid and solid, $\boldsymbol{K}_F$, $\boldsymbol{K}_{uu}$ are stiffness matrixes in the fluid and solid, $\boldsymbol{K}_{u\phi}$, $\boldsymbol{K}_{\phi\phi}$ are piezoelectric coupling matrix and dielectric stiffness matrix, $\boldsymbol{K}_{\Gamma}$ is a matrix derived from slope constraint on the mathematical boundary, $\boldsymbol{K}_{\omega}$ is a matrix for the drilling d. o. f. constraint, $\hat{\boldsymbol{Q}}$, $\hat{\boldsymbol{F}}$ are nodal point charge and nodal force vectors $\boldsymbol{F}_{\Gamma}$ is a vector derived from slope constraint. α is the penalty factor for the slope constraint and γ is a parameter for the drilling d. o. f. constraint. Solid region for the structure can be considered as a special case of piezoelectric material which has no electric field. $\boldsymbol{K}_F$ includes the constraint parameter, C_{22}, in the material property matrix for the irrotational condition of acoustic fluid. At the interface boundary between the solid and fluid, normal displacements in the fluid and the solid are matched and the tangential displacement obeys slip boundary conditions. To slove this problem, we add extra degrees of freedoms for the tangential displacements of the fluid at the interface boundary. Boundary conditions for $\boldsymbol{u}$, $\boldsymbol{v}$, and their derivatives $\bar{\boldsymbol{u}}_{\Gamma_S}$, $\bar{\boldsymbol{v}}_{\Gamma_F}$ are calculated from the solution of the obliquely incident plane wave problem.

2.2 Obliquely Incident Plane Waves

We consider an infinite flat plate in which the upper surface contacts the fluid medium and the bottom is exposed to the vacuum (see Fig. 3). When

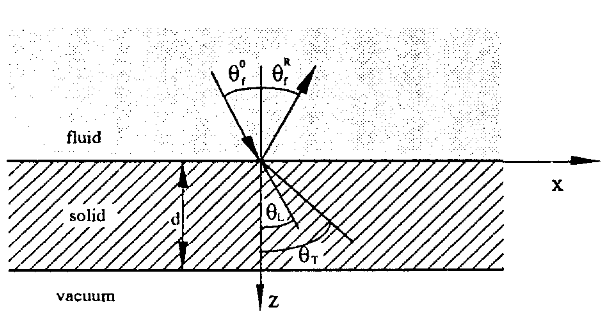


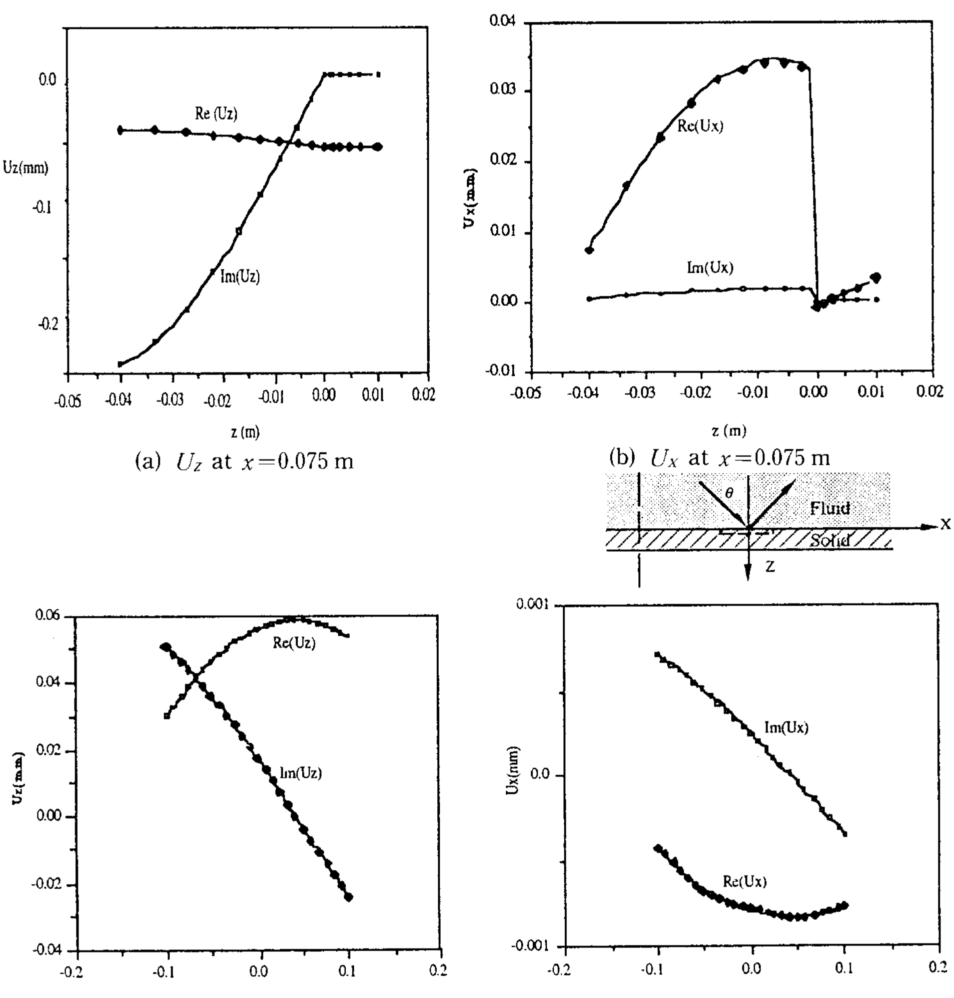
Fig. 3 Infinite flat plate with fluid and vacuum environments

the fluid is considered to be inviscid, by superimposing the incoming and outgoing waves, displacements in the fluid and the solid region can be written as[11]

$$\boldsymbol{v}=\boldsymbol{n}_f^{\circ}A_f^{\circ}\exp(j\boldsymbol{k}_f^{\circ}\cdot\boldsymbol{r})+\boldsymbol{n}_f^{R}A_f^{R}\exp(j\boldsymbol{k}_f^{R}\cdot\boldsymbol{r}) \quad (3)$$

$$\boldsymbol{u}=\boldsymbol{p}_L^{+}A_L^{+}\exp(j\boldsymbol{k}_L^{+}\cdot\boldsymbol{r})+\boldsymbol{p}_L^{-}A_L^{-}\exp(j\boldsymbol{k}_L^{-}\cdot\boldsymbol{r}) +\boldsymbol{p}_T^{+}A_T^{+}\exp(j\boldsymbol{k}_T^{+}\cdot\boldsymbol{r})+\boldsymbol{p}_T^{-}A_T^{-}\exp(j\boldsymbol{k}_T^{-}\cdot\boldsymbol{r}) \quad (4)$$

where A_f° is the amplitude of incident wave and A_f^R is the amplitude of reflected wave. Subscript L refers to the longitudinal wave and T refers to transverse wave. Superscript "+" indicates the positive direction and "−" the negative direction. A_L^+, A_L^-, A_T^+ and A_T^- are amplitudes of incoming, outgoing longitudinal and transverse waves in the solid plate respectively. k_f, k_L and k_T are the wave numbers in the fluid, longitudinal and transverse directions in the solid. $\boldsymbol{k}_f^{\circ}$, $\boldsymbol{k}_f^R$, $\boldsymbol{k}_L^+$ and $\boldsymbol{k}_L^-$ are the wave vectors in the fluid and longitudinally plus and minus and transversely plus and minus directions in the solid respectively. $\boldsymbol{e}_x$, $\boldsymbol{e}_x$, $\boldsymbol{e}_z$ are the unit normal vectors in rectangular coordinate. Snell's law and the boundary conditions at the top and bottom surfaces give five relations. Therefore five unknown variables, A_f^R, A_L^+, A_L^-, A_T^+ and A_T^- can be find. After substituting these unknowns into equations (3) and (4), we can obtain the displacement at a desired certain point, say, at the boundary of the finite element model.



(a) U_Z at x=0.075 m

(b) U_X at x=0.075 m

(c) U_Z at z=0

(d) U_X at z=0

Fig. 4 Infinite plate problem without PZT : plane wave solution (—) vs. FEM(•)

3. Numerical Results and Discusstion

3.1 Infinite Plate without Piezoelectric Sensor

To verify the feasibility of this approach, a flat steel slab without a piezoelectric patch and is exposed to the fluid and vacuum, is considered (Fig. 3). The region surrounding the piezoelectric sensor and its vicinity is treated using finite element modeling (Fig. 2). If the part is substituted with the same material of the steel, this model will be identical with the infinite slab problem which can easily be dealt with using the plane wave representation. The width and height of the finite element model is 200 mm by 100 mm, Total 1347 nodes are provided and 8-node element is used. The plate is steel with 10 mm thick, the frequency of incident wave is 10 kHz, the oblique angle is 10°, and the amplitude of the incident wave is 0.1 mm. The range of constraint parameter in the fluid stiffness matrix is 2-20 (times bulk modulus of the fluid). Penalty factor for the slope constraint is set to 10^6 to 10^{10} and the range of drilling d. o. f. constraint penalty factor is 10^4 to 10^8. In these ranges, optimal parameters are searched by an optimization technique.

The displacements U_Z and U_X at an arbitrary section ($x=75$ mm) are shown in Fig. 4(a) and (b), respectively. Figure 4(c) and (d) represent the displacements of the solid, U_Z and U_X, at the interface boundary. The finite element results show good agreement with plane wave solution in the fluid and the solid regions.

3.2 Infinite Plate with PZT Sensor

For the piezoelectric sensor problem, one PZT patch embedded in the flat plate is considered (Fig. 1). The size of the PZT element is 1 mm × 10 mm in thickness and width respectively. The material properties of PZT-5H is used for the PZT element.[10] The incident wave conditions are in the same as the previous case with different incident angles. The finite element model size and the number of nodes are the same as in the previous case. Fine meshes are introduced in and near the PZT. Before solving the PZT inclusion problem, the constraint parameter in the fluid element and penalty factors for the slope and drilling d. o. f. constraints are searched such that the finite element result without the PZT element is close to the plane wave solution. By using these constraint parameters, the sensor response at the PZT is found.

Figure 5 shows the excited voltage output at the sensor when the incident angle is changed. In this figure, a peak of excited voltage amplitude is shown at 17° and slightly increased after the peak. To explain this result, the wave propagation phenomenon in the infinite flat plate with fluid loading is considered. Generally, there are two wave modes in an infinite plate: symmetric and antisymmetric modes[11]. For the first mode, called longitudinal wave, the vertical displacements on the boundaries of the plate are identical in magnitude and opposite in sign. For the second of these oscillations, called flexural wave, the displacements on the boundaries are identical. The propagation velocity, c_S of the longitudinal waves in the infinite steel plate is 5212 m/s[12]. If the incident wave speed is close to c_S, the coincidence occurs. When the fluid is water, the coincidence angle is 16.73°. This angle is close to the location of the peak in Fig. 5. Therefore, the first peak is related to the coincidence of longitudinal mode in the infinite plate. Also, the flexural wave speed of the plate is dependent on the excitation frequency[12]. For the steel plate with 10 kHz excita-

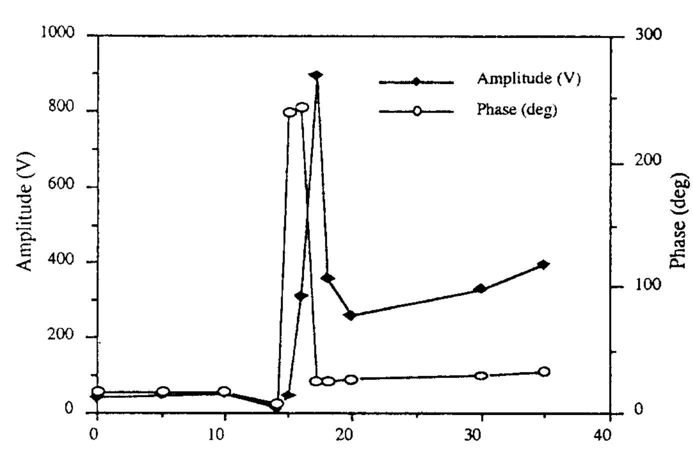


Fig. 5 Excited voltage on the PZT sensor (frequency = 10 kHz)

tion frequency and the previously given conditions, this speed is 972.25 m/s which is smaller than the wave speed of water — 1500 m/s. Therefore, coincidence due to the flexural waves at the given condition, does not happen. This is true in Fig. 5 where there is no second peak, and after the first peak, the amplitude of the excited voltage is gradually increased with the incident angle. If the freuency is increased up to 23.8 kHz, which corresponds to the coincidence angle at the given conditions, then coincidence can be seen.

From the sensor response, it is clear that the sensor is sensitive to the characteristic of the structure.

4. Conclusions

A hybrid finite element program, which is combined with the plane wave solution of the infinite problem and the slope information at the mathematical boundary is included, has been developed for the sensor problem. By comparing the result of this program with the plane wave solution in a flat plate immersed in a fluid, the feasibility of this approach has proven. In the sensor problem, the excited voltage at the electrode due to the obliquely incident wave has been found according to the increased incident angle. This response shows the coincidence of the longitudinal wave in the infinite plate. Thus it is found that the sensor is sensitive to the characteristic of the structure. In this approach, the fluid is considered to be inviscid and this is true as far as the model domain is sufficiently large and the local effect of the interface boundary is negligible.

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