Performance Analysis of Satellite Maneuver and Structure Control Using Risk-Sensitive Control

위성 운동과 건물 진동 제어에 활용된 리스크 센서티브 제어기의 성능 분석

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요 약 : 지구를 원궤도로 돌고 있는 위성 운동과 지진에 흔들리는 건물 진동을 선형 확률적 미분 방정식으로 나타내고 최적화 제어를 위하여 리스크 센서티브 제어기를 사용한다. 리스크 센서티브 파라미터에 따라서 코스트 함수의 평균과 분산이 변하게 된다. 이 파라미터가 무한히 커지면 리스크 센서티브 제어기는 기존의 LQG 제어기와 같아지므로 리스크 센서티브 제어 이론은 LQG 제어 이론을 포함한 종합적인 이론이다. 이 논문에서는 리스크 센서티브 이론을 소개하고, 리스크 센서티브 상태 궤환 제어 시스템의 성능을 분석하기 위하여 상태 벡터와 제어 벡터의 공분산을 구한다. 리스크 센서티브 제어 방식의 성능 측정 및 평가 방법을 도출하기 위하여 공분산을 이용하면 리스크 센서티브 제어기는 기존의 LQG 제어기 보다 우수한 성능을 나타낸다는 것을 보여준다. 시뮬레이션을 통하여 위성의 자세 및 궤도 운동 제어와 건물 진동 제어에 활용된 리스크 센서티브 제어기의 향상된 성능과 안정성을 보여준다.

Keywords: risk-sensitive, LQG, performance, satellite, structure, robust control

I. Introduction

The Risk-Sensitive (RS) control strategies are a generalization of the LQG control strategy, and are related to the dynamic games and the H_∞ control methods. This RS idea seems to be originated from Jacobson in 1973 [1]. Finite horizon linear exponential-of-integral solution (EOI) of the full-state-feedback case has been solved by Jacobson. In 1985 Bensoussan and van Schuppen solved the partially observable RS control problem [2]. In 1991 Whittle introduced the RS maximum principle in [3]. RS control has been shown to be related to H_∞ control through entropy [4]. Furthermore, the relationship among H_∞ optimal control, game theory, and risk-sensitive stochastic control have been established in recent years [5]. Runolfsson has solved the infinite horizon RS case, however just for the risk-aversive choice [5]. The probabilistic performance and stability characteristics of RS controlled structures have been studied using reliability analysis in [6].

This paper recognizes RS control as a generalization of LQG control. RS control is an extension of classical LQG control in the sense that there exists an extra variable parameter, γ_{RS}, and setting this parameter to a particular value recovers the LQG result. The average value derivation of cost function to determine the performance is given as a proposition.

This proposition differs from the LQG derivation by the extra term with the risk-sensitivity parameter, γ_{RS}, and as γ_{RS} approaches infinity we recover the LQG result. Thus, by varying this γ_{RS}, we achieve different performance and stability results. Here, we also propose a method to choose the risk sensitivity parameter in order to achieve better stability margin.

In the satellite and structure control applications the performance is calculated, and the tradeoff between the stability and the average performance is shown. In the next section, full-state feedback RS control of Jacobson is discussed. In Section III, the average performance of an RS controller is derived. Then in Sections IV and V, the applications of RS control are given. First application is for satellite attitude/orbit maneuver and the other is in structural control under seismic disturbances. Finally, conclusions are given in the last section.

II. Full state feedback risk-sensitive control

In his paper "Optimal Stochastic Linear Systems with Exponential Performance Criteria and Their Relationship to Deterministic Differential Games," Jacobson defines and solves the linear exponential quadratic Gaussian control problem [1]. In this setting the LQG method is extended by replacing the quadratic criterion with the exponential of a quadratic function. Then the solution shows that the controller depends on the statistics of the noise. The system considered in the paper is given in the equation

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + \Lambda(t)w(t). \tag{1}$$

The problem is to minimize the equation

$$J^\sigma = \sigma\phi E\left\{ \exp\left(\frac{\sigma\phi}{2} \int_0^{t_F} [x'(t)Qx(t) + u'(t)Ru(t)]dt + \frac{1}{2}x'(t_F)Px(t_F) \right) \right\} \tag{2}$$

with respect to $u(\cdot)$ and subject to (1), where $w(t)$ is a Gaussian white process with $E\{w(t)\} = 0$ and $E\{w(t)w'(\tau)\} = W\delta(t-\tau)$. Here σ denotes a sign operator which is equal to -1 for the risk seeking case and +1 for the risk aversive case, and ϕ denotes a real scalar parameter. Assuming perfect state observation, the solutions are derived using the Hamilton-Jacobi-Bellman equation

$$-\frac{\partial J^\sigma(x,t)}{\partial t} = \min_u \left\{ \left(\frac{\partial J^\sigma(x,t)}{\partial x} \right)'(A(t)x + B(t)u) + \mathrm{tr}\left(\frac{1}{2}\Lambda(t)W\Lambda'(t)\frac{\partial^2 J^\sigma(x,t)}{\partial x^2} \right) + \frac{\sigma\phi}{2}(x'Qx + u'Ru)J^\sigma(x,t) \right\}. \tag{3}$$

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Take derivative of both sides with respect to u to get

$$u^{\sigma}(x,t)=-\frac{R^{-1}B(t)\frac{\partial J^{\sigma}(x,t)}{\partial x}}{\sigma\phi J^{\sigma}(x,t)}. \tag{4}$$

If we assume an optimal cost of the form

$$J^{\sigma}=\sigma\phi F^{\sigma}(t)\exp\left(\frac{\sigma\phi}{2}x'M(t)^{\sigma}x\right) \tag{5}$$

and substitute the above equations into the Hamilton-Jacobi-Bellman (2), we get two differential equations similar to the results in the last section, namely

$$\begin{aligned}-\dot{M}^{\sigma}(t)=&\ Q-M^{\sigma}(t)(B(t)R^{-1}B'(t)\\&-\sigma\phi\Lambda(t)W\Lambda'(t))M^{\sigma}(t)\\&+M^{\sigma}(t)A(t)+A'(t)M^{\sigma}(t)\\-\dot{F}^{\sigma}(t)=&\ \frac{\sigma\phi}{2}F^{\sigma}\operatorname{tr}[E(t)WE'(t)M^{\sigma}(t)]\end{aligned} \tag{6}$$

with the boundary conditions $M^{\sigma}(t_F)=P$ and $F^{\sigma}(t_F)=1$. Finally, then, the minimum cost is

$$\min J^{\sigma}(x,0)=\sigma\phi F^{\sigma}(0)\exp\left(\frac{\sigma\phi}{2}x'(0)M(0)x(0)\right). \tag{7}$$

III. Average performance of an RS controller

In LQG control, we are minimizing the average value of the cost function, $\hat{J}(t_F)$, which is closely related to the minimization of the covariance of the state, x. In RS control this average value of the cost function and the covariance of the state can be varied by choosing the appropriate risk-sensitivity parameter, γ_{RS}.

Consider the system described by the stochastic differential equation,

$$dx(t)=(Ax(t)+Bu(t))dt+\Lambda dw(t),\ x(0)=x_0, \tag{8}$$

where w is a Brownian motion, $x(t)\in R^n$ is the state, and $u(t)\in R^m$ is the control action.

Assume the following:

$$\begin{aligned}E\{w\}&=0,\ E\{dw(t)dw'(t)\}=W\,dt,\\E\{x(0)\}&=0,\ E\{x(0)x'(0)\}=\Sigma_0.\end{aligned} \tag{9}$$

Furthermore assume that w and $x(0)$ are mutually independent. The cost function that is to be minimized is given by

$$J_{RS}(\gamma)=\log\left(E\left\{\exp\left(\frac{1}{2\gamma_{RS}}\hat{J}(t_F)\right)\right\}\right) \tag{10}$$

where

$$\hat{J}(t_F)=\int_0^{t_F}[x'(t)Qx(t)+u'(t)Ru(t)]dt, \tag{11}$$

$Q\geq 0$, and $R>0$. Then the state feedback optimal controller is found to be [5]

$$u(t)=k(x(t))=-R^{-1}B'S(t)x(t)=-K(t)x(t) \tag{12}$$

where $S(t)$ is obtained from

$$\begin{aligned}&\dot{S}(t)+A'S(t)+S(t)A+Q\\&-S(t)\left(BR^{-1}B'-\frac{1}{\gamma_{RS}}\Lambda W\Lambda'\right)S(t)=0.\end{aligned} \tag{13}$$

Now we are ready to present the average performance result which is an extension of the classical LQG case. The average performance results of the LQG case are well known in the literature, see [7] for example, but here we derive the result for the RS control case. Note that there exists an extra term with γ_{RS} in (16). This is an extension of the LQG case in the sense that the extra term disappears as γ_{RS} approaches infinity, and (16) reduces to the classical LQG equation.

Proposition 1 : For a full-state-feedback RS control with the above assumptions, the covariance of the state, $E\{x(t)x'(t)\}=\Sigma(t)$, is given by

$$\begin{aligned}\dot{\Sigma}(t)=&\ (A-BR^{-1}B'S(t))\Sigma(t)\\&+\Sigma(t)(A-BR^{-1}B'S(t))'+\Lambda W\Lambda',\end{aligned} \tag{14}$$

with $\Sigma(0)=\Sigma_0$ the covariance of the control force is obtained from

$$E\{u(t)u'(t)\}=R^{-1}B'S(t)\Sigma(t)S(t)BR^{-1}; \tag{15}$$

and the average value of the cost function, $\hat{J}(t_F)$ is given by

$$\begin{aligned}E\{\hat{J}(t_F)\}=\operatorname{tr}\Big[S(0)\Sigma(0)+\int_0^{t_F}\Big(&S(t)\Lambda W\Lambda'\\&-\frac{1}{\gamma_{RS}}S(t)\Lambda W\Lambda'S(t)\Sigma(t)\Big)dt\Big].\end{aligned} \tag{16}$$

Proof : The covariance of the state (7) is derived in a standard way; see [7] for example. And (8) is a direct consequence of (5). To get the average value of the cost function, take the trace of (4) and obtain

$$\begin{aligned}&E\{\hat{J}(t_F)\}\\&=E\left\{\operatorname{tr}\left(\int_0^{t_F}[x'(t)Qx(t)+u'(t)Ru(t)]\ dt\right)\right\}\\&=\operatorname{tr}\left(\int_0^{t_F}[\Sigma(t)Q+RK\Sigma(t)K']\ dt\right).\end{aligned} \tag{17}$$

Add the differential, $\frac{d}{dt}(S\Sigma)$ into the integrand of (9), compensating by adding $S(0)\Sigma(0)-S(t_F)\Sigma(t_F)$ outside the integral to obtain,

$$\begin{aligned}&E\{\hat{J}(t_F)\}=\operatorname{tr}(S(0)\Sigma(0)-S(t_F)\Sigma(t_F))+\\&\operatorname{tr}\left(\int_0^{t_F}[\Sigma(t)Q+RK\Sigma(t)K'+\dot{S}\Sigma(t)+S\dot{\Sigma}(t)]dt\right).\end{aligned} \tag{18}$$

Now use (6) and (7), then simplify to get

$$\begin{aligned}&E\{\hat{J}(t_F)\}=\operatorname{tr}(S(0)\Sigma(0)\\&+\int_0^{t_F}[S(t)\Lambda W\Lambda'-\frac{1}{\gamma_{RS}}S(t)\Lambda W\Lambda'S(t)\Sigma(t)]\ dt).\end{aligned} \tag{19}$$

Thus, we note that by varying the risk-sensitivity parameter, γ_{RS}, we can change the average performance. As γ_{RS} approaches ∞ we get the classical linear quadratic Gaussian (LQG) average performance.

IV. Satellite applications of an RS controller

As the applications, satellite attitude and orbit maneuvers are considered here. Firstly, the attitude of a geostationary satellite such as the KOREASAT and Experimental Communication Satellite (ECS) of ETRI is modeled and controlled using the RS controller. Then RS attitude and orbit maneuver of a satellite are considered as applications. The purpose of these examples is to show the applicability of RS control in spacecraft maneuver, and to compare RS control with existing LQG control. In the orbit control application, we apply the performance analysis method discussed in Section III.

1. Attitude maneuver application

This section shows the simulation results associated with the model of a geostationary satellite equipped with a bias momentum wheel on the third axis of body frame. This model assumes that the disturbance torque is Gaussian white noise. Then a stochastic RS controller is applied. For this model we assume small attitude angle, and roll/yaw dynamics are assumed to be decoupled from the pitch dynamics.

A roll/yaw attitude model of the geostationary satellite such as the KOREASAT and the ECS is simplified [8] as the following linear differential equation when $h_w \gg \max\{I_i, \omega_c\}$,

$$dx(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{h_w \omega_c}{I_1} & 0 & 0 & -\frac{h_w}{I_1} \\ 0 & -\frac{h_w \omega_c}{I_2} & \frac{h_w}{I_2} & 0 \end{bmatrix} x(t)\, dt + \begin{bmatrix} 0 \\ 0 \\ \frac{B_e}{I_1}\cos(\theta) \\ \frac{B_e}{I_2}\sin(\theta) \end{bmatrix} m(t)\, dt + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{I_1} \\ \frac{1}{I_2} \end{bmatrix} dw(t) \tag{20}$$

$$dy(t) = I_{4\times4}\, x(t)\, dt + dv(t) \tag{21}$$

where $\frac{dw}{dt}$ is white Gaussian noise representing the disturbance torque, $\frac{dv}{dt}$ is white Gaussian noise representing the measurement noise, h_w is the wheel momentum, θ is the angle that the positive roll axis makes with the magnetic torquer, ω_c is the orbital rate, I_i is the moment of inertia of the i-th axis, $x = [yaw, roll, \dot{yaw}, \dot{roll}]$ is the state, m is a dipole moment of the magnetic torquer (control), $B_e = 1.07\times10^{-7}$ telsa is the nomial magnetic field strength, and $I_{4\times4}$ is an identity matrix with dimension four. The expected value of $\frac{dw}{dt}$ is zero with $E\left\{\frac{dw}{dt}\frac{dw'}{dt}\right\} = 0.7B_e$, and the expected value of $\frac{dv}{dt}$ is zero with $E\left\{\frac{dv}{dt}\frac{dv'}{dt}\right\} = 1\times10^{-7}$. Here we chose $\gamma_{RS} = 5\times10^{-2}$ for the demonstration purpose, but this risk-sensitivity parameter, γ_{RS}, should be viewed as another design parameter just like the weighting matrices Q and R. By varying this γ_{RS}, we can obtain different performance and stability results. Theoretically, all γ_{RS} that give a solution to the Riccati (13) are possible. In the next example, we show how we choose this risk-sensitivity parameter to obtain larger stability margin. The constants for the operational mode are given as

$$I_1 = 1988\,kg\cdot m^2,\ I_2 = 1876\,kg\cdot m^2,\ h_w = 55\,kg\cdot m^2/s,$$

$$\omega_c = 0.00418\ \mathrm{deg}/s, \text{ and } \theta = 60\,\mathrm{deg}.$$

These values are actual parameters of the geostationary satellite, ECS-1.

The initial condition is $[0.5\,\mathrm{deg}, 0, 0, 0.007\,\mathrm{deg}/s]$. Finally the weighting matrices are chosen to be $Q = I_{4\times4}$ and $R = 1\times10^{-10}$.

In this model, the states are measured with the sensor noise, $\frac{dv}{dt}$. Then a Kalman filter is used to estimate the states. The following simulations are performed using MATLAB, a software package. Figure 1 shows the phase plot of RS controlled ECS-1. The RS controller is found using (12). Note that both yaw and roll angles reduce to a value close to the origin. The second graph, Figure 2 shows the roll and yaw angles with respect to time variation. After about 3 hours, both roll and yaw angles stay below 0.1 degree. Figure 3 shows the control action needed to control the satellite. Initially large control action is needed, but after 3 hours or so, less than $300\,Atm^2$ magnetic torque is required. It is important to note that despite the external disturbances, RS control law produces good performance.

To compare the results with well known LQG controller, we simulated the system with an LQG controller. Here we let γ_{RS} approach infinity in (13). Note that (13) becomes classical Riccati equation as γ_{RS} goes to infinity. This is shown in Figure 4. Note that in LQG case, it takes longer for yaw and roll angles to fall below 0.1 degree, and the variation in the angles are larger than the RS case. Thus in this sense, RS controller outperforms the LQG controller.

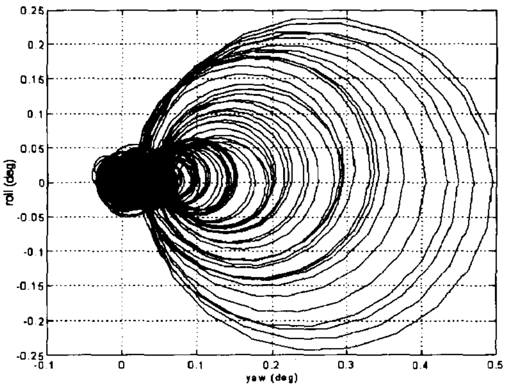

Fig. 1. Roll/yaw phase plot, RS control.

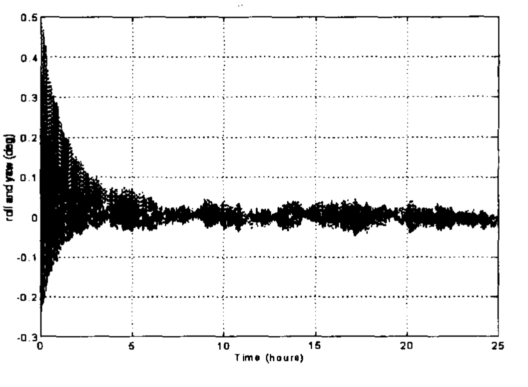

Fig. 2. Roll/yaw versus time, RS control.

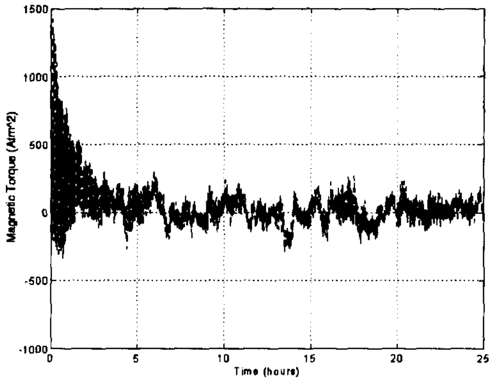

Fig. 3. Control action, RS control.

2. Orbit maneuver application

To show the possible use of RS control in space technology application, we ran a computer simulation of the orbital maneuver of a satellite. Due to the nature of the application and large experimental costs involved, actual experiment is not yet performed. But it is possible to do an experiment using the satellite simulator to be developed in ETRI by 1999 [9].

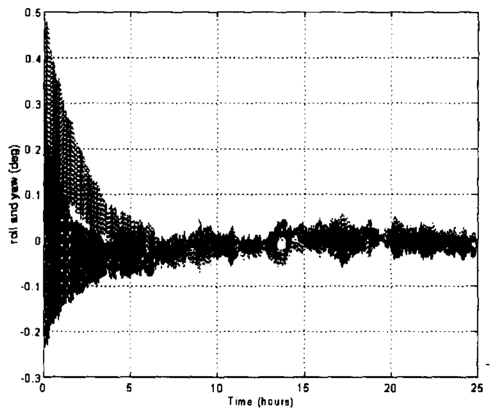

Fig. 4. Roll/yaw versus time, LQG.

A linear perturbation model of a satellite in orbit about a planet with an ideal inverse-square gravity field is given by (see [10])

$$\frac{dx(t)}{dt} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3\Omega^2 & 0 & 0 & 2R_0\Omega \\ 0 & 0 & -\frac{2\Omega}{R_0} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & \frac{1}{R_0} \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} w(t) \qquad (22)$$

where $x=[r\ \theta\ \dot{r}\ \dot{\theta}]'$, r and θ describe the position and the angle of the satellite in the polar coordinate. The control, u, represents the acceleration due to thrust. The constant $R_0=7063$km is the orbit radius, μ is the gravitational constant, angular rate, $\Omega=\sqrt{\frac{\mu}{R_0^3}}=1.064\times10^{-3}$rad/s, and $w(t)$ is the white noise with zero mean and $E\{w(t)w'(\tau)\}=\delta(t-\tau)$. Here $w(t)$ is perturbation in the space which accounts for modeling inaccuracies such as the oblateness of the Earth, incomplete knowledge of the atmosphere, aerodynamic forces, spherical symmetry, and rotational forces. The weighting matrices are taken to be and $Q=I_{4\times4}$ $R=1$. Once again, these values are chosen by trial and error method. The terminal time t_F is taken to be 10 seconds. Here we assume that the all the state are known, If the states are not known then we can use a Kalman filter to estimate the states as in the previous example.

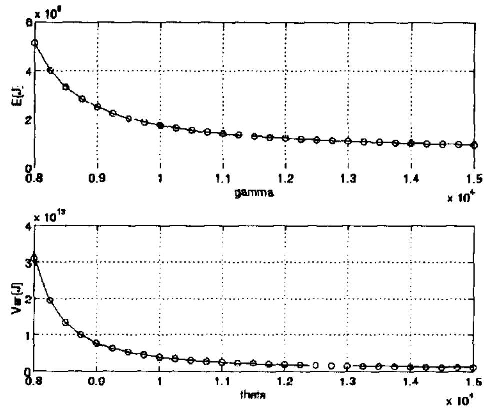

Fig. 5. Full-state-feedback; optimal mean and variance.

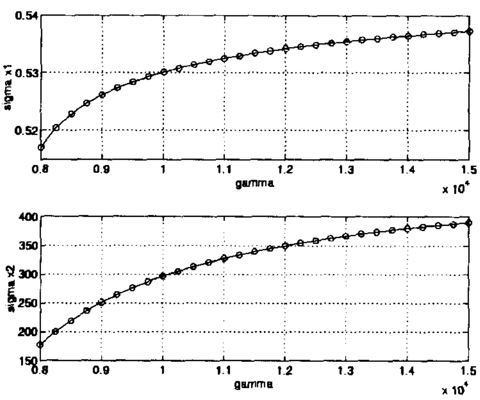

Fig. 6. Full-state-feedback; RMS of r and theta.

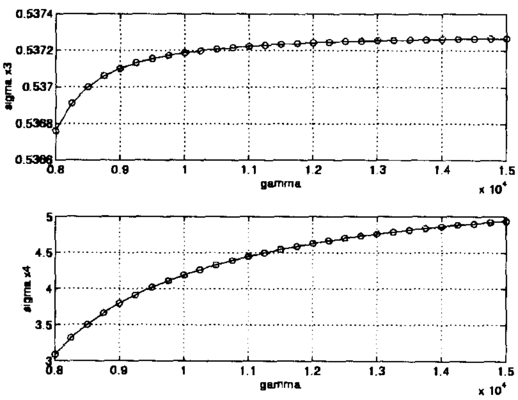

Fig. 7. Full-state-feedback; RMS rdot and theta_dot.

Figure 5 shows the graphs of the optimal mean and variance with respect to γ_{RS}. Note that both mean and variance decrease as γ_{RS} increases. The mean and the variance of the cost function is related to the energy of the system. Thus, smaller mean and variance correspond to saving valuable fuel of the satellite, and are preferred. Since the RMS value(square root of the covariance) has more intuitive meaning than the covariance, we present the results in terms of the RMS values. Figures 6 and 7, which are calculated using (14), show that all four RMS values of the states increase as γ_{RS} increases. Thus to reduces the RMS values of r and θ smaller γ_{RS} is preferred. The RMS values are calculated using the results of Proposition 1. Because large γ_{RS} corresponds to LQG performance value, we note that RS control gives better performance in terms of the RMS values of the state. Figure 8 shows the control actions required. These RMS values are one way to measure performance of the controller. Smaller RMS values imply more precise satellite maneuver. Thus, there is a tradeoff between the amount of fuel used and the precision of the maneuver.

When the state x is measured, the RS controller are implemented in the manner $u=-Kx$ for an appropriate matrix K of feedback $A_{cl}=A-BK$ gain, which leads to a closed loop matrix, . Since the asymptotic stability of a system depends upon its eigenvalues of A_{cl}, the system is stable if all the eigenvalues are in the open left half plane. Furthermore if the maximum real part of the eigenvalues are farther away from the imaginary axis, we say the system has more stability margin.

Figure 9 shows the change of maximum real part of the eigenvalue with respect to γ_{RS}. Since smaller maximum real part of the eigenvalue corresponds to more stability margin, the best margin occurs when γ_{RS} is around 1.024×10^4. The variation in the maximum real part of the eigenvalue is not monotonic, thus it is necessary to obtain these simulation results in order to choose appropriate γ_{RS} value. It is important to note that RS control gives an extra degree of freedom in choosing this stability margin through γ_{RS}. For the classical LQG case, the maximum real part of the eigenvalue would be fixed around -0.01, and for the RS case the maximum real part of the eigenvalues could be in the range of -0.014 to -0.0065, depending on the risk sensitivity parameter. There exists tradeoff with the stability and performance, so these graphs will help in the engineering decision process.

V. RS control of a structure under the earthquake

As another application area of RS control, we propose structural control under seismic disturbances. Using active structure control in the civil engineering application is an actively researched area; see [6] and [11] for example. Here we consider a 3DOF, single-bay structure model with an active tendon controller as shown in Figure 10; see [11]. The structure is subject to an one-dimensional earthquake excitation.

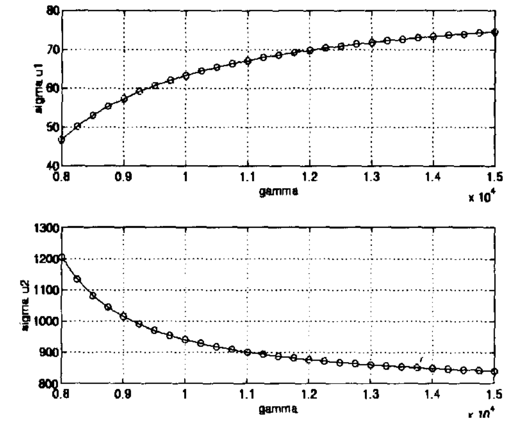

Fig. 8. Full-state-feedback; RMS of the inputs, u1 and u2.

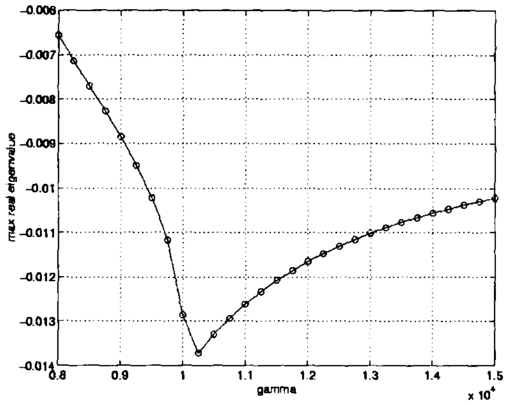

Fig. 9. RS full-state-feedback, maximum eigenvalue.

Let the state x consist of the displacements of first, second, and third floors, augmented with the velocities of first, second, and third floors respectively. If we assume a simple shear frame model for the structure, then we can write the governing building equations of motion in state space form as

$$dx(t)=\begin{bmatrix} 0 & I \\ -M_s^{-1}K_s & -M_s^{-1}C_s \end{bmatrix} x(t)\ dt + \begin{bmatrix} 0 \\ M_s^{-1}B_s \end{bmatrix} u(t)\ dt + \begin{bmatrix} 0 \\ -\Gamma_s \end{bmatrix} dw(t) \tag{23}$$

where

$$M_s=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix},\ B_s=\begin{bmatrix} -4k_c\cos\alpha \\ 0 \\ 0 \end{bmatrix},$$
$$C_s=\begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} \tag{24}$$
$$\Gamma_s=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ K_s=\begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

and m_i, c_i, k_i are the mass, damping, and stiffness, respectively, associated with the i-th floor of the building. The values are given in Table 1. The k_c is the stiffness of the tendon and α is the angle that the tendon makes with respect to the floor as shown in the Figure 10. The disturbance $\frac{dw}{dt}$ is a Gaussian white noise representing the earthquake with $E\{dw(t)\}=0$ and $E\{dw(t)dw(t)'\}=W\,dt$; in this example, $W=1.00$ in^2/ sec^3. The terminal time, t_F, is taken to be 10 seconds, and the weighting matrices are given as $R=k_c$ and $Q=\begin{bmatrix} K_s & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}$, where $0_{3\times3}$ are three-by-three matrix of zeros.

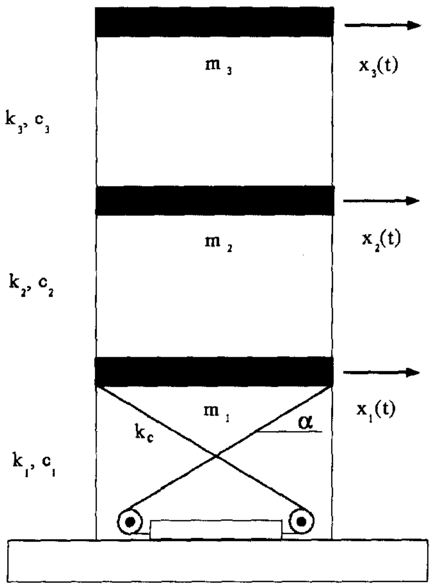

Fig. 10. Schematic diagram for three-degree-of-freedom structure.

Table 1. Model parameters of the 3DOF structure.

Constants	Values
$m_{1,2,3}$ (lb-s^2/in)	5.6000
c_1 (lb-s/in)	2.6000
c_2 (lb-s/in)	6.3000
c_3 (lb-s/in)	0.3500
k_1 (lb/in)	5034.0
k_2 (lb/in)	10965
k_3 (lb/in)	6135.0
kc (lb/in)	2124.0
α (degrees)	36.000

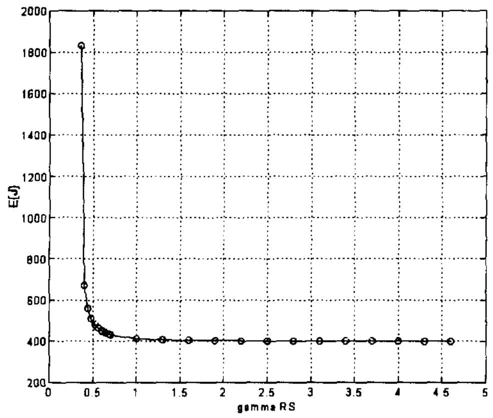

Fig. 11. Optimal mean; full-state-feedback, RS, 3DOF.

We designed the RS controller for this system with parameter values given in Table 1 for the cost function (2). The parameters, mass, damping, and stiffness are chosen to match modal frequencies and dampings of the experimental structure in [11].

Figure 11 shows the average values of the cost function $E\{\hat{J}\}$ decreases as the risk-sensitivity parameter γ_{RS} increases. On the other hand, the variance of the cost function decreases rapidly and then increases back to the classical LQG value. See Figure 12. Thus, we note the variance of the cost function can be smaller than the LQG case for some values of γ_{RS}.

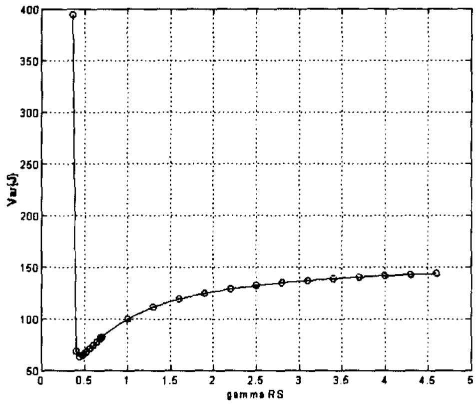

Fig. 12. Optimal variance; full-state-feedback, RS, 3DOF.

Figure 13 shows the RMS displacement responses of first σ_{x_1}, second σ_{x_2}, and third σ_{x_3} floors; and the RMS velocity responses of first σ_{x_4}, second σ_{x_5}, and third σ_{x_6} floors respectively, versus the risk-sensitivity parameter, γ_{RS}. These values are calculated using (14). It is important to note that both third floor displacement and velocity can be decreased by choosing small γ_{RS}. Note that not all the RMS responses decrease as γ_{RS} decreases. If we choose smaller γ_{RS}, note that we require larger control force,

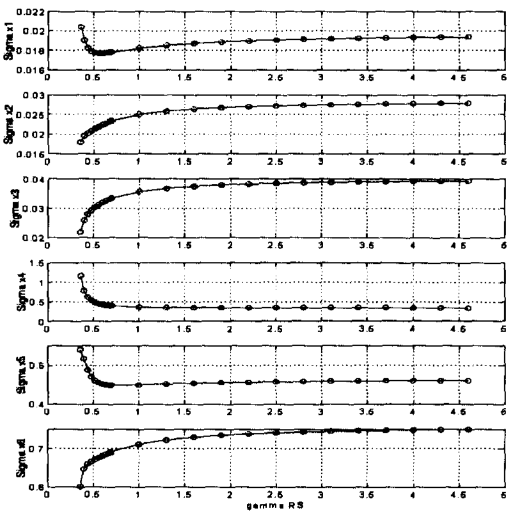

Fig. 13. Displacements and velocities; full-state-feedback, RS, 3DOF.

which means that more effort is needed to reduce the RMS displacement and velocity responses, see Figure 14. Furthermore, as γ_{RS} decreases the maximum real part of the eigenvalue decreases until certain point and starts to increase rather dramatically. See Figure 15. Thus once again, this shows the tradeoff between the stability margin and the performance.

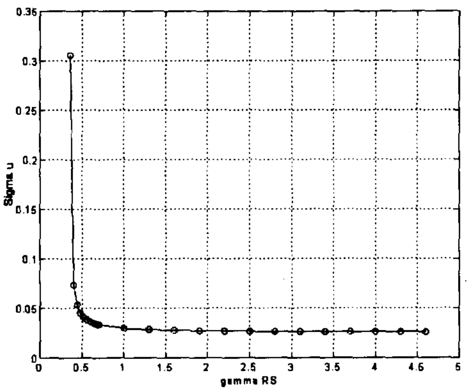

Fig. 14. Control force; full-state-feedback, RS, 3DOF.

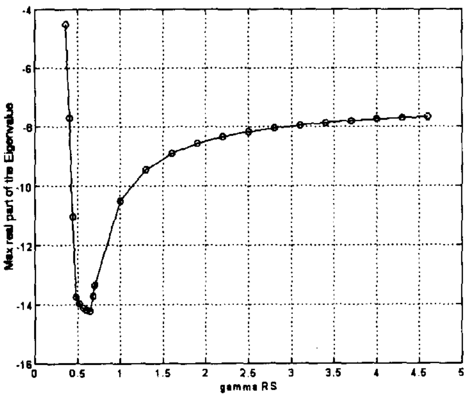

Fig. 15. Stability; full-state-feedback, RS, 3DOF.

Ⅵ. Conclusions

This paper introduced RS control as a generalization of the classical LQG control. Also, it derived the average cost performance of an RS controller in terms of the covariance of the state. As applications, the ECS-1
and a 3DOF structure control were considered. In latter two cases the RMS value of the states were varied by changing the risk-sensitivity parameter, γ_{RS}. Thus, the tradeoffs between fuel, precision of the maneuver, and the stability margin of the system had been shown in the space satellite application. The structural control application is also considered to show the tradeoff between the stability and performance with respect to the risk sensitivity parameter. For the satellite application, the next step may be to apply the RS control method using a real-time satellite simulator [9].

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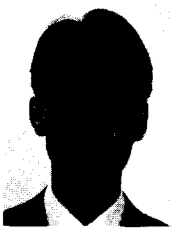

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