Robust H_∞ Controller for State and Input Delayed Systems with Structured Uncertainties

구조화된 불확실성과 상태와 입력에 시간지연이 있는 시스템을 위한 강인 H_∞ 제어기

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요 약 : 본 논문에서는 상태와 입력에 시간지연과 구조화된 불확실성이 있는 시스템을 위한 강인 H_∞ 제어기를 제안한다. 제안된 제어기는 시간지연의 크기에 관계없이 항상 불확실한 시스템을 안정화시키고, 또한 제한된 크기의 어떤 구조화된 불확실성에 대해서도 항상 폐루프 전달함수의 H_∞ 노옴의 크기를 주어진 레벨 이하로 줄인다. 제어기는 볼록 최적화 알고리즘을 이용한 LMI 문제를 풀어서 구한다.

Keywords : robust control, delay system, structured uncertainty, linear matrix inequality

I. Introduction

The last decade has witnessed a significant advances in the H_∞ control theory. However, in case there are parameter uncertainties in the plant model, the stability and the performance of the standard H_∞ control cannot be guaranteed. To deal with both parameter uncertainty and input disturbance, robust H_∞ control has been proposed in recent years[1-4].

For the time-delay systems, there has been considerable research on the H_∞ control for disturbance attenuation[5-7] and the robust stabilization under parametric system uncertainty[8-10]. However, there are only a few publications on the robust H_∞ control for linear systems which are subject to both parameter uncertainty and exogenous disturbance[11-13]. In [11] and [12], robust H_∞ control problem for state delayed systems are addressed and a robust H_∞ controller for the input delayed systems is proposed in [13].

In this paper, a robust H_∞ controller is proposed for state and input delayed linear systems with structured uncertainties. The proposed controller stabilizes the uncertain delay system and reduces the H_∞ norm of the closed loop transfer function to a prescribed level for all bounded structured uncertainties. The proposed controller is a memoryless state feedback controller which can be obtained by solving an LMI problem. The LMI problem in this paper can be reduced to the LMI problems in [8] and [6] as special cases. Several convex optimization algorithms can be applied to solve the proposed LMI problem[14].

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II. Main results

Let us consider a linear system with state and input delays

$$\begin{aligned} \dot{x}(t) &= (A_0+G_0\Delta_{10}H_0)x(t)+\sum_{i=1}^{p}(A_i+G_i\Delta_{1i}H_i)x(t-h_{1i}) \\ &\quad +(B_0+M_0\Delta_{20}N_0)u(t) \\ &\quad +\sum_{k=1}^{q}(B_k+M_k\Delta_{2k}N_k)u(t-h_{2k})+Dw(t) \\ z(t) &= Ex(t) \end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state, $u(t)\in R^m$ is the control, $w(t)\in R^r$ is the disturbance, $z\in R^s$ is the controlled output, and $h_{1i}, h_{2k}>0$ are delays in the system. In addition, $A_i, B_k, G_i, H_i, M_k, N_k, D$ and E are constant matrices with appropriate dimensions. Δ_{1i} and Δ_{2k} are time invariant parameter uncertainties which satisfy the norm bound condition

$$\|\Delta_{1i}\|\le 1, \qquad \|\Delta_{2k}\|\le 1$$

i.e. $\|\Delta\|\le 1$ where

$$\Delta := \begin{bmatrix} \Delta_{10} & 0 & \cdots & & \cdots & 0 \\ 0 & \Delta_{20} & & & & 0 \\ \vdots & & \Delta_{11} & & & \vdots \\ & & & \ddots & & \\ & & & \Delta_{1p} & & \\ \vdots & & & & \Delta_{21} & \vdots \\ & & & & \ddots & \vdots \\ 0 & 0 & \cdots & & \cdots & \Delta_{2q} \end{bmatrix}$$

We shall design a memoryless linear state feedback control

$$u(t)=Fx(t) \tag{2}$$

where $F\in R^{m\times n}$ is a constant matrix. The closed loop transfer function T_{zw} from the disturbance w to the output z is given by

$$T_{zw}(s) := E\Big\{sI-\widehat{A}_0-\sum_{i=1}^{p}\widehat{A}_i e^{-sh_{1i}} - \sum_{k=1}^{q}\widehat{B}_k F e^{-sh_{2k}}\Big\}^{-1} D \tag{3}$$

where $\widehat{A}_0 := A_0+G_0\Delta_{10}H_0+(B_0+M_0\Delta_{20}N_0)F,$

$\widehat{A_i} := A_i + G_i\Delta_{1i}H_i$, and $\widehat{B_k} := B_k + M_k\Delta_{2k}N_k$ for all $i,\ k$.

Our aim is to find the robust H_∞ controller which stabilizes the delayed system (1) and guarantees the H_∞ norm bound γ of the transfer function T_{zw}, namely, $\| T_{zw} \|_\infty < \gamma$ for all uncertainties $\| \Delta \| \leq 1$. The following lemma gives a sufficient condition for the robust stability of the closed loop system with the control (2).

Lemma 1 : If there exist positive definite matrices P_0, $\widetilde{P_{1i}}$, $\widetilde{P_{2k}}$ and positive constants μ_{1i}, μ_{2k} which satisfy the matrix inequality

$$\begin{aligned}(A_0+B_0F)'P_0 + P_0(A_0+B_0F) + \sum_{i=0}^{p}\mu_{1i}P_0G_iG_i'P_0 \\ + \sum_{k=0}^{q}\mu_{2k}P_0M_kM_k'P_0 + \sum_{i=0}^{p}\frac{1}{\mu_{1i}}H_i'H_i \\ + \sum_{k=0}^{q}\frac{1}{\mu_{2k}}F'N_k'N_kF + \sum_{i=1}^{p}P_0A_i\widetilde{P_{1i}}^{-1}A_i'P_0 \\ + \sum_{k=1}^{q}P_0B_kF\widetilde{P_{2k}}^{-1}F'B_k'P_0 + \sum_{i=1}^{p}\widetilde{P_{1i}} + \sum_{k=1}^{q}\widetilde{P_{2k}} < 0\end{aligned} \tag{4}$$

then the delayed system (1) with the control (2) is stable for all $\| \Delta \| \leq 1$ and $h_{1i}, h_{2k} > 0$.

Proof : Let's assume that $w(t)$ is equal to zero for all t. Define a Lyapunov functional $V(x_t)$ as follows:

$$\begin{aligned}V(x_t) := & \ x'(t)P_0x(t) + \sum_{i=1}^{p}\int_{t-h_{1i}}^{t}x'(s)P_{1i}x(s)ds \\ & + \sum_{k=1}^{q}\int_{t-h_{2k}}^{t}x'(\tau)P_{2k}x(\tau)d\tau.\end{aligned}$$

$$\frac{dV(x_t)}{dt} = y'(t)\widehat{W}y(t) \tag{5}$$

where

$$\begin{aligned}y(t) = [& x(t)\ x(t-h_{11}) \cdots x(t-h_{1p}) \\ & x(t-h_{21}) \cdots x(t-h_{2q})]'\end{aligned}$$

and

$$\widehat{W} := \begin{bmatrix} \hat{Z} & P_0\widehat{A_1} & \cdots & P_0\widehat{A_p} & P_0\widehat{B_1}F & \cdots & P_0\widehat{B_q}F \\ \widehat{A_1}'P_0 & -P_{11} & & & \cdots & & 0 \\ \vdots & & \ddots & & & & \\ \widehat{A_p}'P_0 & & & -P_{1p} & & & \\ F'\widehat{B_1}'P_0 & \vdots & & & -P_{21} & & \vdots \\ \vdots & & & & & \ddots & \\ F'\widehat{B_q}'P_0 & 0 & & & \cdots & & -P_{2q} \end{bmatrix}$$

The submatrix $\hat{Z}$ in the above matrix $\widehat{W}$ is defined by

$$\hat{Z} := \widehat{A_0}'P_0 + P_0\widehat{A_0} + \sum_{i=1}^{p}P_{1i} + \sum_{k=1}^{q}P_{2k}.$$

The matrix $\widehat{W}$ can be denoted by $\widehat{W} = W + \Phi\Delta\Psi + \Psi'\Delta'\Phi'$ where

$$W := \begin{bmatrix} Z & P_0A_1 & \cdots & P_0A_p & P_0B_{1F} & \cdots & P_0B_qF \\ A_1'P_0 & -P_{11} & & & \cdots & & 0 \\ \vdots & & \ddots & & & & \\ A_p'P_0 & & & -P_{1p} & & & \\ F'B_1'P_0 & \vdots & & & -P_{21} & & \vdots \\ \vdots & & & & & \ddots & \\ F'B_q'P_0 & 0 & & & \cdots & & -P_{2q} \end{bmatrix}$$

$$\Phi := \begin{bmatrix} P_0G_0 & P_0M_0 & P_0G_1 & \cdots & P_0G_p & P_0M_1 & \cdots & P_0M_q \\ 0 & 0 & & \cdots & & & & 0 \\ \vdots & & & & & & & \vdots \\ 0 & 0 & & \cdots & & & & 0 \end{bmatrix}$$

and

$$\Psi := \begin{bmatrix} H_0 & 0 & \cdots & & & & 0 \\ N_0F & 0 & & & & & 0 \\ 0 & H_1 & & & & & \\ \vdots & & \ddots & & & & \vdots \\ & & & H_p & & & \\ & & & & N_1F & & \\ & & & & & \ddots & \\ 0 & 0 & \cdots & & & & N_qF \end{bmatrix}$$

The submatrix Z in the matrix W is defined by

$$\begin{aligned}Z := & \ (A_0+B_0F)'P_0 + P_0(A_0+B_0F) \\ & + \sum_{i=1}^{p}P_{1i} + \sum_{k=1}^{q}P_{2k}.\end{aligned}$$

Let's assume that μ_{1i} and μ_{2k} are positive constants for all $i = 0, \cdots, p$ and $k = 0, \cdots, q$. Define a positive definite matrix M as follows

$$\begin{aligned}M := & \text{ block diag } \{\mu_{10}I_{10}, \mu_{20}I_{20}, \mu_{11}I_{11}, \cdots, \\ & \mu_{1p}I_{1p}, \mu_{21}I_{21}, \cdots, \mu_{2q}I_{2q}\}.\end{aligned}$$

The inequality

$$\Phi\Delta\Psi + \Psi'\Delta'\Phi' \leq \Phi M\Phi' + \Psi' M^{-1}\Psi$$

is then satisfied for all $\| \Delta \| \leq 1$ and positive constants μ_{1i} and μ_{2k}. Hence, we obtain the inequality

$$\widehat{W} \leq \begin{bmatrix} \check{Z} & P_0A_1 & \cdots & P_0A_p & P_0B_1F & \cdots & P_0B_qF \\ A_1'P_0 & -\widetilde{P_{11}} & & & \cdots & & 0 \\ \vdots & & \ddots & & & & \\ A_p'P_0 & & & -\widetilde{P_{1p}} & & & \\ F'B_1'P_0 & \vdots & & & -\widetilde{P_{21}} & & \vdots \\ \vdots & & & & & \ddots & \\ F'B_q'P_0 & 0 & & & \cdots & & -\widetilde{P_{2q}} \end{bmatrix}$$

where

$$\begin{aligned}\check{Z} := & \ (A_0+B_0F)'P_0 + P_0(A_0+B_0F) + \sum_{i=0}^{p}\mu_{1i}P_0G_iG_i'P_0 \\ & + \sum_{k=0}^{q}\mu_{2k}P_0M_kM_k'P_0 + \sum_{i=0}^{p}\frac{1}{\mu_{1i}}H_i'H_i \\ & + \sum_{k=0}^{q}\frac{1}{\mu_{2k}}F'N_k'N_kF + \sum_{i=1}^{p}\widetilde{P_{1i}} + \sum_{k=1}^{q}\widetilde{P_{2k}}\end{aligned}$$

and

$$\widetilde{P_{1i}} := P_{1i} - \frac{1}{\mu_{1i}} H_i' H_i$$
$$\widetilde{P_{2k}} := P_{2k} - \frac{1}{\mu_{2k}} F' N_k' N_k F.$$

Hence, if there exist positive definite matrices $\widetilde{P_{1i}}$, $\widetilde{P_{2k}}$ and positive constants μ_{1i}, μ_{2k} which satisfy the inequality (4), then $\widehat{W}$ is negative definite [15] and the Lyapunov derivative (5) is negative definite. ■

The Riccati inequality (4) can be represented by an LMI as the following lemma.

Lemma 2 : The Riccati inequality (4) is equivalent to the LMI

$$\begin{bmatrix} R & A_1Q_0 & \cdots & A_pQ_0 & B_1Y & \cdots & B_qY & Q_0H_0' & \cdots & Q_pH_p' & Y'N_0 & \cdots & Y'N_q \\ Q_0A_1' & -Q_{11} & & & & \cdots & & & & & & & 0 \\ \vdots & & \ddots & & & & & & & & & & \\ Q_0A_p' & & & -Q_{1p} & & & & & & & & & \\ Y'B_1' & \vdots & & & -Q_{21} & & & & & & & & \\ \vdots & & & & & \ddots & & & & & & & \\ Y'B_q' & & & & & & -Q_{2q} & & & & & & \vdots \\ H_0Q_0 & & & & & & & -\mu_{10}I_{10} & & & & & \\ \vdots & & & & & & & & \ddots & & & & \\ H_pQ_0 & & & & & & & & & -\mu_{1p}I_{1p} & & & \\ N_0Y & & & & & & & & & & -\mu_{20}I_{20} & & \\ \vdots & & & & & & & & & & & \ddots & \\ N_qY & 0 & & & & \cdots & & & & & & & -\mu_{2q}I_{2q} \end{bmatrix} < 0 \qquad (6)$$

where $Q_0 = P_0^{-1}$, $Q_{1i} = P_0^{-1}\widetilde{P_{1i}}P_0^{-1}$, $Q_{2k} = P_0^{-1}\widetilde{P_{2k}}P_0^{-1}$, $Y = FP_0^{-1}$

and

$$R = A_0Q_0 + Q_0A_0' + B_0Y + Y'B_0' + \sum_{i=1}^{p} Q_{1i} + \sum_{k=1}^{q} Q_{2k} + \sum_{i=0}^{p} \mu_{1i}G_iG_i' + \sum_{k=0}^{q} \mu_{2k}M_kM_k'.$$

The following theorem provides a delay independent robust stabilizer for the state and input delayed system (1).

Theorem 1 : If there exist Y, $Q_0 > 0$, $Q_{1i} > 0$, $Q_{2k} > 0$ and positive constants μ_{1i}, μ_{2k} which satisfy the LMI (6), then the state feedback control

$$u(t) = YQ_0^{-1}x(t) \qquad (7)$$

stabilizes the system (1) for all $\|\Delta\| \le 1$ and $h_{1i}, h_{2k} > 0$.

Proof : If there exist Y, Q_0, Q_{1i}, Q_{2k} and positive constants μ_{1i}, μ_{2k} which satisfy the LMI (6), then the state feedback control gain F is equal to YQ_0^{-1} as in the lemma 2. Hence we obtain the theorem. ■

We have obtained the robust stability conditions and the robust controller for the state and input delayed system (1). The following theorem provides a robust H_∞ controller which guarantees the H_∞ norm bound gamma of the closed loop transfer function of the system (1) for all bounded structured uncertainties.

Theorem 2 : If there exist Y, $Q_0 > 0$, $Q_{1i} > 0$, $Q_{2k} > 0$ and positive constants γ, μ_{1i}, μ_{2k} which satisfy the LMI

$$\begin{bmatrix} R & A_1Q_0 & \cdots & A_pQ_0 & B_1Y & \cdots & B_qY & Q_0H'_0 & \cdots & Q_pH'_p & Y'N_0 & \cdots & Y'N_q & D & Q_0E' \\ Q_0A'_1 & -Q_{11} & & & & & & & & & & & & & \\ \vdots & & \ddots & & & & & & & & & & & & \\ Q_0A'_p & & & -Q_{1p} & & & & & & & & & & & \\ Y'B'_1 & & & & -Q_{21} & & & & & & & & & & \\ \vdots & & & & & \ddots & & & & & & & & & \\ Y'B'_q & & & & & & -Q_{2q} & & & & & & & & \\ H_0Q_0 & & & & & & & -\mu_{10}I_{10} & & & & & & & \\ \vdots & & & & & & & & \ddots & & & & & & \\ H_pQ_0 & & & & & & & & & -\mu_{1p}I_{1p} & & & & & \\ N_0Y & & & & & & & & & & -\mu_{20}I_{20} & & & & \\ \vdots & & & & & & & & & & & \ddots & & & \\ N_qY & & & & & & & & & & & & -\mu_{2q}I_{2q} & & \\ D' & & & & & & & & & & & & & -\gamma I & \\ EQ_0 & & & & & & & & & & & & & & -\gamma I \end{bmatrix} < 0 \qquad (8)$$

then the state feedback control

$$u(t) = YQ_0^{-1}x(t)$$

stabilizes the system (1) and guarantees the H_∞ norm bound of the closed loop transfer function T_{zw} in (3), i.e. $\|T_{zw}\|_\infty < \gamma$ for all uncertainties $\|\Delta\| \le 1$.

Proof : If Y, $Q_0 > 0$, $Q_{1i} > 0$ and $Q_{2k} > 0$ are the solutions of the LMI (8), then the solutions also satisfy the LMI (6). Hence from the theorem 1, the state feedback control (7) stabilizes the state and input delayed system (1) for all norm bounded uncertainties.

Using the relations, $P_0 = Q_0^{-1}$, $\widetilde{P_{1i}} = Q_0^{-1}Q_{1i}Q_0^{-1}$, $\widetilde{P_{2k}} = Q_0^{-1}Q_{2k}Q_0^{-1}$ and $F = YQ_0^{-1}$, it can be shown that the LMI (8) is equal to the following matrix inequality

$$W + \Phi M \Phi' + \Psi' M^{-1} \Psi + \begin{bmatrix} \frac{1}{\gamma}P_0DD'P_0 + \frac{1}{\gamma}E'E & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} < 0$$

where W, Φ, Ψ and M are defined as in the proof of lemma 1. Since the inequality

$$\Phi \Delta \Psi + \Psi' \Delta' \Phi' \le \Phi M^{-1} \Phi' + \Psi' M \Psi$$

is satisfied for all $\|\Delta\| \le 1$, we obtain the matrix inequality

$$\widehat{W} + \begin{bmatrix} \frac{1}{\gamma}P_0DD'P_0 + \frac{1}{\gamma}E'E & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} < 0$$

where $\widehat{W}$ is defined as in the proof of lemma 1. Therefore, the following inequality

$$P_0\widehat{A}_0 + \widehat{A}_0'P_0 + \sum_{i=1}^{p} P_{1i} + \sum_{k=1}^{q} P_{2k} + \sum_{i=1}^{p} P_0\widehat{A}_iP_{1i}^{-1}\widehat{A}_i'P_0 + \sum_{k=1}^{q} P_0\widehat{B}_kFP_{2k}^{-1}F'\widehat{B}_k'P_0 + \frac{1}{\gamma}P_0DD'P_0 + \frac{1}{\gamma}E'E < 0$$

is satisfied for all uncertainty $\|\Delta\| \le 1$. Let's define a positive definite matrix S as follows:

$$S := -(P_0\widehat{A}_0 + \widehat{A}_0'P_0 + \sum_{i=1}^{p} P_{1i} + \sum_{k=1}^{q} P_{2k} + \sum_{i=1}^{p} P_0\widehat{A}_iP_{1i}^{-1}\widehat{A}_i'P_0 + \sum_{k=1}^{q} P_0\widehat{B}_kFP_{2k}^{-1}F'\widehat{B}_k'P_0 + \frac{1}{\gamma}P_0DD'P_0 + \frac{1}{\gamma}E'E).$$

Then we have the equation

$$P_0\widehat{A}_0 + \widehat{A}_0'P_0 + \sum_{i=1}^{p} P_{1i} + \sum_{k=1}^{q} P_{2k} + \sum_{i=1}^{p} P_0\widehat{A}_iP_{1i}^{-1}\widehat{A}_i'P_0 + \sum_{k=1}^{q} P_0\widehat{B}_kFP_{2k}^{-1}F'\widehat{B}_k'P_0 + \frac{1}{\gamma}P_0DD'P_0 + \frac{1}{\gamma}E'E + S = 0$$

and

$$(-jwI-\widehat{A}_0'-\sum_{i=1}^{p}\widehat{A}_i'e^{jwh_{1i}}-\sum_{k=1}^{q}F'\widehat{B}_k'e^{jwh_{2k}})P_0+$$
$$P_0(jwI-\widehat{A}_0-\sum_{i=1}^{p}\widehat{A}_ie^{-jwh_{1i}}-\sum_{k=1}^{q}\widehat{B}_kFe^{-jwh_{2k}})-S-$$
$$\frac{1}{\gamma}P_0DD'P_0-\frac{1}{\gamma}E'E = \sum_{i=1}^{p}W_i(jw)+\sum_{k=1}^{q}V_k(jw)$$

where

$$\begin{aligned}W_i(jw) :=& P_{1i}+P_0\widehat{A}_iP_{1i}^{-1}\widehat{A}_i'P_0 \\ &-\widehat{A}_i'P_0e^{jwh_{1i}}-P_0\widehat{A}_ie^{-jwh_{1i}} \\ =& [P_0\widehat{A}_ie^{-jwh_{1i}}-P_{1i}]P_{1i}^{-1}[\widehat{A}_i'P_0e^{jwh_{1i}}-P_{1i}]\end{aligned}$$

and

$$\begin{aligned}V_k(jw) :=& P_{2k}+P_0\widehat{B}_kFP_{2k}^{-1}F'\widehat{B}_k'P_0 \\ &-F'\widehat{B}_k'P_0e^{jwh_{2k}}-P_0\widehat{B}_kFe^{-jwh_{2k}} \\ =& [P_0\widehat{B}_kFe^{-jwh_{2k}}-P_{2k}]P_{2k}^{-1}[F'\widehat{B}_k'P_0e^{jwh_{2k}}-P_{2k}].\end{aligned}$$

From the definition, $W_i(jw)$ and $V_k(jw)$ are nonnegative definite for all $i=1,\cdots,p$, $k=1,\cdots,q$. Let's define a matrix

$$X(jw) := (jwI-\widehat{A}_0-\sum_{i=1}^{p}\widehat{A}_ie^{-jwh_{1i}}-\sum_{k=1}^{q}\widehat{B}_kFe^{-jwh_{2k}})^{-1}.$$

Then the equation (9) can be rewritten as

$$X'(-jw)^{-1}P_0+P_0X(jw)^{-1}-\sum_{i=1}^{p}W_i(jw)-\sum_{k=1}^{q}V_k(jw)$$
$$-S-\frac{1}{\gamma}P_0DD'P_0-\frac{1}{\gamma}E'E=0$$

Hence, following the similar procedure as in the proof of the theorem 1 in [5], we can show that γ is the H_∞ norm bound of the closed loop transfer function T_{zw} (3). ■

The theorem 2 provides an LMI problem to obtain a robust H_∞ controller which guarantees the H_∞ norm bound γ for all bounded uncertainties. In order to find the robust H_∞ controller which provides the smallest possible H_∞ norm bound, we must solve the minimization problem for γ subject to conditions $Q_0>0$, $Q_{1i}>0$, $Q_{2k}>0$, $\mu_{1i}>0$, $\mu_{2k}>0$ and the LMI (8). It is also noted that the LMI (8) is reduced to the LMI (6) when the H_∞ norm bound γ approaches infinity.

Ⅲ. Conclusions

In this paper, a robust H_∞ controller for state and input delayed systems with structured uncertainties ispresented. The proposed robust H_∞ controller does not only stabilize the state and input delayed system, but also guarantees the H_∞ norm bound for all uncertainties $\|\Delta\|\le 1$. The proposed controller is a memoryless state feedback controller which can be obtained by solving the LMI (8) with convex optimization algorithms.

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