Output Feedback Control for Robot Manipulator Using Variable Structure Control

위치만을 이용한 가변 구조 제어 방법에 의한 로봇 동작부 제어기 설계

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요 약 : 모델 불확실성이 있고 n 축 자유도(degree of freedom)를 갖고 있는 로봇 동작부(manipulator)에 대해서 위치정보만을 이용하여 가변 구조 제어기(variable structure controller)를 설계하였다. 모델의 불확실성이 존재하는 경우에도 제어기에 사용되는 속도를 잘 예측하기 위해 고이득 관찰기를 사용 하였으며 고이득 관찰기를 사용할 때 발생할 수 있는 상태변수의 피킹현상(peak phenomenon)를 적게 하게 하기위하여 제어기의 값을 제한(globally bounded)하여 제어기를 설계하였다. 부하(payload)의 범위만 알고 있는 2 축 자유도를 갖는 로봇 동작부에 대해서 제안한 제어 방법에 따라 제어기를 설계하여 그 성능를 확인 하였다.

Keywords : variable structure control, output feedback, globally bounded control, high-gain observer

I. Introduction

Robust state feedback control schemes for robot manipulators have been developed to overcome modeling uncertainties and/or disturbances [1]. A number of these schemes use variable structure control(VSC) to track the desired path for robot manipulators. Most of the work on VSC assume that measurements of positions and velocities are available for feedback [1]. However, the velocity, obtained by tachometers, is easily contaminated by measurement noise[2]. One way to overcome this problem is to estimate velocity from position measurement using observers [3],[4],[5],[6],[12]. The papers [4],[6] used a discontinuous sliding observer with a continuous control scheme, while [3],[5],[12] used a continuous observer with a continuous control scheme. The paper [18] used a continuous observer with a VSC scheme. A common theme of observer design is the use of high gain observers which reject disturbance due to modeling uncertainty and imperfect feedback cancellation of nonlinearities. The VSC used in [18] is similar to the one described in this paper, but [18] used a circular argument in the design of the observer. In fact, [18] required the observer gain to be greater than a function of the estimates of the angular velocities, which itself is dependent on the observer gain. The paper [13], motivated by [8], that the specially scaled observer with globally bounded VSC can reject disturbances due to modeling uncertainty and imperfect feedback cancellation of nonlinearity. Motivated by the study of a 2 DOF manipulator control problem, the controller design is slightly different than the design of [13]. Uncertainty on the input coefficient matrix differs from that of the paper [13], so is the design of controller. We show, via an example, that the current controller is less conservative than that of [13]. To illustrate the performance of the controller, we consider the tracking control of a 2 DOF manipulator with unknown payload, but with known payload range, and design the discontinuous controller. To reduce chattering, a continuous approximate controller replaces the discontinuous one. The continuous approximation reduces chattering, but results in an increase in tracking error.

II. Manipulator dynamic model and problem statement

Consider the equation describing the dynamics of an n DOF rigid robot manipulator [16]

$$H(\theta)\ddot{\theta}+N(\theta,\dot{\theta})\dot{\theta}+g(\theta)=\tau \tag{1}$$

where $\theta\in R^n$ is a vector of generalized coordinates (joint positions), $H(\theta)\in R^{n\times n}$ is the positive definite inertia matrix, $N(\theta,\dot{\theta})\dot{\theta}\in R^n$ is a vector of Coriolis and centripetal torques, $g(\theta)\in R^n$ is a vector of gravitational torque, τ is a vector of applied joint torques. The manipulator model (1) has the following inherent properties [16], which are useful in control design. For all $\theta\in R^n$,

(i) $H(\theta)=H(\theta)^T>0$, (ii) $\|H(\theta)\|\le q_h$,

(iii) $\|N(\theta,\dot{\theta})\dot{\theta}\|\le q_c\|\dot{\theta}\|^2$, (iv) $\|g(\theta)\|\le q_g$

where q_h, q_c, and q_g are some positive constants. Property (i) implies that $H^{-1}(\theta)$ always exists, and property (ii) implies that the elements of the inertia matrix $H(\theta)$ are bounded. Property (iii) means that the Coriolis and centripetal torques are quadratic in the velocity $\dot{\theta}$. Property (iv) means that the gravitational torques are bounded. Let the desired path of θ be θ_d, and $(\theta_d, \dot{\theta}_d, \ddot{\theta}_d)\in\Theta_D$ assume that $(\theta_d, \dot{\theta}_d)\in\Theta_a$ and where Θ_a and Θ_D are compact subsets of R^{2n} and R^{3n} respectively. The objective is to design the applied torque τ using only joint position θ such that joint position θ tracks the desired path θ_d when the payload is unknown, but its range is known. We use VSC to design the torque τ.

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III. Controller design

This section is devided into three(3) subsections. The first subsection describes a design method of hign-gain observer and the property of a globally bounded controller. We state the result through Lemma 1 in which errors between state variables and their estimates can be make arbitrary small while a high-gain observer rejects disturbances during a short time period. In the second subsection, we illustrate a design method of a globally bounded controller and show that the controller can satisfy the sliding mode condition after errors between state variables and their estimates become small enough. Using a sliding mode condition, we estabilish that the tracking errors can be make arbitrary small in the third subsection.

1. A high-gain observer design and the property of gloablly bounded controller

Introducing state variables in error coordinates, $e_1^i=\theta_i-\theta_{di}$, $e_2^i=\dot\theta_i-\dot\theta_{di}$, for $i=1,\ldots,n$, and setting $e_1=[e_1^1,e_1^2,\ldots,e_1^{n-1},e_1^n]^T$, $e_2=[e_2^1,e_2^2,\ldots,e_2^{n-1},e_2^n]^T$, the dynamic equation (1) has the following state-space representation

$$\dot e_1^i = e_2^i$$
$$\dot e_2^i = -\ddot\theta_{di}+F^i(e_1,e_2,\theta_d,\dot\theta_d)+\sum_{j=1}^{n}G_j^i(e_1,\theta_d)\tau_j,\quad 1\le i\le n$$

where $F^i(e,\theta_d,\dot\theta_d)$ denotes the ith component of the vector $H^{-1}(e_1+\theta_d)\{-C(e_1+\theta_d,e_2+\dot\theta_d)(e_2+\dot\theta_d)-g(e_1+\theta_d)\}$ and $G_j^i(e_1,\theta_d)$ denotes the ith row and jth column entry of the matrix $H^{-1}(e_1+\theta_d)$. Notice that from property (i), the matrix $G(e_1,\theta_d)=\{G_j^i(e_1+\theta_d)\}$ exists for $\forall e_1+\theta_d\in R^n$. After setting $e=[e_1^1,e_2^1,\ldots,e_1^n,e_2^n]^T$, we rewrite the state equations in the compact form

$$\dot e = Ae+B[-\ddot\theta_d+F(e,\theta_d,\dot\theta_d)+G(e_1,\theta_d)\tau] \tag{2}$$

where $A=block\ diag[A_1,\cdots,A_n]$, $A_i=\begin{bmatrix}0&1\\0&0\end{bmatrix}$

$$B=block\ diag[B_1,\ldots,B_n],\ B_i=\begin{bmatrix}0\\1\end{bmatrix},F=\begin{bmatrix}F^1\\ \vdots\\ F^n\end{bmatrix}$$

$C=block\ diag[C_1,\ldots,C_n]$, $C_i=[1\ \ 0]$. Let $F_0(\cdot)$ and $G_0(\cdot)$ be the known nominal models of $F(\cdot)$ and $G(\cdot)$, respectively. From property (i), we see that $G(\cdot)$ is nonsingular $\forall(e_1+\theta_d)\in R^n$; moreover, from property (ii), $G(\cdot)$ is globally bounded. We choose $G_0(\cdot)$ to have the same properties. We assume that $F_0(\cdot)$ is globally bounded. This can be always achieved by saturating the given nominal functions outside a bounded domain of interest, as it will be illustrated later on. Let N be an $n\times n$ matrix such that each component of N is an upper bound on the absolute value of the corresponding component of the matrix $[G(\theta)G_0^{-1}(\theta)-I]$, $\forall\theta\in R^n$. Notice that since $G(\cdot)$ and $G_0^{-1}(\cdot)$ are globally bounded, the matrix N is well defined.

Assumption 1 : *The matrix [I-N] is an M-matrix.*

Note that Definition of M-matrix can be found in [14]. Assumption 1 restricts uncertainty on the input coefficient matrix. To estimate the derivative of the tracking error, $e_2=\theta-\dot\theta_a$, we construct the observer

$$\dot{\hat e}_1^i = \hat e_2^i+(\alpha_1^i/\varepsilon)(e_1^i-\hat e_1^i) \tag{3}$$

$$\dot{\hat e}_2^i = -\ddot\theta_{di}+(\alpha_2^i/\varepsilon^2)((e_1^i-\hat e_1^i)+F_0^i(\cdot)+\sum_{j=1}^{n}G_{0j}^i(\cdot)\tau_j, \tag{4}$$

for $i=1,\cdots,n$, where all α_j^i are positive constants, $\hat e_j^i$ is the estimate of the state variable e_j^i, and ε is a positive constant to be specified. We rewrite the observer equation (3)-(4) in the compact form

$$\dot{\hat e}=A\hat e+B[-\ddot\theta_d+F_0(\cdot)+G_0(\cdot)\tau)]+D(\varepsilon)LC(e-\hat e) \tag{5}$$

where

$L=block\ diag[L_1,\ldots,L_n]$, $L_i=[\alpha_1^i\ \alpha_2^i]^T$ $D(\varepsilon)=block\ diag[D_1(\varepsilon),\ldots,D_n(\varepsilon)]$, $D_i(\varepsilon)=diag[1/\varepsilon\ \ 1/\varepsilon^2]$ $C=block\ diag[C_1,\ldots,C_n]$, $C_i=[1\ \ 0]$. Let $\xi_j^i=e_j^i-\hat e_j^i$ be the estimation error, and define the scaled variables $\zeta_j^i=(1/\varepsilon^{2-j})\xi_j^i$, for $j=1,2$. The closed-loop equation can be rewritten as

$$\dot e = Ae+B[-\ddot\theta_d+F(e,\theta_d,\dot\theta_d)+G(e_1,\theta_d)\tau] \tag{6}$$

$$\begin{aligned}\varepsilon\dot\zeta = {} & (A-LC)\zeta+\varepsilon B[F(e,\theta_d,\dot\theta_d)-F_0(\hat e,\theta_d,\dot\theta_d)\\ & +(G(e_1,\theta_d)-G_0(\hat e_1,\theta_d))\tau]\end{aligned} \tag{7}$$

where $\zeta=[\zeta_1^1,\zeta_2^1,\ldots,\zeta_1^n,\zeta_2^n]^T$ We choose the sliding surface $\sigma(\hat e)=[\sigma_1(\hat e),\ldots,\sigma_n(\hat e)]^T$ such that $\sigma_i(\hat e)=\hat e_2^i+m_1^i\hat e_1^i$, $1\le i\le n$, where m_1^i are positive constants. Rewrite $\sigma(\hat e)$ as

$$\sigma(\hat e)=M\hat e \tag{8}$$

where $M=block\ diag[M_1,\ldots,M_n]$, $M_i=[m_1^i\ 1]$. Define $\bar\zeta=[\zeta_1^1,\ldots,\zeta_1^n]^T$, and rewrite equation (3) as

$$\dot{\hat e}_1=\bar A\hat e_1+\sigma(\hat e)+\hat L\bar\zeta \tag{9}$$

where $\bar A=diag[-m_1^1,\ldots,-m_1^n]$, $\hat L=diag[\alpha_1^1,\ldots,\alpha_1^n]$. Consider a control input of the form $\tau_j=\varphi_j(\hat e,\theta_d,\dot\theta_d,\ddot\theta_d)+\upsilon_j(\hat e,\theta_d,\dot\theta_d)sgn(\sigma_j(\hat e))$ where $\varphi_j(\cdot)$and $\upsilon_j(\cdot)$ are continuous and globally bounded functions. We will specify τ, later on. Let $P=diag[1/(2m_1^1),\ldots,1/(2m_1^n)]$, and take $V(e_1)=e_1^TPe_1$. Notice that P is a positive definite matrix since all m_1^i are positive constants. Choose the positive constants c_{sr} and r such that $\Omega_r\equiv\{e\in R^{2n}\mid \|Me\|\le c_{sr},\sqrt{V(e_1)}\le c_{zr}\}$ where $c_{zr}=r(a_2/a_1)c_{sr}$, $a_1=1/\lambda_{\max}(P)$, $a_2=2\|P\|/\sqrt{\lambda_{\min}(P)}$, and $r>1$. The set Ω_r is taken as the region of interest in our analysis. Define

$\Omega_s\equiv\{e\in R^{2n}\mid \|Me\|\le c_{ss},\sqrt{V(e_1)}\le c_{zs}\}$ where $c_{ss}>c_{sr}$, and $c_{zs}>c_{zr}$. Achieving globally bounded functions $F_0(e,\theta_d,\dot\theta_d)$ can be done by saturating $F_0(\cdot)$ outside the set $\Omega_s\times\Theta_a$. Let

$$\Omega_0 = \{(e\in R^{2n}\mid \|Me\|\le c_{so},\ \sqrt{V(e_1)}\le c_{zo}\},$$
$$\Omega_1 = \{\zeta\in R^{2n}\mid \|\zeta\|<c/\varepsilon\}$$
$$\Omega = \Omega_0\times\Omega_1 \tag{10}$$

where $c_{so}<c_{sr}$, $c_{zo}<c_{zr}$ and c is an arbitrary positive constant. Notice that $\Omega_0\subset\Omega_r$. The following lemma states that the fast variables decays very rapidly during a shorttime period. The proof of the lemma is the same as the proof of Lemma 1 in [13], hence it is omitted.

Lemma 1 : Consider the singularly perturbed system (6)-(7) and suppose that the torque τ is globally bounded, Then, for all $(e(0),\zeta(0))\in\Omega$, there exist ε_1 and $T_1=T_1(\varepsilon)\le T_3$ such that for all $0<\varepsilon<\varepsilon_1$, $\|\zeta\|<k\varepsilon$ for all $t\in[T_1,T_4)$ where T_3 is a finite time and $T_4>T_3$ is the first time e(t) exits from the set Ω_r.

Remark 1 : From (6)-(7), one can observe that *e(t)* is the slow variable since we use the globally bounded control input τ while ζ is the fast one. Lemma 1 implies that the fast variable ζ becomes $O(\varepsilon)$ before the slow variable $e(t)$ exits the domain of interest. Lemma 1 can be established due to the use of a specially scaled observer and a globally bounded control.

2. Sliding mode condition and a design method of globally bounded controller

We design the control input τ such that a sliding mode condition is satisfied when $\|\zeta\| < \bar{k}\varepsilon$ and $(e, \hat{e}) \in \Omega_r \times \Omega_r$. This will be done by showing that $\sigma(\hat{e})^T \dot{\sigma}(\hat{e}) < 0$ as long as $\sigma(\hat{e}) \neq 0$. We will show $\sigma(\hat{e})^T \dot{\sigma}(\hat{e}) < 0$ using control input τ different from the control input in [13]. This is motivated by studying a 2 DOF manipulator control problem which is studied in the Example section. We have

$$\begin{aligned}\sigma^T(\hat{e})\dot{\sigma}(\hat{e}) &= \sigma^T(\partial\sigma/\partial\hat{e})\dot{\hat{e}} \\ &= \sigma^T M[A\hat{e} + B(-\ddot{\theta}_d + F_0(\hat{e}, \theta_d, \dot{\theta}_d) \\ &\quad + G_0(\hat{e}_1, \theta_d)\tau) + (1/\varepsilon)\tilde{D}(\varepsilon)LC\zeta]\end{aligned} \tag{11}$$

where $\tilde{D}(\varepsilon) = block\ diag[\ \tilde{d}_1(\varepsilon), \ldots, \tilde{d}_n(\varepsilon)]$, $\tilde{d}_i(\varepsilon) = diag[\varepsilon, 1]$. For simplicity, we set $\phi_1(\hat{e}, \theta_d, \dot{\theta}_d) \equiv F(\hat{e}, \theta_d, \dot{\theta}_d) - F_0(\hat{e}, \theta_d, \dot{\theta}_d)$ and let $\eta = \zeta + \varepsilon(A - LC)^{-1}B\phi_1(\hat{e}, \theta_d, \dot{\theta}_d)$. Using the fact that $\tilde{D}(0)LC(A - LC)^{-1}B = -B$, it can be shown that equation (11) is given by

$$\begin{aligned}\sigma^T(\hat{e})\dot{\sigma}(\hat{e}) &= \sigma^T M[A\hat{e} + B(-\ddot{\theta}_d + F_0(\cdot) + G_0(\cdot)\tau) \\ &\quad + (1/\varepsilon)\tilde{D}(\varepsilon)LC\eta + B\phi_1(\cdot) + O(\varepsilon)]\end{aligned} \tag{12}$$

We need an estimate of $(1/\varepsilon)M\tilde{D}(\varepsilon)LC\eta$ to design the control input τ such that the sliding mode condition is satisfied. It can be also shown that

$$\begin{aligned}(1/\varepsilon)M\tilde{D}(\varepsilon)LC\eta &= (1/\varepsilon)M\tilde{D}(0)L\int_{T_1}^{t} Ce^{(A-LC)(t-l)/\varepsilon}B \\ &\quad \times \{G(\cdot)G_0^{-1}(\cdot) - I\} \times G_0(\hat{e}_1, \theta_d)v(l)dl + O(\varepsilon)\end{aligned}$$

for $t \in [T_1 + \varepsilon\ln(1/\varepsilon), T_4)$ where $v(t) \in K\{\tau(t)\}$ for almost all t and the convex hull $K\{\tau(t)\}$ is defined in [9]. Define $k_{ti} = \alpha_2^i \int_0^\infty |h_{ii}(t)|dt \geq 1$ where $h_{ii}(t)$ is the ith diagonal element of diagonal matrix $Ce^{(A-LC)t}B$, for $i = 1, \ldots, n$. Let k_{ui} be an upper bound of the absolute value of ith component for vector $G_0(\cdot)\tau$, to be specified. It can be verified that

$$\left|\left[(1/\varepsilon)M\tilde{D}(0)L\int_{T_1}^{t} Ce^{(A-LC)(t-l)/\varepsilon}B\ \{G(\cdot)G_0^{-1}(\cdot) - I\}G_0(\cdot)v(l)dl\right]_i\right| \leq [K_t N k_u]_i \tag{13}$$

where $[\cdot]_i$ denotes the ith component of a vector, $K_t = diag[k_{t1}, \ldots, k_{tn}]$, and $k_u = [k_{u1}, \ldots, k_{un}]^T$. Choose the observer gain α_j^i such that all eigenvalues of $(A-LC)$ are real and negative. Then all $k_{ti} = 1$ [10]. Hence inequality (13) becomes

$$\left|\left[(1/\varepsilon)M\tilde{D}(0)L\int_{T_1}^{t} Ce^{(A-LC)(t-l)/\varepsilon}B\ \{G(\cdot)G_0^{-1}(\cdot) - I\}G_0(\cdot)v(l)dl\right]_i\right| \leq [N k_u]_i \tag{14}$$

From properties (ii) and (iii), we can alway find a locally Lipschitz function $\rho_i(\hat{e}, \theta_d, \dot{\theta}_d)$ such that $|F^i(\hat{e}, \theta_d, \dot{\theta}_d) - F_0^i(\hat{e}, \theta_d, \dot{\theta}_d)| \leq \rho_i(\hat{e}, \theta_d, \dot{\theta}_d)$ for $(\hat{e}, \theta_d, \dot{\theta}_d) \in \Omega_r \times \theta_a$. Define the constant k_{si} by the inequality

$$|[\ \ddot{\theta}_d - M(A\hat{e} + BF_0(\hat{e}, \theta_d, \dot{\theta}_d) - \rho(\hat{e}, \theta_d, \dot{\theta}_d)sgn(\sigma)]_i| \leq k_{si} \tag{15}$$

almost everywhere for $(\hat{e}, \theta_d, \dot{\theta}_d, \ddot{\theta}_d) \in \Omega_r \times \Theta_D$ where $\rho(\cdot) = diag[\rho_1(\cdot), \ldots, \rho_n(\cdot)]$, $sgn(\sigma) = [sgn(\sigma_1), \ldots, sgn(\sigma_n)]^T$

Notice that k_{si} can be calculated since $F_0(\cdot)$ and $\rho(\cdot)$ are known. Define the vector

$$\bar{\beta} = (I - N)^{-1}\ Nk_s + \gamma \tag{16}$$

where $\gamma > 0$ is a vector such that $(I - N)\gamma > 0$ and $k_s = [k_{s1}, \ldots, k_{sn}]^T$. Since the matrix $(I-N)$ is an M-matrix, such a vector γ always exists [14]. Consider the function

$$\begin{aligned}\psi(\hat{e}, \theta_d, \dot{\theta}_d, \ddot{\theta}_d) &= G_0^{-1}(\cdot)[\ \ddot{\theta}_d - MA\hat{e} - F_0(\cdot) \\ &\quad - (\rho(\cdot) + \beta)sgn(\sigma(\hat{e}))]\end{aligned}$$

where $\beta = diag[\ \bar{\beta}_1, \ldots, \bar{\beta}_n]^T$ and $\bar{\beta}_i$ is the ith component of the vector $\bar{\beta}$.. We take the control input τ as $\psi(\cdot)$, saturated outside the set $\Omega_r \times \Theta_D$. In particular, let $\psi_a(\cdot) = G_0^{-1}(\cdot)[\ \ddot{\theta}_d - MA\hat{e} - F_0(\cdot)]$ and $\psi_b(\cdot) = -G_0^{-1}(\cdot)(\bar{\rho}(\cdot) + \bar{\beta})$where $\bar{\rho}(\cdot) = [\rho_1(\cdot), \ldots, \rho_n(\cdot)]^T$. Define

$$\psi_a^{si}(\hat{e}, \theta_d, \dot{\theta}_d) = \begin{cases} S_{amax}^i & \psi_a^i(\cdot) > S_{amax}^i \\ \psi_a^i(\cdot) & S_{amin}^i \leq \psi_a^i(\cdot) \leq S_{amax}^i \\ S_{amin}^i & \psi_a^i(\cdot) < S_{amin}^i \end{cases}$$

$$\psi_b^{si}(\hat{e}, \theta_d, \dot{\theta}_d, d\dot{\theta}_d) = \begin{cases} S_{bmax}^i & \psi_b^i(\cdot) > S_{bmax}^i \\ \psi_b^i(\cdot) & S_{bmin}^i \leq \psi_b^i(\cdot) \leq S_{bmax}^i \\ S_{bmin}^i & \psi_b^i(\cdot) < S_{bmin}^i \end{cases}$$

for $i = 1, \ldots, n$ where $\psi_a^i(\cdot)$ and $\psi_b^i(\cdot)$ denote the ith components of the vectors $\psi_a(\cdot)$ and $\psi_b(\cdot)$, respectively, $S_{amax}^i = \max_{(e, \theta_d, \dot{\theta}_d, d\dot{\theta}_d) \in \Omega_r \times \theta_D} \psi_a^i(\hat{e}, \theta_d, \dot{\theta}_d, \ddot{\theta}_d)$ $S_{amin}^i = \min_{(e, \theta_d, \dot{\theta}_d, \ddot{\theta}_d) \in \Omega_r \times \theta_D} \psi_a^i(\hat{e}, \theta_d, \dot{\theta}_d, \ddot{\theta}_d)$, S_{bmax}^i and S_{bmin}^i are similarly defined. Note that since we know a domain of interest $\Omega_r \times \Theta_a$ before an implementation of the controller, we can calculate the values of $S_{\max}^i$ or $S_{\min}^i$ in an off-line manner for an implementation. Take $\tau_i = \psi_a^{si}(\hat{e}, \theta_d, \dot{\theta}_d, \theta_d) + \psi_b^{si}(\hat{e}, \theta_d, \dot{\theta}_d)sgn(\sigma_i)$

Inside the set $\Omega_r \times \Theta_D$, we have

$$\tau(\cdot) = \psi(\cdot) \tag{17}$$

where $\tau(\cdot) = [\tau_1(\cdot), \ldots, \tau_n(\cdot)]^T$. Hence, k_u in inequality (13) can be taken by

$$k_u = k_s + \bar{\beta}. \tag{18}$$

Using (12), (14), (17) and (18), we obtain

$$\begin{aligned}\sigma^T\dot{\sigma} &= \sigma^T[-\rho(\cdot)sgn(\sigma) - \beta sgn(\sigma) + (1/\varepsilon)M\tilde{D}(0)LC\eta \\ &\quad + \phi_1(\cdot) + O(\varepsilon)] \\ &\leq -\bar{\sigma}^T[\ \bar{\beta} - N(k_s + \bar{\beta})] + \varepsilon k\sum_{i=1}^{n}|\sigma_i| \\ &= -\bar{\sigma}^T[(I - N)\ \bar{\beta} - Nk_s] + \varepsilon k\sum_{i=1}^{n}|\sigma_i| \\ &= -\bar{\sigma}^T(I - N)\gamma + \varepsilon k\sum_{i=1}^{n}|\sigma_i| \\ &< -\bar{\sigma}^T\bar{\gamma}\end{aligned}$$

for sufficiently small ε, where $\bar{\sigma} = [|\sigma_1|, \ldots, |\sigma_n|]^T$, $\bar{\gamma} > 0$, and k is some positive constant. We summarize our findings in the following lemma.

Lemma 2 : Consider the singularly perturbed system (6)-(7) with the applied torque τ defined by (18). Suppose that Assumption 1 is satisfied, $\|\zeta\| < \bar{k}\varepsilon$, and $(e, \hat{e}) \in \Omega_r \times \Omega_r$ for $t \in [T_1, T_4)$, and ε is small enough. Then as long as $\sigma(\hat{e}) \neq 0$, the sliding mode condition $\sigma^T(\hat{e})\dot{\sigma}(\hat{e}) < 0$ is satisfied for sufficiently small ε and for all $t \in [T_1 + \varepsilon\ln(1/\varepsilon), T_4)$.

Remark 2 : The smaller value of ε is used, the time $T_1 + \varepsilon(1/\varepsilon)$ becomes smaller. However, the time is always definded since we use the finite value of $1/\varepsilon$ in Lemma 1.

3. Tracking of the desired path

So far we showed that $\|\xi\| < \tilde{k}\varepsilon$ $\|\xi\| < \tilde{k}\varepsilon$ for $t \in [T_1, T_4)$ and the torque τ (18) satisfies the sliding mode condition when $\|\xi\| < \tilde{k}\varepsilon$ and $(e, \hat{e}) \in \Omega_r \times \Omega_r$ for all $t \in [T_1 + \varepsilon \ln(1/\varepsilon), T_4)$. It is shown in [13, Lemma 3] that for sufficiently small ε, $\|\xi\| < \tilde{k}\varepsilon$ and $(e, \hat{e}) \in \Omega_r \times \Omega_r$ for all $t \geq T$, where $T = T_1(\varepsilon) + \varepsilon \ln(1/\varepsilon)$. This allows us to arrive at the following theorem.

Theorem 1 : *Consider the closed-loop system (6)-(7). Suppose that Assumption 1 is satisfied. Let the observer gain be chosen as in (3)-(4), all eigenvalues of (A-LC) be real and negative, and the applied torque be chosen as in (17). Then there exist* $\varepsilon_3 > 0$ *and a finite time* t_1 *such that for all* $0 < \varepsilon < \varepsilon_3$, $\{\|e\| \leq k_p \sqrt{\varepsilon}, \|\xi\| < \tilde{k}\varepsilon\}$ $\forall t \geq t_1$, *for some positive constants* k_p *and* $\tilde{k}$, *and* Ω, *defined by (10), is an estimate of the region of attraction.*

Proof : See the proof of Theorem 1 [13].

Remark 3 : The existence of a finite time t_1 was proven by a conservative way in [13]. Even if it is calculated , the actual ultimately bounded time could be smaller than the calculated one in a given system.

Ⅳ. Example

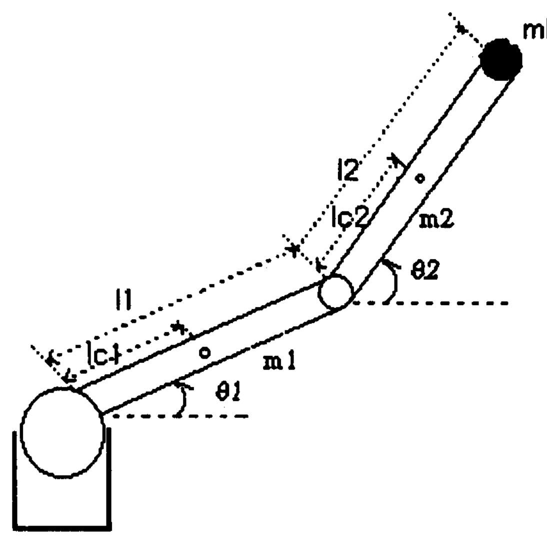

Fig. 1. Two link manipulator.

As an example we consider the problem of controlling the 2 DOF planar manipulator shown in Figure 1, where m_i is the mass of each link, m_l is the mass of payload, l_i is the length of each link, l_{ci} is the length of center of mass, and I_i is the moment of inertia for each link. The matrices $H(\theta)$, $M(\theta, \dot{\theta})$, and $g(\theta)$ take the forms

$$H(\theta) = \begin{bmatrix} h_{11}(\theta) & h_{12}(\theta) \\ h_{12}(\theta) & h_{22}(\theta) \end{bmatrix}, \quad M(\theta, \dot{\theta}) = \begin{bmatrix} c\dot{\theta}_2 & c\dot{\theta}_2 + c\dot{\theta}_1 \\ -c\dot{\theta}_1 & 0 \end{bmatrix}$$

$$g(\theta) = \begin{bmatrix} \{m1 l_{c1} + (m_2 + m_l)\} l_1 g cos(\theta_1) + (m_2 + m_l) l_{c2} g cos(\theta_1 + \theta_2) \\ (m_2 + m_l) l_{c2} g cos(\theta_1 + \theta_2) \end{bmatrix}$$

where

$$h_{11}(\theta) = m_1 l_{c1}^2 + (m_2 + m_l)[l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} \cos(\theta_2)] + I_1 + I_2,$$

$$h_{12}(\theta) = (m_2 + m_l)(l_{c2}^2 + l_1 l_{c2} \cos(\theta_2) + I_2,$$

$$h_{22}(\theta) = (m_2 + m_l) l_{c2}^2 + I_2, \text{ and } c = -(m_2 + m_l) l_1 l_{c2} \sin(\theta_2).$$

Let the desired path of $\theta_1(t)$ and $\theta_2(t)$ be

$$\theta_{d1}(t) = -90^{\circ} + 52.5^{\circ}(1 - \cos 1.26t)$$
$$\theta_{d2}(t) = 170^{\circ} - 60^{\circ}(1 - \cos 1.26t),$$

respectively. The control task is to design the applied torque, τ_i, such that $\theta_1(t)$ and $\theta_2(t)$ track the desired path, $\theta_{d1}(t)$ and $\theta_{d2}(t)$, respectively, with unknown payload. We take the parameters values as m_1=12.3 kg, m_2=10.9 kg, l_1=0.36 m, l_2=0.25 m from [17] and assume that the range of payload is 0 kg-1.2 kg. We use the formulas, $I_i = (1/12) m_i l_i^2$, lc1=0.18, and lc2=(10.9(l2/2)+mll2) /(10.9+ml). Defining state variables in error coordinates, $e_1^i = \theta_i - \theta_{di}$, $e_2^i = \dot{\theta}_i - \dot{\theta}_{di}$, $i = 1, 2$, and set-ting $e_1 = [e_1^1 \ e_1^2]^T$, $e_2 = [e_2^1 \ e_2^2]^T$, $e = [e_1^1 \ e_2^1 \ e_1^2 \ e_2^2]^T$, the state-space model is

$$\dot{e} = Ae + B[-\ddot{\theta}_d + F(e, \theta_d, \dot{\theta}_d) + g(e_1, \theta_d)\tau]$$

where $g(e_1, \theta_d) = H^{-1}(e_1 + \theta_d)$ and

$$F(e, \theta_d, \dot{\theta}_d) = H^{-1}(\cdot)\{-C(\cdot)(e_2 + \dot{\theta}_d) - g(\cdot)\}$$

$$A = block\ diag[A_1, A_2], \quad A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$B = block\ diag[B_1, B_2]$, $B_i = [0 \ 1]^T$. Take the nominal functions $F_0(\cdot)$and $G_0(\cdot)$ when the payload is $m_l = 0.6$ kg. The observer is constructed as $\dot{\hat{e}} = A\hat{e} + B[-\ddot{\theta}_d + F_0(\cdot) + G_0(\cdot)\tau)] + D(\varepsilon)LC(e - \hat{e})$ where $L = block\ diag[L_1, L_2]$, $L_i = [3 \ 2]^T$, $C = block\ diag[C_1, C_2]$, $C_i = [1 \ 0]$, $D(\varepsilon) = block\ diag[D_1(\varepsilon), D_2(\varepsilon)]$, $D_i(\varepsilon) = diag[1/\varepsilon, 1/\varepsilon^2]$. The sliding sur-faces are chosen by $\sigma_1(\hat{e}) = 2\hat{e}_1^1 + \hat{e}_2^1$, and $\sigma_2(\hat{e}) = 2\hat{e}_1^2 + \hat{e}_2^2$. Let the region of interest Ω_r be $\Omega_r = \{e \in R^4 | \ \|Me\| \leq 1, \sqrt{V(e)} \leq 0.5\}$ where $M = block\ diag[M_1, M_2]$, $M_i = [2 \ 1]$, and $V(e) = (e_1^1)^2 + (e_1^2)^2$. The globally bounded nominal functions $F_0(\cdot)$ and $G_0(\cdot)$ are taken by saturating $F_0(\cdot)$ and $G_0(\cdot)$ outside the set Ω_s, where $\Omega_s = \{\hat{e} \in R^4 | \ \|M\hat{e}\| \leq 1.2, \sqrt{V(\hat{e})} \leq 0.5\}$. It can be verified that $N = \begin{bmatrix} 0.06 & 0.02 \\ 0.57 & 0.19 \end{bmatrix}$ is the matrix such that each component of N is the upper bound of absolute value of the corresponding component of the matrix $[G(e_1 + \theta_d) G_0^{-1}(e_1 + \theta_d) - I]$, $\forall e_1 + \theta_d \in R^n$; moreover, the matrix [I-N] is an M-matrix.

Remark 4 : It can be verified that the maximum of absolute values of each component of $\{G(\cdot)G_0^{-1}(\cdot) - I\}$ is obtained when $\theta_2 = 0$ for 2 link case. Using [11, Exercise 5.35], it can be easily shown that $\|G(\cdot)G_0^{-1}(\cdot) - I\|_\infty < 1$ implies that (I-N) is an M-matrix when the matrix is constant, however, converse is not true. As an example, consider that the range of payload is 0-3 kg and the nominal value of $G(\cdot)$ is taken when $m_l = 1.5$ kg. One can verify that $G(\cdot)G_0^{-1}(\cdot) - I = \begin{bmatrix} 0.11 & 0.03 \\ -0.89 & -0.32 \end{bmatrix}$ when $\theta_2 = 0$. One can also verify that Assumption 1 is satisfied, but not the assumption, $\|G(\cdot)G_0^{-1}(\cdot) - I\|_\infty < 1$, in [13].

For the comparison of controller design, we consider the case that both assumptions are satisfied in the rest of paper.

It can be also verified that

$$|F^1(e, \theta_d, \dot{\theta}_d) - F_0^1(e, \theta_d, \dot{\theta}_d)| \leq \rho_1(e, \theta_d, \dot{\theta}_d) \text{ for } e \in \Omega_r$$
$$|F^2(e, \theta_d, \dot{\theta}_d) - F_0^2(e, \theta_d, \dot{\theta}_d)| \leq \rho_2(e, \theta_d, \dot{\theta}_d)$$

where $\rho_1(\cdot) = 0.06|(e_2^1 + \dot{\theta}_{d1})(e_2^2 + \dot{\theta}_{d2})| + 0.05(e_2^2 + \dot{\theta}_{d2})^2 + 0.8$ and

$\rho_2(\cdot)=0.1|(e_2^1+\dot{\theta}_{d1})(e_2^2+\dot{\theta}_{d2})|+0.4(e_2^2+\dot{\theta}_{d2})^2+1.6$. Notice that since we chose all eigenvalues of (A-LC) to be real and negative, $k_{ti}=1$. It can be verified that $k_{s1}=52$ and $k_{s2}=103.5$ in inequality (15). Hence using equation (16), we obtain $\overline{\beta}=[7\ 67]^T$ for $\gamma=[0.7\ 1.5]^T$. Define $u_i=\ddot{\theta}_{di}-F_{02}^i(\cdot)-\psi_a^{si}(\cdot)-\psi_b^{si}(\cdot)\,sgn(\sigma_i)$ for i=1, 2, where $F_{02}^i(\cdot)=-[H^{-1}(\cdot)\,g(\cdot)]_i$ when $m_l=0.6$, and

$$\psi_a^{si}(\hat{e},\theta_d,\dot{\theta}_d)=\begin{cases} S_{amax}^i & 2\hat{e}_{2^i}+F_{01}^i(\cdot)>S_{amax}^i \\ 2\hat{e}_{2^i}+F_{01}^i(\cdot) & S_{amin}^i\le 2\hat{e}_{2^i}+F_{01}^i(\cdot)\le S_{amax}^i \\ S_{amin}^i & 2\hat{e}_{2^i}+F_{01}^i(\cdot)\le S_{amin}^i \end{cases}$$

$$\psi_b^{si}(\hat{e},\theta_d,\dot{\theta}_d)=\begin{cases} S_{bmax}^i & \rho_i(\cdot)+\overline{\beta}_i>S_{bmax}^i \\ \rho_i(\cdot)+\overline{\beta}_i & S_{bmin}^i\le\rho_i(\cdot)+\overline{\beta}_i\le S_{bmax}^i \\ S_{bmin}^i & \rho_i(\cdot)+\overline{\beta}_i<S_{bmin}^i \end{cases}$$

for $i=1,2$ where $F_{01}^i(\cdot)=-[H^{-1}(\cdot)C(\cdot)(e_2+\dot{\theta}_d)]_i$, and

$$\begin{aligned} S_{amin}^i &= \min_{(\hat{e},\theta_d,\dot{\theta}_d)\in\Omega_r\times\theta_d}\{2\hat{e}_{2^i}+F_{01}^i(\hat{e},\theta_d,\dot{\theta}_d)\} \\ S_{amax}^i &= \max_{(\hat{e},\theta_d,\dot{\theta}_d)\in\Omega_r\times\theta_d}\{2\hat{e}_{2^i}+F_{01}^i(\hat{e},\theta_d,\dot{\theta}_d)\} \\ S_{bmin}^i &= \min_{(\hat{e},\theta_d,\dot{\theta}_d)\in\Omega_r\times\theta_d}\{\rho_i(\hat{e},\theta_d,\dot{\theta}_d)+\overline{\beta}_i\} \\ S_{amax}^i &= \max_{(\hat{e},\theta_d,\dot{\theta}_d)\in\Omega_r\times\theta_d}\{\rho_i(\hat{e},\theta_d,\dot{\theta}_d)+\overline{\beta}_i\} \end{aligned}$$

Using the optimization toolbox of MATLAB, we determine these maxima and minima to be $S_{amax}^1=11.34$, $S_{amin}^1=-7.64$, $S_{amax}^2=5.1$, $S_{amin}^2=-27.48$, $S_{bmax}^1=9.22$, $S_{bmin}^1=7.64$, $S_{bmax}^2=74.19$, and $S_{bmin}^2=68.6$. Take the torque $\tau=G_0^{-1}(\hat{e},\theta_d)u$ where $u=[u_1\ u_2]^T$. Let

$$\begin{aligned} \Omega_0 &= \{(e\in R^{2n}\mid \|Me\|\le 0.8,\ \sqrt{V(e_1)}\le 0.3\}, \\ \Omega_1 &= \{\zeta\in R^{2n}\mid \|\zeta\|<1/\varepsilon\} \\ \Omega &= \Omega_0\times\Omega_1 \end{aligned}$$

where $\zeta_1^i=(1/\varepsilon)(e_1^i-\hat{e}_1^i)$, $\zeta_2^i=(e_2^i-\hat{e}_2^i)$, $i=1,2$ and $\zeta=[\zeta_1^1\ \zeta_2^1\ \zeta_1^2\ \zeta_2^2]^T$. According to Theorem 1, the tracking error e is uniformly ultimately bounded. We assume a payload of $m_l=1.2$ kg and simulate the response for $e(0)=[0.2\ 0\ 0\ 0]^T$, $\hat{e}(0)=[0\ 0\ 0\ 0]^T$ with ε = 0.005. Notice that the initial conditions belong to $\Omega_r\times\Omega_r$.

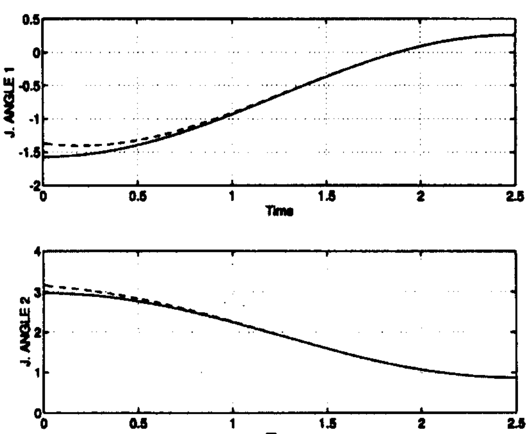

Fig. 2. Tracking of the desired joint angles θ_i (the solid lines are the desired joint angles and the dashed lines are the actual joint angles).

Figure 2 shows that the actual path tracks the desired path with small error. The tracking error can be reduced as the design parameter ε is decreased. One can choose a different controller parameters, i.e., sliding surface $\sigma=0$, ρ, and β, to reduce a tracking time as it does in state feedback control. Figure 3 shows that attractivity of the sliding surfaces is achieved after errors between tracking errors and their estimates become small enough. One way to reduce chattering is the use of continuous approximation of the discontinuous signum nonlinearity [15], i.e., sgn(x) is replaced by sat(x) where

$$sat(x)=\begin{cases} 1 & x>\mu \\ x/\mu & -\mu\le x\le\mu \\ -1 & x<-\mu \end{cases}$$

However, the continuous approximation results in uniform ultimate boundness around origin in the state feedback stabilization problem [7].

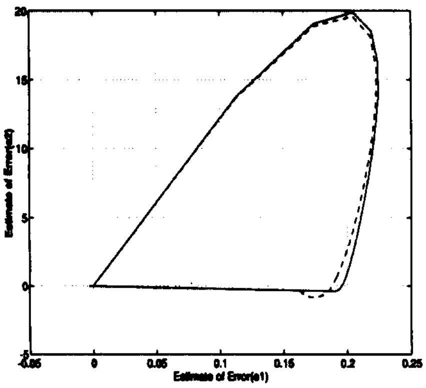

Fig. 3. The phase portraits of estimates of errors $\hat{e}$(the solid line is for $(\hat{e}_1^1, \hat{e}_2^1)$ and the dashed line is for $(\hat{e}_1^2, \hat{e}_2^2)$).

Hence tracking error in the continuous approximation is larger than that of the discontinuous controller. We simulate the system with $e(0)=[0.2\ 0\ 0\ 0]^T$, $\hat{e}(0)=[0\ 0\ 0\ 0]^T$, and $\mu=0.05$ for continuous approximation. Comparing Figure 2 and Figure 4, one can observe that tracking error in the continuous approximation is larger than that of the discontinuous control case.

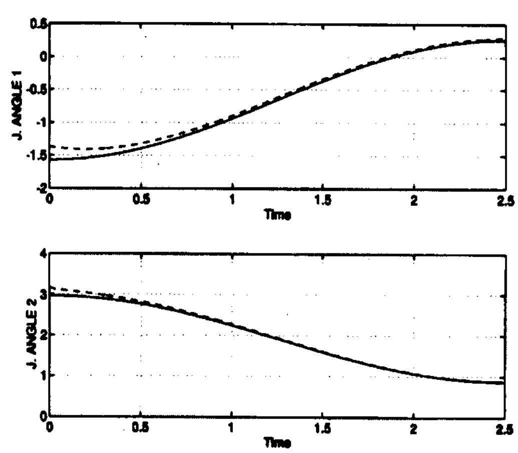

Fig. 4. Tracking of the desired joint angles θ_i with continuous approximation, $\mu=0.05$ (the solid lines are the desired the joint angles and the dashed lines are the actual joint angles).

Figures 5 and 6 show that chattering does not appear in the angular velocities $\dot{\theta}$ and the control input u.

Figure 6 shows that the control input u is saturated during a short transient period, which is a consequence of saturating u outside the set $\Omega_r \times \Omega_r$. If we use the control scheme in [13], it can be verified that

$$\bar{u}_i = \dot{\theta}_{di} - F^i_{02}(\cdot) - sat(2\hat{e}^i_2 + F^i_{01}(\cdot)) - sat(\rho_2(\cdot) + 106)\, sgn(\sigma_i),$$

for *i=1, 2*. One can observe that the magnitude of the discontinuous coefficient terms are significantly increased if we use the control scheme in [13].

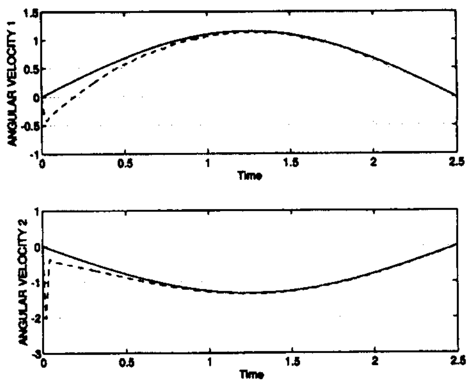

Fig. 5. Tracking of the desired angular velocities $\dot{\theta}_i$(the solid lines are the desired velocities and the dashed lines are the actual velocities).

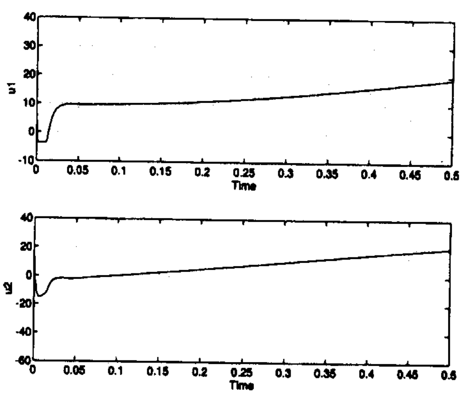

Fig. 6. The control input u with continuous approximation ($\mu = 0.05$).

V. Concluding remark

We have designed an output feedback variable structure controller that ensures tracking of a desired path, with arbitrarily small error, for an n DOF manipulator. The desired accuracy of tracking can be achieved by choosing the design parameter ε. We require that the diagonal components of $(G(\theta)G_0^{-1}(\theta) - I)$ dominate the off diagonal components by requiring *(I-N)* to be an M-matrix. This is different from the requirement $\|G(\theta)G_0^{-1}(\theta) - I\|_\infty < 1$ in [13]. We show, via an example, that the assumption on the input coefficient matrix uncertainty and the corresponding controller in this paper are less conservative than those of [13]. We also give the complete regional analysis of the tracking problem for n DOF robot manipulator in the presence of modeling uncertainty. One may consider an adaptive output feedback scheme to achieve the same objective of this paper since adaptive scheme give the better performance for state feedback control case in general when a model contains parametric uncertainty [1]. However, the properties achieved by state feedback control does not neccessary hold for output feedback control. Hence, the design of adaptive output feedback controller will be involved a lot of subjects, e.g., stability issue, robustness issue, and parameter convergence. This can be a future subject of research.

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