Robust Control Design Applicable to General Flexible Joint Manipulators

일반적인 유연조인트 로봇에 부합되는 견실제어설계

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요 약 : 불확실한 변수와 비선형성을 가지는 유연조인트 로봇의 견실제어 방안을 제시한다. 그리고 본 시스템에서 불확실구조는 일치성을 유지하지 않는 불일치성 불확실 시스템이다. 제어기는 리아프노프의 방안에 근거를 두고 있다. 견실제어는 연산토크법을 사용하고 삽입제어기법을 통하여 좌표변환을 통해 구성된다. 제어기 설계과정은 우선 연산토크방법에 의해 시스템 동역학에서 정격부분을 선형으로 2개의 부분시스템으로 구성한다. 이후 좌표변환을 이용하여 각 부분시스템에 제어기를 구축한다. 이 방안을 통하여 관성 행렬이 알려진 값인 경우 이 행렬의 상위한계 조건 없이 제어기를 설계할 수 있다. 따라서 임의의 형태의 로봇에도 적용 가능한 제어알고리즘이 된다. 설계된 견실제어는 변환된 시스템이나 원시스템 모두 실용적 안정성을 보장한다. 이 변환은 단지 불확실변수의 최대 한계 값의 정보만을 요구한다.

Keywords: robust control, quadratic Lyapunov function, practical stability, flexible joint manipulators, computed torque method, uncertainty

I. Introduction

We design control scheme for flexible joint manipulators in the presence of nonlinearity and uncertainty. For this nonlinear system dynamics, feedback compensation must be considered to achieve good performance. It has been shown that joint flexibility has a significant influence on system performance compared with rigid manipulators [1][2]. So far there have been various efforts devoted to the study of control for flexible joint manipulators. References of these efforts are cited in [3].

The exact model based approach includes singular perturbation [4], feedback linearization scheme [5][6] and invariant manifold scheme [7][8]. Since the control schemes which are designed using this approach require exact knowledge of the robot parameters, it is necessary to study the control design issue in the presence of uncertainty. The use of adaptive control for flexible joint manipulators has been reported in numerous literatures. The control allows the existence of uncertainty in system models. The adaptive control schemes developed in [9]-[11] also adopted the singular perturbation and linear parameterization technique. We are concerned about the possible excessive transient response before adaptive parameter converges in adaptive control. This paper is based on the design paradigm often used in robust control based on Lyapunov approach for the uncertainty issue. This approach is the only approach whose theoretical basis has been established and which is applicable to nonlinear systems with time-varying parameters. Contributions which are based on robust control in the area of flexible joint manipulator have been done in [12][13]. These schemes need acceleration and jerk feedbacks. As for robust control, numerous control schemes have been introduced based on a structural condition on the uncertainty, namely, the matching condition [14][15]. However, this condition does not hold in certain cases which include the flexible joint manipulator with uncertainty since that system does not have a control input for each node.

In this paper we propose a control scheme by introducing *implanted control* which utilizes a state transformation as shown in [16]. Since the control schemes proposed in [16][17] rely on system geometry in inertia matrix, it is sometimes necessary to propose assumption on inertia matrix such as positive definiteness and upper-boundness.

This paper aims at developing a robust control by using computed torque scheme. With this algorithm practical stability is guaranteed. The control offers a more feasible tool in designing control for the flexible-link manipulator system. We demonstrate procedure of designing robust control with computed torque scheme and show simulation results by applying to 2-link flexible joint manipulator.

II. Flexible joint manipulators

Consider an n serial link mechanical manipulator. The links are assumed rigid. The joints are however flexible. All joints are revolute or prismatic and are directly actuated by DC-electric motors. For the flexible joint robot define vectors $q_l=[q^2\ q^4 \cdots q^{2n-2}\ q^{2n}]^T$ and $q_j=[q^1\ q^3 \cdots q^{2n-3}\ q^{2n-1}]^T$, where q^2, $q^4 \cdots$ are link angles and q^1, $q^3 \cdots$ are joint angles. Let be the $2n$-vector of generalized

$$q=\begin{bmatrix} q_l \\ q_j \end{bmatrix} \tag{1}$$

coordinates for the system. We model the joint flexibility by a linear torsional spring at each joint and denote by

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K the diagonal matrix of joint stiffness. We assume that the rotors are modeled as uniform cylinders so that the gravitational potential energy of the system is independent of the rotor position and is therefore a function only of link position. The dynamic equation of motion of the flexible joint manipulator can be expressed in terms of the partition of the generalized coordinates [2]:

$$\begin{bmatrix} D(q_l) & 0 \\ 0 & J \end{bmatrix}\begin{bmatrix} \ddot{q}_l \\ \ddot{q}_j \end{bmatrix}+\begin{bmatrix} C(q_l,\dot{q}_l)\dot{q}_l \\ 0 \end{bmatrix}+\begin{bmatrix} G(q_l) \\ 0 \end{bmatrix} + \begin{bmatrix} K(q_l-q_j) \\ -K(q_l-q_j) \end{bmatrix}=\begin{bmatrix} 0 \\ u \end{bmatrix}, \tag{2}$$

where $D(q_l)$ is the link inertia matrix and J is a constant diagonal matrix representing the inertia of actuator. $C(q_l, \dot{q}_l)\dot{q}_l$ represents the Coriolis and centrifugal force, $G(q_l)$ represents the gravitational force, and u denotes the input force from the actuators. K is a constant diagonal matrix representing the torsional stiffness between links and joints (hence K^{-1} exists).

III. System description

Since the controllers introduced in [16][17] depend on the condition of boundness of inertia matrix $D(q_l)$, which refers to uniformly positive definiteness and uniformly boundedness from above and below. Thus, we need to investigate whether the current system satisfies that conditions. For instance, consider the manipulator with one revolute and one prismatic manipulator in [18]. There does not exist constant upper-bound of inertia matrix. Moreover, if we use Lyapunov function based on $D(q_l)$ we need the information of $\dot{D}(q_l)$ in case the system has time-varying uncertain parameters. This causes more cost in implementing controller than system with constant uncertain parameters, which only utilizes the skew-symmetric property. With this, we try to construct a different control scheme by introducing computed torque scheme, which invokes only quadratic Lyapunov function.

In this section we construct a system description. The system is decomposed into nominal part and uncertain part and is expressed in matrix form. The flexible joint manipulator system is shown as follows:

$$\begin{aligned} &D(q_l)\ddot{q}_l+N(q_l,\dot{q}_l)+G(q_l)+Kq_l=Kq_j, \\ &J\ddot{q}_j+K(q_j-q_l)=u. \end{aligned} \tag{3}$$

Let $X_1=q_l$, $X_2=\dot{q}_l$, $X_3=q_j$ and $X_4=\dot{q}_j$ also let $x_1=[X_1^T\ X_2^T]^T$, $x_2=[X_3^T\ X_4^T]^T$ and $x=[x_1^T\ x_2^T]^T$.

We rewrite the first part of (3) by using state variables defined above:

$$\begin{aligned} \dot{X}_1 &= X_2, \\ \dot{X}_2 &= \ddot{q}_l=-D^{-1}(q_l)N(q_l,\dot{q}_l) \\ &\quad -D^{-1}(q_l)G(q_l)-D^{-1}(q_l)Kq_l \\ &\quad +D^{-1}(q_l)Kq_j \\ &=-L_{11}q_l-L_{21}\dot{q}_l+L_{11}q_l+L_{21}\dot{q}_l \\ &\quad -D^{-1}(q_l)N(q_l,\dot{q}_l)-D^{-1}(q_l)G(q_l) \\ &\quad -D^{-1}(q_l)Kq_l+D^{-1}(q_l)Kq_j, \end{aligned} \tag{4}$$

where $L_{11}, L_{21} \in R^{n\times n}$.

From now on, "overbar" on parameters represents the nominal portion and Δ represents the uncertainty portion. Here, we consider uncertainties in parameters and express (4) in matrix form as

$$\begin{aligned} N_1: \dot{x}_1(t) &= A_1x_1(t)+f_{1x}(x_1(t),\sigma_1(t)) \\ &\quad +B_1(x_1(t),\sigma_1(t))q_j(t) \\ &= A_1x_1(t)+f_{1x}(x_1(t),\sigma_1(t)) \\ &\quad +\overline{B}_1(x_1(t))q_j(t) \\ &\quad +\Delta B_1(x_1(t),\sigma_1(t))q_j(t), \end{aligned} \tag{5}$$

where

$$\begin{aligned} A_1 &:= \begin{bmatrix} 0 & I \\ -L_{11} & -L_{21} \end{bmatrix}, \\ f_{1x}(x_1,\sigma_1) &:= \begin{bmatrix} 0 \\ f_{21x}(x_1,\sigma_1) \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} f_{21x}(x_1,\sigma_1) &= L_{11}q_l+L_{21}\dot{q}_l \\ &\quad -D^{-1}(q_l,\sigma_1)N(q_l,\dot{q}_l,\sigma_1) \\ &\quad -D^{-1}(q_l,\sigma_1)G(q_l,\sigma_1) \\ &\quad -D^{-1}(q_l,\sigma_1)K(\sigma_1)q_l, \end{aligned}$$

$$\begin{aligned} \overline{B}_1(x_1) &:= \begin{bmatrix} 0 \\ [D^{-1}(q_l)K]_n \end{bmatrix}, \\ \Delta B_1(x_1,\sigma_1) &:= \begin{bmatrix} 0 \\ \Delta(D^{-1}(q_l,\sigma_1)K(\sigma_1)) \end{bmatrix}, \end{aligned} \tag{6}$$

and $\sigma_1(t)$ is the uncertainty parameter vector in the subsystem N_1. $[D^{-1}K]_n$ denotes the nominal value of the designated matrix, which is identical to $\overline{D}^{-1}\overline{K}$. For the second part of (3) we rewrite it as follows:

$$\begin{aligned} \dot{X}_3 &= X_4, \\ \dot{X}_4 &= \ddot{q}_j=-J^{-1}Kq_j+J^{-1}Kq_l+J^{-1}u \\ &=-L_{12}q_j-L_{22}\dot{q}_j+L_{12}q_j+L_{22}\dot{q}_j \\ &\quad -J^{-1}Kq_j+J^{-1}Kq_l+J^{-1}u, \end{aligned} \tag{7}$$

where $L_{12}, L_{22} \in R^{n\times n}$.

The above equation can be expressed in the matrix form as

$$\begin{aligned} N_2: \dot{x}_2(t) &= A_2x_2(t)+f_{2x}(x(t),\sigma_2(t)) \\ &\quad +B_2(\sigma_2(t))u(t) \\ &= A_2x_2(t)+f_{2x}(x(t),\sigma_2(t)) \\ &\quad +\overline{B}_2u(t)+\Delta B_2(\sigma_2(t))u(t), \end{aligned} \tag{8}$$

where

$$\begin{aligned} A_2 &= \begin{bmatrix} 0 & I \\ -L_{12} & -L_{22} \end{bmatrix}, \\ f_{2x}(x,\sigma_2) &= \begin{bmatrix} 0 \\ L_{12}q_j+L_{22}\dot{q}_j-J^{-1}(\sigma_2)K(\sigma_2)q_j \\ +J^{-1}(\sigma_2)K(\sigma_2)q_l \end{bmatrix}, \\ \overline{B}_2 &= \begin{bmatrix} 0 \\ \overline{J}^{-1} \end{bmatrix}, \quad \Delta B_2(\sigma_2)=\begin{bmatrix} 0 \\ \Delta(J^{-1}(\sigma_2)) \end{bmatrix}, \end{aligned} \tag{9}$$

and $\sigma_2(t)$ is the uncertainty vector in the subsystem N_2.

Assumption 1 : For each subsystem the mappings $\sigma_1(\cdot): R\rightarrow\Sigma_1\subset R^{o_1}$, $\sigma_2(\cdot): R\rightarrow\Sigma_2\subset R^{o_2}$ are Lebesgue measurable with Σ_1, Σ_2 prescribed and compact.

The above assumption implies that the uncertainties $\sigma_1(\cdot), \sigma_2(\cdot)$ are the functions of time and the range of them remains within the bounded set and they have dimensions of o_1, o_2, respectively. Also, the values of $\sigma_1(\cdot)$ and $\sigma_2(\cdot)$ are almost everywhere continuous with respect to time.

IV. Controller design procedure

We can rewrite (5) as follows:

$$
\begin{aligned}
N_1: \dot{x}_1 &= A_1x_1 + f_{1x}(x_1,\sigma_1) \\
&\quad + \overline{B}_1(x_1)q_j + \Delta B_1(x_1,\sigma_1)q_j \\
&= A_1x_1 + f_{1x}(x_1,\sigma_1) \\
&\quad + (\overline{B}_1(x_1) + \Delta B_1(x_1,\sigma_1))u_1 \\
&\quad + (\overline{B}_1(x_1) + \Delta B_1(x_1,\sigma_1))(q_j - u_1).
\end{aligned} \tag{10}
$$

Here, we introduce an *implanted control* u_1 without changing the dynamics in the subsystem N_1. Here, we see that (10) has $q_j(t) - u_1(t)$ term on right-hand side. We substitute this $q_j - u_1$ with other state variable. Thus, a state transformation is introduced.

Let $z_1 = [Z_1^T\ Z_2^T]^T$, $z_2 = [Z_3^T\ Z_4^T]^T$ and $z = [z_1^T\ z_2^T]^T$, where

$$
\begin{aligned}
Z_1 &= X_1, \\
Z_2 &= X_2, \\
Z_3 &= q_j - u_1 = X_3 - u_1, \\
Z_4 &= \dot{q}_j - \dot{u}_1 = X_4 - \dot{u}_1.
\end{aligned} \tag{11}
$$

This implies that $z_1 = x_1$ and $z_2 = x_2 - [u_1^T\ \dot{u}_1^T]^T$. By this state transformation we construct the trans- formed subsystem for the subsystem N_1:

$$
\begin{aligned}
\widehat{N}_1: \dot{z}_1 &= A_1z_1 + B_1u_1 \\
&\quad + f_{1z}(z_1,\sigma_1) + B_1(q_j - u_1) \\
&= A_1z_1 + \overline{B}_1(z_1)u_1 + \Delta B_1(\sigma_1)u_1 \\
&\quad + f_{1z}(z_1,\sigma_1) + B_1(q_j - u_1),
\end{aligned} \tag{12}
$$

where $f_{1z}(\cdot)$ is identical to $f_{1x}(\cdot)$ since $z_1 = x_1$ from the above state transformation. We see that uncertainties in f_{1z} and ΔB_1 satisfy the matching condition for $h_1 \in R^{n\times n}$, $E_1 \in R^{n\times n}$.

$$
\begin{aligned}
f_{1z}(z_1,\sigma_1) &= \overline{B}_1(z_1)h_1(z_1,\sigma_1), \\
\Delta B_1(z_1,\sigma_1) &= \overline{B}_1(z_1)E_1(z_1,\sigma_1).
\end{aligned} \tag{13}
$$

From above matching conditions, we can express (16) as follows:

$$
\begin{aligned}
\widehat{N}_1: \dot{z}_1 &= A_1z_1 + \overline{B}_1(z_1)u_1 \\
&\quad + \overline{B}_1(z_1)h_1(z_1,\sigma_1) \\
&\quad + \overline{B}_1(z_1)E_1(z_1,\sigma_1)u_1 \\
&\quad + B_1(z_1,\sigma_1)(q_j - u_1).
\end{aligned} \tag{14}
$$

From (13) we obtain $h_1(\cdot)$ and $E_1(\cdot)$:

$$
\begin{aligned}
h_1 &= [D^{-1}K]_n^{-1}(L_{11}Z_1 + L_{21}\dot{Z}_1 \\
&\quad - D^{-1}N - D^{-1}G - D^{-1}KZ_1),
\end{aligned} \tag{15}
$$

$$
E_1 = [D^{-1}K]_n^{-1}\Delta(D^{-1}K). \tag{16}
$$

Since $h_1(\cdot)$ is continuous and $\sigma_1(t)$ is in a compact set we see that there exists a bounding function $\overline{\rho_1}(\cdot)$: $R^n \to R_+$ such that

$$
\|h_1(z_1,\sigma_1)\| \le \overline{\rho_1}(z_1). \tag{17}
$$

Assumption 2 : There exists a constant ρ_{E_1} such that

$$
\begin{aligned}
\rho_{E_1} &:= \max_{\sigma_1\in\Sigma_1} \|E_1(z_1,\sigma_1)\| \\
&= \max_{\sigma_1\in\Sigma_1} \|[D^{-1}(z_1)K]_n^{-1} \\
&\quad \Delta(D^{-1}(z_1,\sigma_1)K(\sigma_1))\| \\
&< \frac{1}{\sqrt{n}},
\end{aligned} \tag{18}
$$

where n corresponds to number of links.

This assumption implies how much the uncertainty varies over the nominal value and the nominal value $[D^{-1}(z_1)K]_n^{-1}$ is not far from $D^{-1}(z_1,\sigma_1)\ K(\sigma_1)$. If the ratio $\Delta(D^{-1}(z_1,\sigma_1)\ K(\sigma_1))$ to $[D^{-1}(z_1)K]_n^{-1}$ is less than $\frac{1}{\sqrt{n}}$ the condition holds thus, we can compute the value of ρ_{E_1}. This assumption also implies that the nominal value $D^{-1}K$ is not far from $D^{-1}(z_1,\sigma_1)$ $K(\sigma_1)$. In a single link robot, this value becomes $\max_{\sigma_1\in\Sigma_1} \frac{d(\sigma_1)\ k(\sigma_1)}{d\ k}$, where d and k which are scalars represent link inertia and joint stiffness respectively. This can be easily understood.

Remark : We see that the allowable upper bound of ρ_{E_1} depends on n, which may be restrictive in designing the controller in high degree of freedom systems. Moreover, if system consists of prismatic joint manipulator we can not estimate ρ_{E_1} as above. This is since the bound of $D(q_l)$ has to do with z_1. However, if $D(q_l)$ is known then ρ_{E_1} becomes $\max_{\sigma_1\in\Sigma_1} \|\overline{K}^{-1}\Delta K(\sigma_1)\|$. Then we have a different condition for ρ_{E_1}, which does not depend on n. Furthermore, we do not need the upper-bound condition for inertia matrix in designing controller. These will be shown in detail later in Section 6. Let $\rho_1(\cdot): R^{2n} \to R_+$ be chosen such that it is C^2 and

$$
\rho_1(z_1) \ge (1 - \sqrt{n}\rho_{E_1})^{-1}\ \overline{\rho_1}(z_1). \tag{19}
$$

Let

$$
\mu_1 = \overline{B_1}^T P_1 z_1 \rho_1(z_1), \tag{20}
$$

also

$$
\mu_1 = [\mu_{11}\mu_{21}\cdots\mu_{n1}]^T, \tag{21}
$$

$$
u_1 = [u_{11}u_{21}\cdots u_{n1}]^T. \tag{22}
$$

For given scalar $\varepsilon_1 > 0$, u_{1i} is given by

$$
u_{1i} = \begin{cases} -\dfrac{\mu_{1i}}{\|\mu_{1i}\|}\rho_1(z_1), & \text{if}\ \|\mu_{1i}\| > \varepsilon_1 \\ -\sin\left(\dfrac{\pi\mu_{1i}}{2\varepsilon_1}\right)\rho_1(z_1), & \text{if}\ \|\mu_{1i}\| \le \varepsilon_1 \end{cases} \tag{23}
$$

i=1,2,...n. Note that

$$
u_{1i} \begin{cases} \le -\dfrac{\mu_{1i}}{\varepsilon_1}\rho_1, & \text{if}\ 0 \le \mu_{1i} \le \varepsilon_1 \\ \ge -\dfrac{\mu_{1i}}{\varepsilon_1}\rho_1, & \text{if}\ -\varepsilon_1 \le \mu_{1i} \le 0, \end{cases} \tag{24}
$$

and $\|u_{1i}\| \le \rho_1$. $P_1 > 0$ is the solution of

$$
P_1A_1 + A_1^TP_1 = -Q_1,\ \ Q_1 > 0, \tag{25}
$$

and L_{11} and L_{21} in A_1 are selected such that A_1 is Hurwitz.

From (7) and (11) we construct the transformed sub- system for the subsystem N_2:

$$
\begin{aligned}
\dot{Z}_3 &= Z_4 \\
\dot{Z}_4 &= -J^{-1}KZ_3 - \ddot{u}_1 - J^{-1}Ku_1 \\
&\quad + J^{-1}KZ_1 + J^{-1}u \\
&= -L_{12}Z_3 - L_{22}\dot{Z}_3 + L_{12}Z_3 \\
&\quad + L_{22}\dot{Z}_3 - J^{-1}KZ_3 - \ddot{u}_1 \\
&\quad - J^{-1}Ku_1 + J^{-1}KZ_1 + J^{-1}u.
\end{aligned} \tag{26}
$$

The above equation can be expressed in the matrix form as

$$
\begin{aligned}
\widehat{N}_2: \dot{z}_2 &= A_2z_2 + B_2(\sigma_2)u + f_{2z}(z,\sigma_1,\sigma_2) \\
&= A_2z_2 + (\overline{B}_2 + \Delta B_2)u \\
&\quad + f_{2z}(z,\sigma_1,\sigma_2),
\end{aligned} \tag{27}
$$

where

$$\begin{aligned} &f_{2z}(z,\sigma_1,\sigma_2) \\ &:= L_{12}Z_3 + L_{22}\dot{Z}_3 - J^{-1}(\sigma_2)K(\sigma_2)Z_3 \\ &\quad - \ddot{u}_1(z_1,z_2,\sigma_1,\sigma_2) - J^{-1}(\sigma_2)K(\sigma_2)u_1(z_1) \\ &\quad + J^{-1}(\sigma_2)K(\sigma_2)Z_1. \end{aligned} \tag{28}$$

We have the matching condition met for the subsystem $\hat{N}_2$:

$$\begin{aligned} f_{2z}(z,\sigma_1,\sigma_1) &= \overline{B}_2 h_2(z,\sigma_1,\sigma_2), \\ \Delta B_2(\sigma_2) &= \overline{B}_2 E_2(\sigma_2). \end{aligned} \tag{29}$$

From the above matching condition (29), we construct the transformed second subsystem

$$\begin{aligned} \hat{N}_2 : \dot{z}_2 &= A_2 z_2 + \overline{B}_2(\sigma_2)u \\ &\quad + \overline{B}_2 E_2(\sigma_2)u + \overline{B}_2 h_2(z,\sigma_1,\sigma_2). \end{aligned} \tag{30}$$

We now propose the control design. From (29) we have

$$\begin{aligned} &h_2(z_1,z_2,\sigma_1,\sigma_2) \\ &= \overline{J}(L_{12}Z_3 + L_{22}\dot{Z}_3 - \ddot{u}_1(z_1,z_2,\sigma_1,\sigma_2)) \\ &\quad + \overline{J}(-J^{-1}(\sigma_2)K(\sigma_2)Z_3 + J^{-1}(\sigma_2)K(\sigma_2)Z_1 \\ &\quad - J^{-1}(\sigma_2)K(\sigma_2)u_1(z_1)), \end{aligned} \tag{31}$$

$$E_2(\sigma_2) = \overline{J^{-1}}\Delta J(\sigma_2).$$

Since $h_2(\cdot)$ is continuous and $\sigma_1(t)$ and $\sigma_2(t)$ are in compact set, there exists a bounding function $\overline{\rho_2}(\cdot): R^{2n}\times R^{2n}\rightarrow R_+$ such that

$$\|h_2(z_1,z_2,\sigma_1,\sigma_2)\| \le \overline{\rho_2}(z_1,z_2). \tag{32}$$

Assumption 3 : There exist a constant λ_{E_2} such that

$$\begin{aligned} \lambda_{E_2} : &= \min_{\sigma_2\in\Sigma_2} \lambda_{\min}(E_2(\sigma_2)) \\ &= \min_{\sigma_2\in\Sigma_2} \lambda_{\min}(\overline{J}^{-1}\Delta J(\sigma_2)) > -1. \end{aligned} \tag{33}$$

The meaning of the assumption can be explained similarly to Assumption 2. In other words, this assumption implies how the uncertainty of joint inertia ΔJ can be far from the nominal value $\overline{J}^{-1}$. Usually, the joint inertia is of diagonal matrix. Hence, the value λ_{E_2} takes the minimum value out of the n joint.

Let $\rho_2(\cdot): R^{2n}\times R^{2n}\rightarrow R_+$ be chosen such that

$$\rho_2(z_1,z_2) \ge (1+\lambda_{E_2})^{-1}\overline{\rho}_2(z_1,z_2). \tag{34}$$

Let

$$\mu_2 := \overline{B}_2^{\,T} P_2\, z_2\, \rho_2. \tag{35}$$

For given scalar $\varepsilon_2 > 0$, choose the control input u as follows:

$$u(z_1,z_2) = \begin{cases} -\dfrac{\mu_2(z_2)}{\|\mu_2(z_2)\|}\rho_2(z_1,z_2) & \text{if } \|\mu_2(z_2)\| > \varepsilon_2 \\ -\dfrac{\mu_2(z_2)}{\varepsilon_2}\rho_2(z_1,z_2) & \text{if } \|\mu_2(z_2)\| \le \varepsilon_2. \end{cases} \tag{36}$$

P_2 is the solution of

$$A_2^T P_2 + P_2 A_2 = -Q_2,\ Q_2 > 0, \tag{37}$$

and L_{12} and L_{22} in A_2 are selected such that A_2 is Hurwitz.

Assumption 4 : There exists a constant $\underline{\sigma} > 0$ such that

$$D(q_l,\sigma_1) \ge \underline{\sigma}I \quad \forall q_l \in R^n,\ \forall \sigma_1 \in \Sigma_1. \tag{38}$$

Remark : This Assumption implies that inertia matrix is positive definite. This is different from the previous assumption in [16] which requires the inertia matrix to be positive definite and uniformly upper-bounded.

Theorem 1 : Subject to Assumptions 1-4, the system (14, 30) is practically stable [15][16] under the control (36). Furthermore, the uniform ultimate boundedness region can be made arbitrary small by suitable choice of ε_1 and ε_2.

Here, the dynamic system is practically stable if the following properties hold.

i) Existence and continuation of solution, ii) Uniform boundedness, iii) Uniform ultimate boundedness, iv) Uniform stability. The uniform ultimate boundedness means that the response of the system enters and remains within a particular neighborhood of the equilibrium position after some finite time and remains close thereafter.

Proof : Choose the Lyapunov candidate as follows:

$$V(z_1,z_2) = V_1(z_1) + V_2(z_2), \tag{39}$$

where

$$V_1(z_1) = z_1^T P_1 z_1, \tag{40}$$

$$V_2(z_2) = z_2^T P_2 z_2. \tag{41}$$

Here, we see that $V_1(z_1)$ and $V_2(z_2)$ are both positive definite and decrescent since

$$\begin{aligned} \lambda_{\min}(P_1)\|z_1\|^2 \le V_1(z_1) \le \lambda_{\max}(P_1)\|z_1\|^2. \\ \lambda_{\min}(P_2)\|z_2\|^2 \le V_2(z_2) \le \lambda_{\max}(P_2)\|z_2\|^2. \end{aligned} \tag{42}$$

Let $\alpha_1 = \overline{B}_1^{\,T} P_1 z_1$. The derivative of V_1 along (14) is

$$\begin{aligned} \dot{V}_1 &= 2z_1^T P_1 \dot{z}_1 \\ &= 2z_1^T P_1(A_1 z_1 + \overline{B}_1 h_1 + \overline{B}_1 u_1 \\ &\quad + \overline{B}_1 E_1 u_1 + B_1(q_j - u_1)) \\ &= -z_1^T q_l z_1 + 2z_1^T P_1 \overline{B}_1(h_1 + u_1 + E_1 u_1) \\ &\quad + 2z_1^T P_1 B_1(q_j - u_1) \\ &\le -\lambda_{\min}(q_l)\|z_1\|^2 + 2\alpha_1^T h_1 + 2\alpha_1^T u_1 \\ &\quad + 2\alpha_1^T E_1 u_1 + 2z_1^T P_1 B_1(q_j - u_1) \\ &= -\lambda_{\min}(q_l)\|z_1\|^2 + 2\alpha_1^T u_1 \\ &\quad + 2\alpha_1^T E_1 u_1 + 2\alpha_1^T h_1 + 2z_1^T P_1 B_1 Z_3. \end{aligned} \tag{43}$$

For $\|\mu_{1i}\| \ge \varepsilon_1$, the second, third and fourth terms in (43) follow from (17), (19) and (23):

$$\begin{aligned} &2\alpha_1^T u_1 + 2\alpha_1^T E_1 u_1 + 2\alpha_1^T h_1 \\ &\le 2\sum_{i=1}^n \alpha_{1i} u_{1i} + 2\rho_{E_1}\sum_{i=1}^n \|\alpha_{1i}\| \sum_{i=1}^n \|u_{1i}\| \\ &\quad + 2\sum_{i=1}^n \|\alpha_{1i}\|\ \|h_1\| \\ &= 2\sum_{i=1}^n \alpha_{1i}\Big(-\frac{\alpha_{1i}}{\|\alpha_{1i}\|}\rho_1\Big) \\ &\quad + 2\rho_{E_1}\sum_{i=1}^n \|\alpha_{1i}\|\sqrt{n}\rho_1 + 2\overline{\rho_1}\sum_{i=1}^n \|\alpha_{1i}\| \\ &\le -2\sum_{i=1}^n \|\alpha_{1i}\|\rho_1 + 2\rho_{E_1}\sum_{i=1}^n \|\alpha_{1i}\|\sqrt{n}\rho_1 \\ &\quad + 2(1-\sqrt{n}\rho_{E_1})\rho_1\sum_{i=1}^n \|\alpha_{1i}\| \\ &= 0. \end{aligned} \tag{44}$$

When $\|\mu_{1i}\| \le \varepsilon_1$ it follows that

$$\begin{aligned} &2\alpha_1^T u_1 + 2\alpha_1^T E_1 u_1 + 2\alpha_1^T h_1 \\ &= 2\sum_{i=1}^n \alpha_{1i} u_{1i} + 2\rho_{E_1}\sum_{i=1}^n \|\alpha_{1i}\| \sum_{i=1}^n \|u_{1i}\| \\ &\quad + 2\sum_{i=1}^n \|\alpha_{1i}\|\ \|h_1\| \\ &= 2\sum_{i=1}^n \alpha_{1i}\Big(-\frac{\alpha_{1i}}{\varepsilon_1}\rho_1^2\Big) + 2\rho_{E_1}\sum_{i=1}^n \|\alpha_{1i}\|\sqrt{n}\rho_1 \\ &\quad + 2\sum_{i=1}^n \|\alpha_{1i}\|\rho_1(1-\sqrt{n}\rho_{E_1}) \\ &= 2\sum_{i=1}^n\Big(-\frac{\alpha_{1i}^2}{\varepsilon_1}\rho_1^2 + \|\alpha_{1i}\|\rho_1\Big) \\ &\le \frac{n\varepsilon_1}{2}. \end{aligned} \tag{45}$$

Next, by Assumption 4 we have the following inequality condition for any $\tau_1 > 0$:

$$\begin{aligned} &2z_1^T P_1 B_1 Z_3 \\ &\le 2\,\|z_1^T P_1 B_1 Z_3\| \\ &\le \tau_1 \|z_1\|^2\, \|P_1 B_1\| + \tau_1^{-1} \|Z_3\|^2\, \|P_1 B_1\| \\ &\le \tau_1 \rho_k \|z_1\|^2 + \tau_1^{-1} \rho_k \|z_2\|^2 \end{aligned} \tag{46}$$

where

$$\begin{aligned} \|P_1 B_1\| &= \|P_1 D^{-1} K\| \\ &\le \|P_1\|\, \|D^{-1}\|\, \|K\| \\ &\le \|P_1\|\, \underline{\sigma}^{-1} \|K\| \\ &=: \rho_k. \end{aligned} \tag{47}$$

Therefore, it follows from (44, 45) and (46) that

$$\begin{aligned} \dot V_1 &\le -(\lambda_{\min}(Q_1) - \tau_1 \rho_k) \|z_1\|^2 \\ &+ \frac{n\varepsilon_1}{2} + \rho_k \tau_1^{-1}\, \|z_2\|^2. \end{aligned} \tag{48}$$

Next, let $\alpha_2 = \overline{B}_2^T P_2 z_2$. The derivative of V_2 along (30) is given by

$$\begin{aligned} \dot V_2 &= 2z_2^T P_2 \dot z_2 \\ &= 2z_2^T P_2 (A_2 z_2 + \overline{B}_2 u \\ &+ \overline{B}_2 h_2 + \overline{B}_2 E_2 u) \\ &= -z_2^T Q_2 z_2 + 2z_2^T P_2 \overline{B}_2 u \\ &+ 2z_2^T P_2 \overline{B}_2 h_2 + 2z_2^T P_2 \overline{B}_2 E_2 u \\ &\le -\lambda_{\min}(Q_2) \|z_2\|^2 + 2\alpha_2^T u \\ &+ 2\alpha_2^T h_2 + 2\alpha_2^T E_2 u. \end{aligned} \tag{49}$$

If $\|\mu_2\| \ge \varepsilon_2$, then the second, third and fourth terms in (49) follow from (32), (34) and (36):

$$\begin{aligned} &2\alpha_2^T u + 2\alpha_2^T h_2 + 2\alpha_2^T E_2 u \\ &\le 2\alpha_2^T \left(-\frac{\mu_2}{\|\mu_2\|} \rho_2\right) + 2\|\alpha_2\| \overline{\rho_2} \\ &+ 2\alpha_2^T E_2 \left(-\frac{\mu_2}{\|\mu_2\|} \rho_2\right) \\ &\le -2\|\alpha_2\| \rho_2 + 2\|\alpha_2\| (1 + \lambda_{E_2}) \rho_2 \\ &- 2\lambda_{E_2} \|\alpha_2\| \rho_2 \\ &= 0. \end{aligned} \tag{50}$$

When $\|\mu_2\| \le \varepsilon_2$ it follows from Assumption 3 that

$$\begin{aligned} &2\alpha_2^T u + 2\alpha_2^T h_2 + 2\alpha_2^T E_2 u \\ &= 2\alpha_2^T \left(-\frac{\alpha_2}{\varepsilon_2} \rho_2^2\right) + 2\|\alpha_2\| \overline{\rho_2} \\ &+ 2\lambda_{E_2} \alpha_2^T \left(-\frac{\alpha_2}{\varepsilon_2} \rho_2^2\right) \\ &\le -2\left(\frac{\|\alpha_2\|^2}{\varepsilon_2} \rho_2^2\right) + 2\|\alpha_2\| (1 + \lambda_{E_2}) \rho_2 \\ &+ 2\lambda_{E_2} \left(-\frac{\|\alpha_2\|^2}{\varepsilon_2} \rho_2^2\right) \\ &= -2 \frac{\|\alpha_2\|^2}{\varepsilon_2} \rho_2^2 (1 + \lambda_{E_2}) \\ &+ 2\|\alpha_2\| (1 + \lambda_{E_2}) \rho_2 \\ &\le \frac{\varepsilon_2 (1 + \lambda_{E_2})}{2}. \end{aligned} \tag{51}$$

Therefore, we obtain

$$\dot V_2 \le -\lambda_{\min}(Q_2) \|z_2\|^2 + \frac{\varepsilon_2 (1 + \lambda_{E_2})}{2}. \tag{52}$$

From (48) and (52), it can be seen that

$$\begin{aligned} \dot V &= \dot V_1 + \dot V_2 \\ &\le -(\lambda_{\min}(Q_1) - \tau_1 \rho_k) \|z_1\|^2 + \frac{n\varepsilon_1}{2} \\ &- (\lambda_{\min}(Q_2) - \tau_1^{-1} \rho_k) \|z_2\|^2 \\ &+ \frac{\varepsilon_2 (1 + \lambda_{E_2})}{2}. \end{aligned} \tag{53}$$

If we choose matrices Q_1, Q_2 and the constant $\tau_1 > 0$ such that

$$\begin{aligned} &\lambda_{\min}(Q_1) - \tau_1 \rho_k > 0, \\ &\lambda_{\min}(Q_2) - \tau_1^{-1} \rho_k > 0, \end{aligned} \tag{54}$$

then we have

$$\begin{aligned} \dot V &\le -\min\{\lambda_{\min}(Q_1) - \tau_1 \rho_k, \lambda_{\min}(Q_2) - \tau_1^{-1} \rho_k\} \|z\|^2 \\ &+ \overline{\varepsilon} \\ &=: -\gamma_3 \|z\|^2 + \overline{\varepsilon}, \end{aligned} \tag{55}$$

where

$$\overline{\varepsilon} := \frac{n\varepsilon_1}{2} + \frac{\varepsilon_2 (1 + \lambda_{E_2})}{2}. \tag{56}$$ ■

Remark : If we first choose Q_1 and τ_1 to satisfy (54), then this affects the choice of L_{11} and L_{21} in matrix A_1. Consequently, q_l is chosen based on τ_1. However, we do not have specific criterion on choosing "best" gains L_{11}, L_{21}, L_{12} and L_{22} which is related to the system performance. Their "optimal" choice (which certainly depends on the specific "cost" indicated) is left for a further investigation.

Following (55), given $r_z \ge 0$ if $\|z_0\| \le r_z$, we can satisfy the requirements of uniform boundedness, uniform ultimate boundedness and uniform stability by selecting [15]

$$d_z(r_z) = \begin{cases} R_z \sqrt{\dfrac{\gamma_2}{\gamma_1}} & \text{if } r_z \le R_z \\ r_z \sqrt{\dfrac{\gamma_2}{\gamma_1}} & \text{if } r_z > R_z, \end{cases} \tag{57}$$

$$\begin{aligned} &T_z(\overline{d}_z, r_z) \\ &= \begin{cases} 0 & \text{if } r_z \le \overline{d}_z \sqrt{\dfrac{\gamma_1}{\gamma_2}} \\ \dfrac{\gamma_2 r_z^2 - \gamma_1^2 \gamma_2^{-1} \overline{d}_z^2}{\gamma_1 \gamma_2^{-1} \gamma_3 \overline{d}_z^2 - \overline{\varepsilon}} & \text{otherwise}, \end{cases} \end{aligned} \tag{58}$$

$$\delta_z(\overline{d}_z) = R_z, \tag{59}$$

where $\gamma_1 = \min\{\lambda_{\min}(P_1), \lambda_{\min}(P_2)\}$,

$\gamma_2 = \max\{\lambda_{\max}(P_1), \lambda_{\max}(P_2)\}$, $R_z = \sqrt{\dfrac{\overline{\varepsilon}}{\gamma_3}}$,

$\underline{d}_z = R_z \sqrt{\dfrac{\gamma_2}{\gamma_1}}$. Q.E.D.

Remark : Considering the above control scheme, we need less computation in case uncertain parameters are time-varying. The information about $\dot D(q_l)$ is needed in computing bounding function $\rho_1(\cdot)$ when we use Lyapunov function dependent on the inertia matrix. This is since skew-symmetric property is no longer utilized in system with time-varying uncertain parameter.

Based on the stability for the transformed system we can analyze the performance of the original system. The performance analysis of the original system is similar to [16]. Hence, we omit the detail. We can also prove that the original system is practically stable.

V. Illustrative example

Consider a 2-link flexible revolute joint manipulator (Figure 1). Let link angle vectors $q_l = [q^2 q^4]^T$ and joint angle vectors $q_j = [q^1 q^3]^T$. Then we have $D(q_l)$, $C(q_l, \dot q_l)$, $G(q_l)$, J, K, which are unknown as follows:

$$D(q_l)=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22}\end{bmatrix},$$

$$C(q_l,\dot{q}_l)=\begin{bmatrix} -m_2 l_1 l_{c2}\sin q^4 \dot{q}^4 & -m_2 l_1 l_{c2}\sin q^4(\dot{q}^4+\dot{q}^2) \\ m_2 l_1 l_{c2}\sin q^4 \dot{q}^2 & 0\end{bmatrix},$$

$$G(q_l)=\begin{bmatrix} (m_1 l_{c1}+m_2 l_1)g\sin q^2+m_2 l_{c2} g\sin(q^2+q^4) \\ m_2 l_{c2} g\sin(q^2+q^4)\end{bmatrix},$$

$$J=\begin{bmatrix} J_1 & 0 \\ 0 & J_2\end{bmatrix}=\begin{bmatrix} \overline{J_1} & 0 \\ 0 & \overline{J_2}\end{bmatrix}+\begin{bmatrix} \Delta J_1 & 0 \\ 0 & \Delta J_2\end{bmatrix},$$

$$K=\begin{bmatrix} K_1 & 0 \\ 0 & K_2\end{bmatrix}=\begin{bmatrix} \overline{K_1} & 0 \\ 0 & \overline{K_2}\end{bmatrix}+\begin{bmatrix} \Delta K_1 & 0 \\ 0 & \Delta K_2\end{bmatrix}. \tag{60}$$

where

$$\begin{aligned} d_{11} &:= 2a_{11}\cos(q^4)+a_{12}, \\ d_{12} &:= a_{11}\cos(q^4)+a_{22},\ d_{21}=d_{12}, \\ d_{22} &:= a_{22}, \\ a_{11} &= m_2 l_1 l_{c2}, \\ a_{12} &= m_2(l_1^2+l_{c2}^2)+m_1 l_{c1}^2+I_1+I_2, \\ a_{22} &= m_2 l_{c2}^2+I_2, \end{aligned} \tag{61}$$

The inertia matrix $D(q_l)$ entries are bounded with

$$\begin{aligned} |d_{11}| &\le 2a_{11}+a_{12}, \\ |d_{12}| &\le a_{11}+a_{22}, \\ |d_{22}| &\le a_{22}. \end{aligned} \tag{62}$$

$G(q_l)=[g_1\ g_2]^T$ entries are bounded with

$$\begin{aligned} |g_1| &\le g_{11}+g_{12}, \\ |g_2| &\le g_{21}, \\ g_{11} &= (m_1 l_{c1}+m_2 l_1)g, \\ g_{12} &= m_2 l_{c2}\, g, \\ g_{21} &= g_{12}. \end{aligned} \tag{63}$$

We consider first the system with constant uncertainty. Let $q_d^2=\pi$ radian, $q_d^4=0$ be desired position of links. We want links to be placed to the desired position (upside position) with keeping joint angles 0 radian.

Let $\widetilde{q}^2=q^2-q_d^2$ and $\widetilde{q}^4=q^4-q_d^4$ to represent position errors. $A_1, A_2, L_{11}, L_{21}, L_{12}$ and L_{21} have the following forms:

$$A_1=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -L_{111} & 0 & -L_{211} & 0 \\ 0 & -L_{112} & 0 & -L_{212}\end{bmatrix},\quad A_2=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -L_{121} & 0 & -L_{221} & 0 \\ 0 & -L_{122} & 0 & -L_{222}\end{bmatrix}. \tag{64}$$

The boundedness function ρ_1 is computed by

$$\rho_1=(\rho_{11}^2+\rho_{21}^2)^{\frac{1}{2}}, \tag{65}$$

where

$$\begin{aligned} \rho_{11} &= t_{11}+t_{21}\widetilde{q}^{2\,2}+t_{31}\dot{\widetilde{q}}^{2\,2}+t_{41}\widetilde{q}^{4\,2}+t_{51}\dot{\widetilde{q}}^{4\,2}, \\ \rho_{21} &= t_{12}+t_{22}\widetilde{q}^{2\,2}+t_{32}\dot{\widetilde{q}}^{2\,2}+t_{42}\widetilde{q}^{4\,2}+t_{52}\dot{\widetilde{q}}^{4\,2}. \end{aligned} \tag{66}$$

$$\begin{aligned} t_{11} &:= \|\overline{K}^{-1}\|\Big(\tfrac{1}{4}(2a_{11}+a_{12})(L_{111}+L_{211}) \\ &\quad +\tfrac{1}{4}(a_{11}+a_{22})(L_{112}+L_{212}) \\ &\quad +g_{11}+g_{12}+\tfrac{K_1}{4}+K_1 q_d^2\Big), \\ t_{21} &:= \|\overline{K}^{-1}\|((2a_{11}+a_{12})L_{111}+K_1), \\ t_{31} &:= \|\overline{K}^{-1}\|((2a_{11}+a_{12})L_{211}+a_{11}), \\ t_{41} &:= \|\overline{K}^{-1}\|((a_{11}+a_{22})L_{112}), \\ t_{51} &:= \|\overline{K}^{-1}\|((a_{11}+a_{22})L_{212}+2a_{11}), \\ t_{12} &:= \|\overline{K}^{-1}\|\Big(g_{12}+\tfrac{K_2}{4}+K_2 q_d^4 \\ &\quad +\tfrac{1}{4}(a_{11}+a_{22})(L_{111}+L_{211}) \\ &\quad +\tfrac{1}{4}a_{22}(L_{112}+L_{212})\Big), \\ t_{22} &:= \|\overline{K}^{-1}\|((a_{11}+a_{22})L_{111}), \\ t_{32} &:= \|\overline{K}^{-1}\|((a_{11}+a_{22})L_{211}+a_{11}), \\ t_{42} &:= \|\overline{K}^{-1}\|(a_{22}L_{112}+K_2), \\ t_{52} &:= \|\overline{K}^{-1}\|(a_{22}L_{212}). \end{aligned} \tag{67}$$

Here, $\widetilde{q}^{2\,2}$ denotes $\widetilde{q}^2\times\widetilde{q}^2$ and $\dot{\widetilde{q}}^{2\,2}$ denotes $\dot{\widetilde{q}}^2\times\dot{\widetilde{q}}^2$, etc.

This also shows that ρ_1 is C^2. Now, we have the following control:

$$u_1=[u_{11}\, u_{12}]^T, \tag{68}$$

$$u_{11}=\begin{cases} -\dfrac{\mu_{11}}{\|\mu_{11}\|}\rho_1, & \text{if } \|\mu_{11}\|>\varepsilon_1 \\ -\sin\!\left(\dfrac{\pi\mu_{11}}{2\varepsilon_1}\right)\rho_1, & \text{if } \|\mu_{11}\|\le\varepsilon_1, \end{cases} \tag{69}$$

$$u_{12}=\begin{cases} -\dfrac{\mu_{12}}{\|\mu_{12}\|}\rho_1, & \text{if } \|\mu_{12}\|>\varepsilon_1 \\ -\sin\!\left(\dfrac{\pi\mu_{12}}{2\varepsilon_1}\right)\rho_1, & \text{if } \|\mu_{12}\|\le\varepsilon_1, \end{cases} \tag{70}$$

$$\mu_1=[\mu_{11}\ \mu_{12}]^T=\overline{B_1}^T P_1[\widetilde{q}_l\ \ \dot{\widetilde{q}}_l]^T\rho_1, \tag{71}$$

$$\widetilde{q}_l=[q^2-q_d^2\ \ q^4-q_d^4]^T,\ \dot{\widetilde{q}}_l=[\dot{q}^2\ \ \dot{q}^4]^T. \tag{72}$$

We have robust control u:

$$u=\begin{cases} -\dfrac{\mu_2}{\|\mu_2\|}\rho_2 & \text{if } \|\mu_2\|>\varepsilon_2 \\ -\dfrac{\mu_2}{\varepsilon_2}\rho_2 & \text{if } \|\mu_2\|\le\varepsilon_2, \end{cases} \tag{73}$$

$$\mu_2=\overline{B_2}^T P_2[q_j-u_1\ \ \dot{q}_j-\dot{u}_1]^T\rho_2, \tag{74}$$

$$\|h_2\|\le\overline{\rho_2}\le(1+\lambda_{E_2})\rho_2, \tag{75}$$

$$\lambda_{E_2}=\min\left\{\frac{\Delta J_1}{\overline{J_1}},\ \frac{\Delta J_2}{\overline{J_2}}\right\}. \tag{76}$$

For simulations, we choose $m_1=m_2=1+0.2\sin(t)$, $l_1=1$, $l_{c2}=0.5$, $K_1=K_2=1+0.2\sin(t)$, $\overline{K_1}=\overline{K_2}=0.5$, $I_1=I_2=1$, $J_1=J_2=0.5$, $\overline{J_1}=\overline{J_2}=0.25$, $g=1$, $\varepsilon_1=150$, $\varepsilon_2=150$.

Next, choose $Q_l=Q_2=I_{4\times4}$, where $I_{4\times4}$ represents 4×4 identity matrix. Also, Choose $L_{111}=L_{112}=1$, $L_{211}=L_{212}=2$, and $L_{221}=L_{222}=2$. $L_{121}=L_{122}=1$, For these selected values we can get P_1, P_2 as followings:

$$P_1=P_2=\begin{bmatrix} 13.5 & 0 & 4.5 & 0 \\ 0 & 13.5 & 0 & 4.5 \\ 4.5 & 0 & 4.5 & 0 \\ 0 & 4.5 & 0 & 4.5\end{bmatrix}. \tag{77}$$

Simulation results are shown in Figures (2-7). Figures

2-4 show the system response with feedback linearization control which is partly adaptable to uncertain nonlinear system. We assume that the uncertainty portion takes 20% and the nominal part takes the rest 80% of the system. The system performance is not satisfactory due to a large steady state error. The feedback linearization control does not compensate the uncertainty portions. Figures 5-7 show the improvement of the performance by using the robust control. With the use of the proposed robust control, an improved system performance with respect to smaller settling time and steady state error is achieved in comparing to the feedback linearization controlled case.

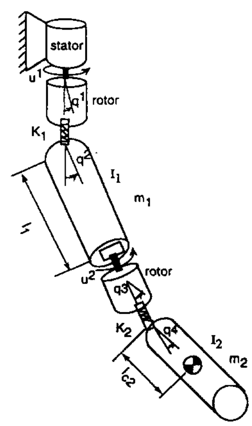

Fig. 1. 2-link flexible joint manipulator mechanism.

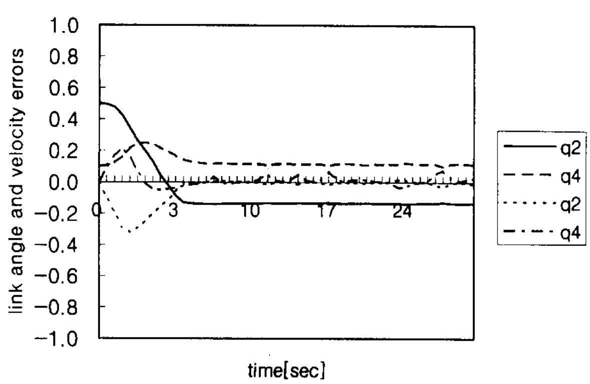

Fig. 2. Response history of link angles and angular velocity errors with feedback linearization control.

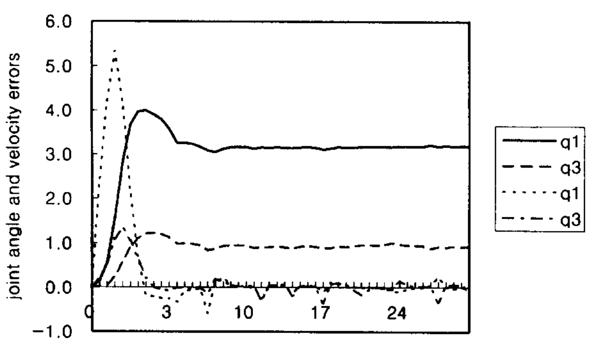

Fig. 3. Response history of joint angles and angular velocity errors with feedback linearization control.

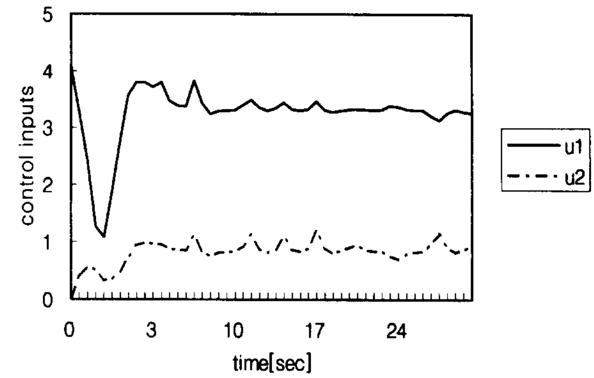

Fig. 4. Control input torques history with feedback linearization control.

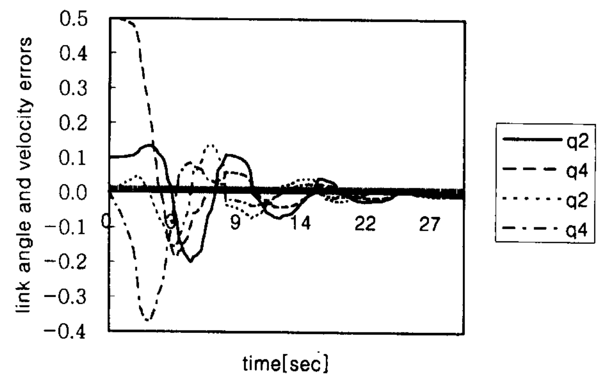

Fig. 5. Response history of link angle and angular velocity errors with robust control.

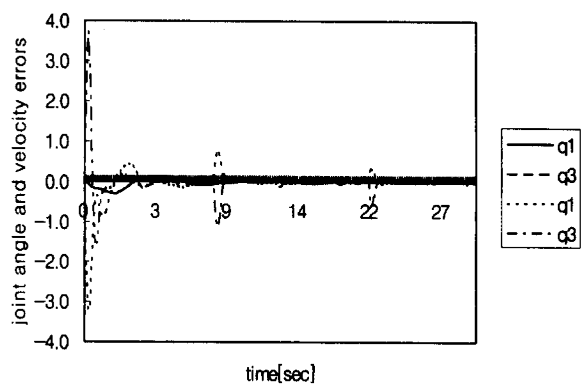

Fig. 6. Response history of joint angle and angular velocity errors with robust control.

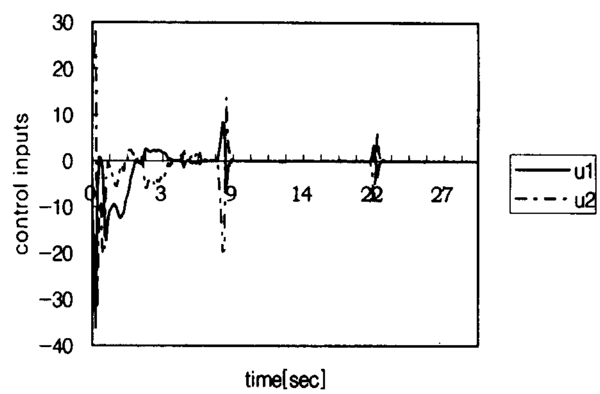

Fig. 7. Control input torques history with robust control.

VI. Conclusion

We have considered robust control schemes for the flexible joint manipulators the presence of nonlinearity and mismatched uncertainty by adopting computed torque scheme and state transformation via implanted control. By this way we overcome the difficulty of designing control in mismatched system. This scheme enables us to choose quadratic Lyapunov function candidates. Thus, for the system with time-varying or constant uncertain parameters we use the same bounding function. This fact overcomes the complexity of computation in system with time-varying parameter system. Furthermore, we can construct a control scheme based on the condition of the bounding function without using the bounding function explicitly. However, this scheme has some drawbacks in applications. There are constraints imposed on the boundedness of inertia matrix and stiffness matrix. Furthermore, the bound of those matrices needs to be constant and that bound depends on the dimension of system. However, these problems can be overcome by the assumption such that inertia matrix is known.

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