Journal of KSNVE, Vol. 8, No. 4, pp. 741~748, 1998.

〈Orignal Paper〉

Response Characteristics of Secondary Structures Subjected to Stationary Random Base Excitation

랜덤 기저가진을 받는 부-구조물의 응답특성

Choon-Eon Jin* and Chon-Wook Kim**
진 춘 언 · 김 천 욱

(Received March 14, 1998 : Accepted July 6, 1998)

Key Words : Primary-Secondary System (주-부구조계), Random Base Excitation (랜덤기저가진)

ABSTRACT

This paper examines a system consisting of a primary structure supporting relatively light multiple secondary structures. The primary structure is subjected to a stationary random base excitations modeled as a white noise. The response characteristics of the secondary structures are investigated in this paper. Proposed are the optimal tuning frequencies of the secondary structures at which the responses of the secondary structures are more evenly distributed resulting in the reduction of the maximum responses of the secondary structures while keeping the response of the primary structure near the minimum point.

요 약

본 연구에서는 랜덤 기저가진을 받는 주 구조물과 여러 개의 부 구조물로 구성된 계의 응답특성을 분석하고 특히, 부 구조물의 응답분포에 관하여 연구하였다. 주 구조물의 응답이 최소가 되도록 설계변수를 최적화 할 경우, 부 구조물간의 응답분포가 균일하지 않음을 확인하고, 부 구조물간의 응답분포의 폭이 최소가 되는 진동수 비를 제안하였다.

1. Introduction

There are many structures consisting of a primary structure supporting one or more relatively light secondary structures. Examples include a light water reactor vessel (RV) supporting a number of the control element drive mechanisms (CEDMs)[1], buildings containing equipment and piping, and any structure controlled by a vibration absorber or tuned mass damper (TMD).

One of the earliest studies of the primary-secondary systems was by Den Hartog[2] using a simple two-degree-of-freedom (2 DOF) model. He has formulated and resolved analytically the classical problem, in which an undamped single DOF primary structure, subjected to harmonic force excitation is equipped with a single DOF absorber. Kelly and Sackman[3] analyzed the modal properties of the two DOF system in detail, developing simple closed form expressions which provide insight into the

* 정회원, 한국전력기술주식회사 원자로설계개발단
** 연세대학교 공과대학 기계공학과

systems dynamic properties. Igusa and Der Kiureghim[4] extended their model to include the effects of damping and analyzed the response to a stationary input. Warburton and Ayorinde[5] have derived the optimum damping parameters for the undamped primary structure subjected to an harmonic support motion, where the acceleration amplitude is fixed for all input frequencies and other kind of harmonic excitation sources. The explicit formulae for the optimum parameters of the 2 DOF system are available under different types of system excitation[6~12].

Recently multiple tuned mass dampers (MTMD) with distributed natural frequencies have been proposed by Xu and Igusa[13,14], and also studied by Yamaguchi and Harnpornchai[15], Abe and Fujino[16], Jangid[17], Abe and Igusa[18] and Jangid and Datta[19]. Joshi and Jangid[20] presented the optimum parameters of the MTMD system minimizing the response of the primary structure subjected to a white noise stationary random process.

Previous studies referred above mainly focus on the response of the primary structure. It is due to the fact that the primary purpose of the systems they studied is to absorb vibration energy by interacting with the substructures, called TMD, to reduce the response of primary structure. For the secondary structures other than TMD, however, the responses of the secondary structures should also be kept low while reducing vibration of the primary structure. It should be considered that the secondary structures might be designed for the largest responses, even though the other substructures will not suffer from those high responses. In the present study, the responses of the multiple secondary structures are investigated and the optimal tuning parameters are proposed to distribute evenly the responses of the secondary structures while still reducing the response of the primary structure significantly. The system excitation is modeled as a ideal white noise stationary random process excited at the base.

2. Mathematical Modeling of the Primary-Secondary System

The system configuration consists of a primary structure supporting n-secondary structures as shown in Fig. 1. The primary structure and each secondary structure are modeled as a single-degree-of-freedom (SDOF) system.

The equation of motion of the system for the case of base excitation is expressed as

$$\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\mathbf{x}(t)=-\mathbf{M}_g\ddot{x}_g(t), \tag{1}$$

where $\mathbf{x}(t)$ is a displacement vector,

$$\mathbf{x}(t)=\{x_p(t)\ x_1(t)\ x_2(t)\ \cdots\ x_j(t)\ \cdots x_n(t)\}^T, \tag{2}$$

where x_p is a relative displacement of the primary structure to the ground, $x_j (j=1,2,\cdots,n)$ is a relative displacement of the j-th secondary structure to the primary mass, $x_g(t)$ is a ground displacement and **M**, **C**, and **K** are mass, damping, and stiffness matrices of the system, respectively, expressed

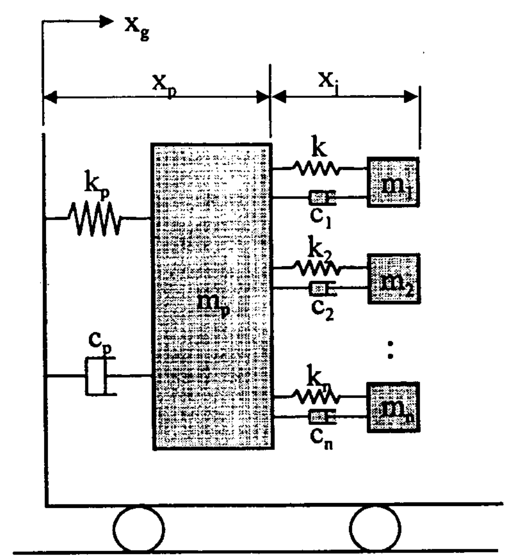

Fig. 1 The system composed of a SDOF primary structure multiple SDOF secondary structures

as:

$$\mathbf{M}=\begin{bmatrix} m_p+\sum_{j=1}^{n}m_j & m_1 & m_2 & \cdots & m_n \\ & m_1 & & 0 & \\ & & m_2 & & \\ & (Sym.) & & \ddots & \\ & & & & m_n \end{bmatrix}, \quad (3)$$

$$\mathbf{C}=\begin{bmatrix} c_p & & & & \\ & c_1 & & 0 & \\ & & c_2 & & \\ & 0 & & \ddots & \\ & & & & c_n \end{bmatrix}, \quad (4)$$

$$\mathbf{K}=\begin{bmatrix} k_p & & & & \\ & k_1 & & 0 & \\ & & k_2 & & \\ & 0 & & \ddots & \\ & & & & k_n \end{bmatrix}, \quad (5)$$

and

$$\mathbf{M}_g=\begin{Bmatrix} m_p+\sum_{j=1}^{n}m_j \\ m_1 \\ m_2 \\ \vdots \\ m_n \end{Bmatrix}. \quad (6)$$

The primary structure is characterized by natural frequency ω_p, damping ratio ζ_p and mass m_p while natural frequency, damping ratio and mass of the j-th secondary structure are ω_j, ζ_j and m_j, respectively. The complex frequency response functions for the relative displacement response of the primary structure $H_p(\omega)$ and the secondary structures $H_{s,j}(\omega)$ to the base acceleration can be derived from the expression in the Ref. (1) after some manipulation,

$$H_p(\omega)=\frac{1+\mu_t+\omega^2\sum_{j=1}^{n}\frac{\mu_j}{\omega_j^2-\omega^2+2i\varsigma_j\omega\omega_j}}{\begin{bmatrix}\omega_p^2-(1+\mu_t)\omega^2+2i\varsigma_p\omega\omega_p \\ -\omega^4\sum_{j=1}^{n}\frac{\mu_j}{\omega_j^2-\omega^2+2i\varsigma_j\omega\omega_j}\end{bmatrix}}. \quad (7)$$

$$H_{s,j}(\omega)=\frac{\omega_p^2+2i\varsigma_p\omega\omega_p}{\begin{bmatrix}\begin{Bmatrix}\omega_p^2-(1+\mu_t)\omega^2+2i\varsigma_p\omega\omega_p \\ -\omega^4\sum_{k=1}^{n}\frac{\mu_k}{\omega_k^2-\omega^2+2i\varsigma_k\omega\omega_k}\end{Bmatrix} \\ \left(\omega_j^2-\omega^2+2i\varsigma_j\omega\omega_j\right)\end{bmatrix}}, \quad (8)$$

where ω is an exciting harmonic frequency, μ_j is a ratio of the mass of the jth secondary structure to the mass of primary structure and μ_t is a ratio of the total mass of the secondary structures to the mass of primary structure, i.e.,

$$\mu_j=\frac{m_j}{m_p}, \qquad \mu_t=\sum_{j=1}^{n}\mu_j. \quad (9),(10)$$

and $i=\sqrt{-1}$.

3. Responses to a Stationary Random Excitation

If the base excitation is modeled as a stationary random process characterized by its power spectral density function (PSDF), then the PSDF of the relative displacement responses of the system are given by

$$S_{x_p}(\omega)=|H_p(\omega)|^2 S_{\ddot{x}_g}(\omega), \quad (11)$$

and

$$S_{x_j}(\omega)=|H_{s,j}(\omega)|^2 S_{\ddot{x}_g(\omega)}, \quad j=1,\ 2,\ \cdots,\ n \quad (12)$$

where $S_{\ddot{x}_g}(\omega)$ is the PSDF of the base acceleration. If the base excitation is assumed as an ideal white noise random process, then PSDFs of the relative displacements of the system become

$$S_{x_p}(\omega)=|H_p(\omega)|^2 S_0, \quad (13)$$

$$S_{x_j}(\omega)=|H_{s,j}(\omega)|^2 S_0. \quad (14)$$

The mean square relative displacements of the system are expressed as

$$E\left[x_p^{\,2}\right]=S_0\int_{-\infty}^{\infty}\left|H_p(\omega)\right|^2 d\omega,\quad -\infty\le\omega\le\infty, \tag{15}$$

for the primary structure and

$$E\left[x_j^{\,2}\right]=S_0\int_{-\infty}^{\infty}\left|H_{s,j}(\omega)\right|^2 d\omega,\quad -\infty\le\omega\le\infty, \tag{16}$$

for the secondary structures.

It can be seen that the mean square responses depend on the parameters of ζ_p, ζ_j μ_t, μ_j, f, β and n. In this paper, the response characteristics of the system are investigated for these parameters.

For simplicity of the parametric study, it is assumed that the masses and the damping factors of the subsystems are all the same, ie., m_s and ζ_s. It is also assumed that the natural frequencies of the secondary structures are distributed from ω_1 to ω_n with equal spacing. The natural frequency of the jth secondary structure is expressed as

$$\omega_j=\omega_p\left[f+\left(j-\frac{n+1}{2}\right)\frac{\beta}{n-1}\right], \tag{17}$$

where f is the ratio of the average frequency of the secondary structures ω_0 to the natural frequency of the primary structure, i.e.,

$$f=\frac{\omega_0}{\omega_p}, \tag{18}$$

where

$$\omega_0=\sum_{j=1}^{n}\frac{\omega_j}{n}. \tag{19}$$

The parameter β is the non-dimensional frequency bandwidth of the secondary structures defined as

$$\beta=\frac{\omega_n-\omega_1}{\omega_p}. \tag{20}$$

3.1 Mean Square Response of the Primary Structure

Among the parameters of the secondary structures, μ_t, f, β, and n are varied such that the mean square displacement of the primary structure attains the minimum value for given ζ_p and ζ_j. In Fig. 2 and Fig. 3, the variations of the optimal frequency bandwidth β^{opt}, and the optimal frequency ratio f_p^{opt}, minimizing the response of the primary structure, are plotted for the variation of the mass ratio μ_t with the number of secondary structure n as a parameter. The subscript p is used to indicate the parameter minimizing the primary response in contrary to f_s^{opt} minimizing the secondary responses presented in Section 3.2 of this paper.

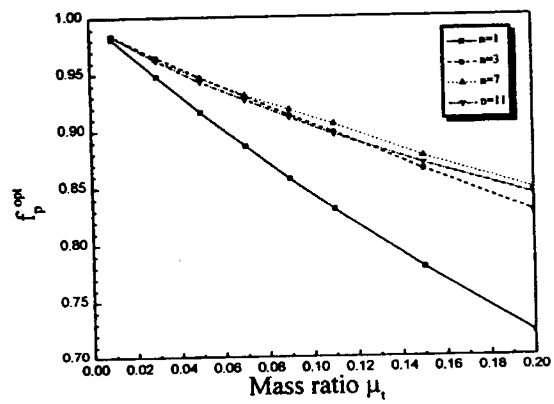

Fig. 2 Variation of the optimal frequency ratio, f_p^{opt} versus the mass ratio, μ_t, for $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

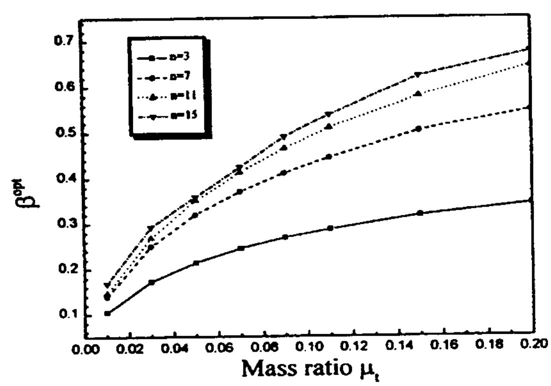

Fig. 3 Variation of the optimal frequency bandwidth, β^{opt} versus the mass ratio, μ_t, for $f=f_p^{opt}$, $\zeta_p=\zeta_s=0.02$

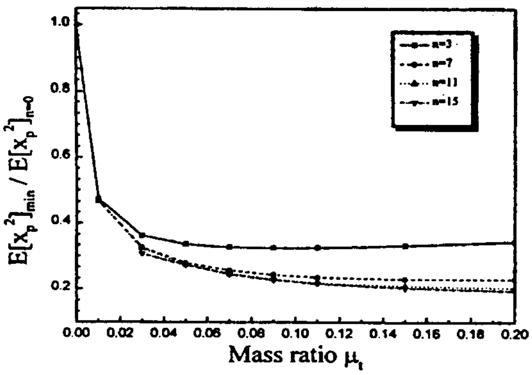

Fig. 4 Variation of the minimum response of the primary structure versus the mass ratio, μ_t, for $f=f_p^{opt}$, $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

Fig. 4 shows the variation of the mean square displacements of the primary structure versus the mass ratios for the different numbers of secondary structures when the system is at $\beta=\beta^{opt}$ and $f=f_p^{opt}$. The response is normalized by dividing the mean square response that the primary structure would have had under that same excitation if secondary structures had been removed completely, i.e., $n=0$. It can be seen from Fig. 4 that the primary response decreases with an increase in both the number of secondary structures and the total mass ratio. It should be noted that the increment of the number of secondary structures contributes in the reduction of the primary response with the total mass ratio μ_t is kept the same. There is an initial steep decrease in the response of the primary structure. As the number of secondary structures increases, however, the response remains almost constant beyond a certain values of the mass ratio and the number of secondary structures.

3.2 Mean Square Responses of Secondary Structures

Figure 5 through Fig. 7 show the variations of the responses of the secondary structures versus the variation of the frequency ratio f

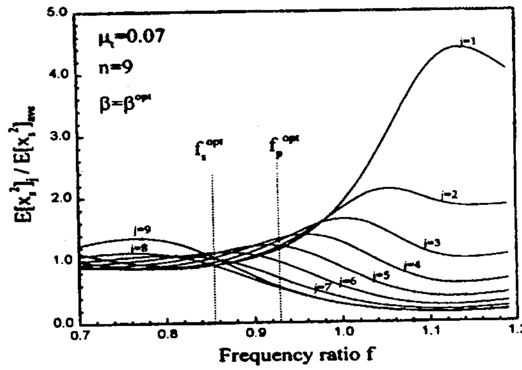

Fig. 5 Variation of the responses of the secondary structures versus the frequency ratio, f for $\mu_t=0.07$, $n=9$, $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

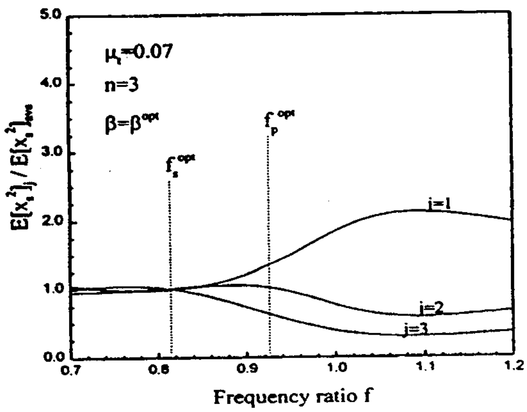

Fig. 6 Variation of the responses of the secondary structures versus the frequency ratio, f for $\mu_t=0.07$, $n=3$, $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

for different μ_t and n. The responses of the secondary structures are normalized by dividing the average responses of the secondary structures. The figures give insight into how the secondary structures are interacting with the primary structure. To analyze the distribution of the secondary responses, the response bandwidth B is defined as

$$B = E[x_s^2]_{max} - E[x_s^2]_{min}. \quad (21)$$

Larger B means that less numbers of the secondary structures are participating in the interaction with the primary structure, resulting in higher responses for more-participating secondary structures while lower responses for less-participating secondary structures. Narrower B indicates that most secondary structures are evenly sharing the interaction effects resulting in almost equal responses of the secondary structures. The optimal frequency ratio, f_s^{opt}, minimizing the response bandwidth of the secondary structure, B, is indicated in Fig. 5 through 7. In Fig. 7 there is an optimal point of P, near $f=0.65$. It should be noted, however, that the point P is far from the f_p^{opt}, which means that the response of the primary structure would increase greatly. Thus, f_s^{opt} should be taken as the first minimum point near f_p^{opt} as indicated in Fig. 5 through 7. In Fig. 8 is shown the variation of response bandwidth B versus the mass ratio μ_t. It can be seen that the optimal mass ratio exists minimizing response bandwidth for given damping factors of the system.

The optimal frequency ratio, f_s^{opt} is plotted in Fig. 9 for the variation of the mass ratios. Unlike f_p^{opt} presented in Fig. 2, f_s^{opt} increases beyond a certain value of the mass ratio. Fig. 10 shows the variation of the maximum responses of the secondary structure versus the mass ratio for different numbers of the secondary structures. The secondary responses

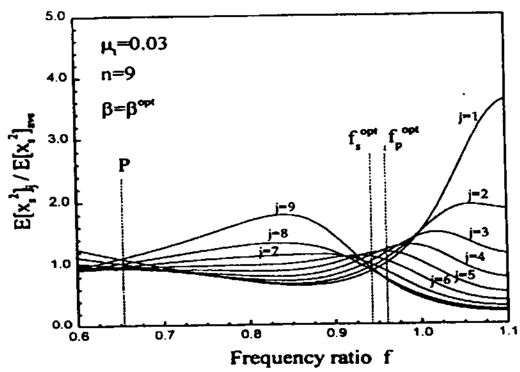

Fig. 7 Variation of the responses of the secondary structures versus the frequency ratio, f for $\mu_t=0.03$, $n=9$, $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

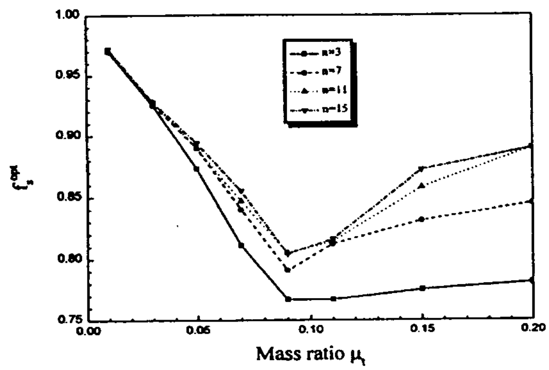

Fig. 9 Variation of the f_s^{opt} versus the mass ratio, μ_t, for $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

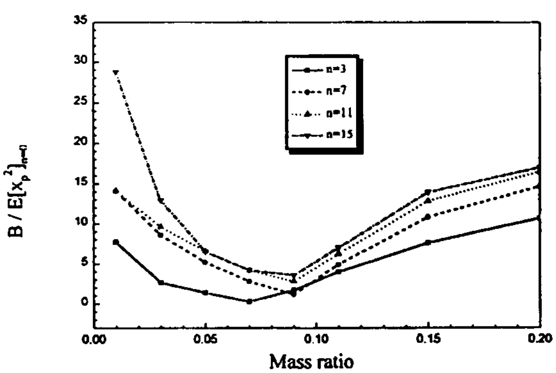

Fig. 8 Variation of the responses bandwidth of the secondary structures versus the mass ratio, μ_t for $f=f_s^{opt}$, $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

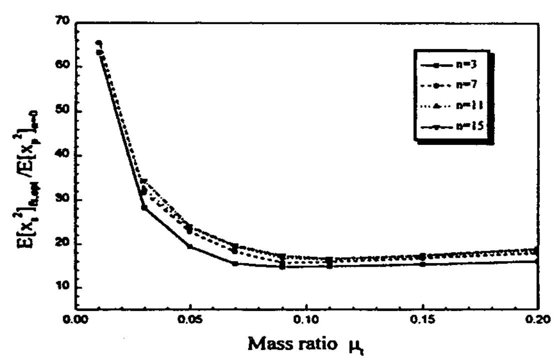

Fig. 10 Variation of the maximum responses of the secondary structures versus the mass ratio, μ_t, for $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$.

decrease with an increase of the mass ratio and remain constant above a certain value of the mass ratio.

Figure 11 shows the ratio of the maximum responses of the secondary structure at f_s^{opt} to those at f_p^{opt} versus the mass ratio. The ratio indicates how much the secondary responses are reduced from the responses when the system would result at the condition of minimizing primary response. It can be seen from Fig. 11 that the maximum response ratios decrease as the mass ratios increase. However as the mass ratio increases further after a certain value, the maximum response ratio increase. Figure 12 plots the responses of

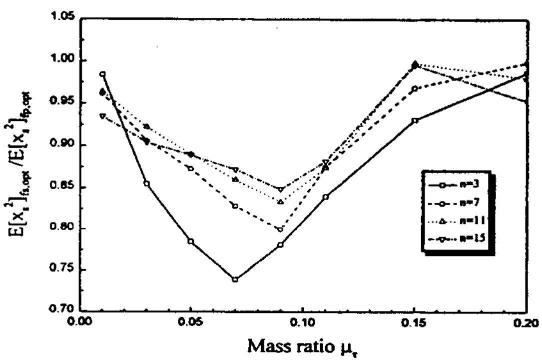

Fig. 11 Variation of the ratio of the maximum secondary responses at f_s^{opt} to those at f_s^{opt} for $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

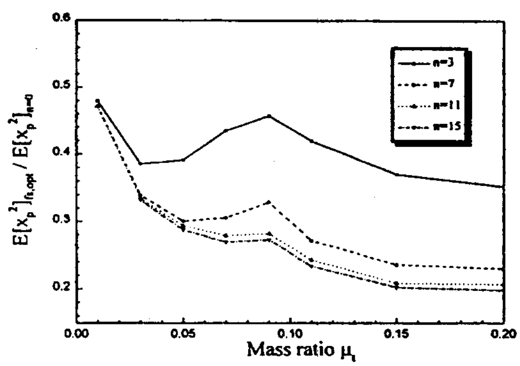

Fig. 12 Primary response when the system is tuned to f_s^{opt} instead of f_s^{opt} for $\beta=\beta^{opt}$, $\zeta_p=\zeta_s=0.02$

the primary structure when the system is tuned to f_s^{opt} instead of f_p^{opt}. Comparing with the plots shown in Fig. 4, the responses of the primary structure increase from the optimal condition. However it should be noted that the responses of the primary structure shown in Fig. 12 are still much lower than the responses without tuning by the secondary structures.

4. Conclusion

The dynamic responses of a system consisting of a primary structure supporting multiple secondary structures are investigated. The system excitation is modeled as a base excited stationary white noise random process. The response characteristics of the secondary structures are investigated to show that how the secondary structures are participating in the interaction with the primary structure. It is shown in this paper that the responses of the secondary structures are not evenly distributed at the optimal frequency ratio minimizing the responses of the primary structure. The frequency ratios at which the responses of the secondary structures evenly distributed resulting in reduction of the maximum responses of the secondary structures are proposed for different mass ratios and number of secondary structures. By tuning the system to the frequency ratio proposed in this paper, the responses of the secondary structures are reduced by around 30 percent depending on the mass ratio and the number of secondary structures, while still reducing the response of the primary structure 50 percent lower than the responses without secondary structures.

References

(1) Jin, C.E. and Kim, C.W., 1997, Dynamic Interactions between the Reactor Vessel and the CEDM of the Pressurized Water Reactor," J. of KSNVE, Vol. 7, No. 5, pp.

837～845.
(2) Den Hartog, J.P., 1956, Mechanical Vibration, McGraw-Hill, New York, fourth edition.
(3) Kelly, J.M. and Sackman, J.L., 1979, "Seismic Analysis of Internal Equipment and Components in Structures," Engineering Structures, Vol.1, No. 4, pp. 179～190.
(4) Igusa, T. and Der Kiureghian, 1985, "Dynamic Characterization of Two-degree-of-freedom Equipment-structure Systems," Journal of Engineering Mechanics, Vol. 111, pp. 1～19.
(5) Warburton, G.B. and Ayorinde, E.O., 1980, "Optimum Absorber Parameters for Simple System," Earthquake Engineering and Structural Dynamics, Vol. 8, pp. 197～217.
(6) Ayorinde, E.O. and Warburton, G.B., 1980, "Minimizing Structural Vibration with Absorbers," 1980, Earthquake Engineering and Structural Dynamics, Vol. 8, pp. 219～236.
(7) Warburton, G.B., 1981, "Optimum Absorber Parameters for Minimizing Vibration Response," Earthquake Engineering and Structural Dynamics, Vol. 9, pp. 251～262.
(8) Warburton, G.B., 1982, "Optimum Absorber Parameters for Various Combinations of Response and Excitation Parameters," Earthquake Engineering and Structural Dynamics, Vol. 10, pp. 381～401.
(9) Tsai, H. C. and Lin, G. C., 1993, "Optimum Tuned Mass Dampers for Minimizing Steady-State Response of Support Excited and Damped System," Earthquake Engineering and Structural Dynamics, Vol. 22, pp. 957～973.
(10) Tsai, H. C. and Lin, G. C., 1993, "Explicit Formulae for Optimum Absorber Parameters for Force Excited and Viscously Damped Systems," Journal of Sound and Vibration, Vol. 89, pp. 385～396.
(11) Thompson, A. G., 1981, "Optimum Tuning and Damping of a Dynamic Absorber Applied to a Force Excited and Damped Primary System," Journal of Sound and Vibration, Vol. 77, pp. 403～415.
(12) Fujino, Y. and Abe, M., 1993, "Design Formulas for Tuned Mass Dampers Based on a Perturbation Technique," Earthquake Engineering and Structural Dynamics, Vol. 22, pp. 833～854.
(13) Xu, K. and Igusa, T., 1992, "Dynamic Characteristics of Multiple substructures with Closely Spaced Frequencies," Earthquake Engineering and Structural Dynamics, Vol. 21, pp. 1059～1070.
(14) Igusa, T. and Xu, K., 1994, "Vibration Control using Multiple Tuned Mass Dampers," Journal of Sound and Vibration, Vol. 175, pp. 491～503.
(15) Yamaguchi, H. and Harnpornchai, N., 1993, "Fundamental Characteristics of Multiple Tuned Mass Dampers for Suppressing Harmonically Forced Oscillations," Earthquake Engineering and Structural Dynamics, Vol. 22, pp. 51～62.
(16) Abe, M. and Fujino, Y. 1994, "Dynamic Characterization of Multiple Tuned Mass Dampers and Some Design Formulae," Earthquake Engineering and Structural Dynamics, Vol. 23, pp. 813～835.
(17) Jangid, R. S., 1995, "Dynamic Characteristics of Structures with Multiple Tuned Mass Dampers," Structural Engineering and Mechanics, Vol. 3, pp. 497～509
(18) Abe, M. and Igusa, T. 1995, "Tuned Mass Dampers for Structures with Closely Spaced Frequencies," Earthquake Engineering and Structural Dynamics, Vol. 24, pp. 247～261.
(19) Jangid, R. S. and Datta, T. K., 1997, "Performance of Multiple Tuned Mass Dampers for Torsionally Coupled System," Earthquake Engineering and Structural Dynamics, Vol. 26, pp. 307～317.
(20) Joshi, A. S. and Jangid, R. S., 1997, "Optimum Parameters of Multiple Tuned Mass Dampers for Base-Excited Damped Systems," Journal of Sound and Vibration, Vol. 202, pp. 657～667.