Robust H_∞ Control of Discrete Uncertain Systems with Time Delays in States and Control Inputs

상태와 제어입력에 시간지연을 가지는 이산 불확실성 시스템의 견실 H_∞ 제어

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요 약 : 본 논문에서는 상태와 제어입력에 시간지연을 가지는 이산 불확실성 시스템의 견실 H_∞ 상태궤환 제어기 설계문제를 다룬다. 동일한 제어기에 대해서, 파라미터 불확실성을 가지는 시간지연 시스템이 자승적 안정성(quadratic stability)과 폐루프 시스템의 H_∞ 노옴의 한계를 유지하면서 파라미터 불확실성이 없는 등가의 시스템으로 변형된다. 그리고 주어진 이산 불확실성 시간지연 시스템의 견실 H_∞ 상태궤환 제어기가 존재할 충분조건과 제어기 설계 알고리듬을 제시한다. 또한 변수치환과 Schur 여수(complement) 정리를 이용하면 구한 충분조건은 LMI(linear matrix inequality) 형태로 쓸 수 있다. 예제를 통하여 제시한 결과의 타당성을 보인다.

Keywords : robust H_∞ control, discrete state feedback, time delay, LMI

I. Introduction

The dynamic behaviour of many physical processes contains inherent time delays and uncertainties and can be modeled by an uncertain system with time delay. For the case of parameter uncertain systems, it has been shown that quadratic stabilization with an H_∞ norm bound is equivalent to the existence of positive definite matrix to a certain ARE(algebraic Riccati equation)[1,2] or LMI (linear matrix inequality) [3]. Also, some works considered a robust stabilization of discrete time linear systems with norm-bounded time-varying uncertainty[4], and a robust H_∞ control for linear discrete time systems with norm-bounded time-varying uncertainty[5].

Recently time delay is main concerns because time delays often are the causes for instability and poor performance of control systems. Since some works of robust H_∞ controller design methods and software toolbox have been developed, many robust H_∞ state feedback controller design algorithms of uncertain time delay systems[6,7] were presented. However, there are some disadvantages in their works. Firstly, the results were conservative in pre-selection of some starting values. Secondly, their works did not consider delayed state and control input in the controlled output and parameter uncertainties in all system matrices. Finally, many related works treated robust H_∞ state feedback controller design algorithms in continuous time case only. Therefore our objective is to find solutions at a time without pre-determination of some variables using LMI technique in discrete uncertain time delay systems. Recently, Song and Kim[8] proposed an H_∞ control method of discrete time linear systems with norm-bounded uncertainties and time delay in state. However, their result is conservative in solving discrete ARE. Also they just considered parameter uncertainty in A matrix and delayed state. Therefore we deal with uncertain time delay system of parameter uncertainties in all system matrices and time delays in all states and control inputs.

In this paper, we propose a robust H_∞ state feedback controller design algorithm of generalized discrete time linear systems with time delays in both states and control inputs. It is shown that parameter uncertain delay systems with parameter uncertainties are equivalent to the auxiliary system without parameter uncertainties under preserving the quadratic stability and H_∞ norm bound of closed loop system. The existence condition and the design method of robust H_∞ state feedback controller are given. Through some changes of variables and Schur complement, the obtained sufficient condition can be rewritten as an LMI form in terms of all variables. Using LMI toolbox[9], the solutions can be easily obtained at a time.

II. Problem formulation

Consider a discrete uncertain time delay systems described by the difference delay equation

$$
\begin{aligned}
x(k+1) &= [A+\Delta A(k)]x(k)+[A_d+\Delta A_d(k)]x(k-d_1) \\
&\quad +[B_u+\Delta B_u(k)]u(k)+[B_d+\Delta B_d(k)]u(k-d_2) \\
&\quad +[B_w+\Delta B_w(k)]w(k) \\
z(k) &= [C+\Delta C(k)]x(k)+[C_{zd}+\Delta C_{zd}(k)]x(k-d_1) \\
&\quad +[D_{zu}+\Delta D_{zu}(k)]u(k)+[D_{zd}+\Delta D_{zd}(k)]u(k-d_2) \\
&\quad +[D_{zw}+\Delta D_{zw}(k)]w(k)
\end{aligned} \tag{1}
$$

where $x(k)\in\mathbf{R}^n$ is the state, $u(k)\in\mathbf{R}^m$ is the control input, $w(k)\in\mathbf{R}^l$ is the exogenous input, which belongs to $l_2[0,\infty)$, and $z(k)\in\mathbf{R}^p$ is the controlled

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output. All matrices have appropriate dimensions and we assume that all states are measurable for state feedback. The d_1 and d_2 are the positive integer time delay terms satisfying

$$0 \le d_i < \infty, \quad i=1,2. \tag{2}$$

The admissible parameter uncertainties are assumed to be the form

$$\begin{bmatrix} \Delta A(k) & \Delta B_u(k) & \Delta B_w(k) & \Delta A_d(k) & \Delta B_d(k) \\ \Delta C_z(k) & \Delta D_{zu}(k) & \Delta D_{zw}(k) & \Delta C_{zd}(k) & \Delta D_{zd}(k) \end{bmatrix} = \begin{bmatrix} H_x \\ H_z \end{bmatrix} F(k)[E_x \ E_u \ E_w \ E_{dx} \ E_{du}] \tag{3}$$

where unknown real time-varying matrix $F(k)$ is defined as

$$F(k) \in \Omega := \{F(k): F(k)^T F(k) \le I, \tag{4}$$

the elements of $F(k)$ are Lebesgue measurable}.

Definition 1 : The discrete uncertain time delay system (1)-(3) is quadratically stabilizable with H_∞ norm bound if there exists a linear memoryless state feedback control law

$$u(k) = Kx(k) \tag{5}$$

such that the resulting closed-loop system is quadratically stable for all admissible parameter uncertainties and time delays, and the H_∞ norm of the closed loop system is bounded by given value γ.

Now, we introduce a norm of signals that are sequences. We write $f=\{f_k\}_{-\infty}^{\infty}$, in which each $f_k \in \mathbf{R}^q$. The spaces $l_2(-\infty,\infty)$ is defined by

$$l_2(-\infty,\infty) = \{ f: \|f\|_2 < \infty \} \tag{6}$$

in which the norm is defined by

$$\|f\|_2 = \{ \sum_{-\infty}^{\infty} f_k^T f_k \}^{1/2} \tag{7}$$

and the space $l_2[0,\infty)$ is

$$l_2[0,\infty) = \{f \in l_2(-\infty,\infty): f_k = 0 \text{ for } k \le -1\}. \tag{8}$$

Therefore, the H_∞ norm of the closed-loop system T_{zw} in discrete time case is

$$\|T_{zw}\|_\infty = \sup_{w(k) \ne 0 \in l_2[0,\infty)} \frac{\|z(k)\|_2}{\|w(k)\|_2}. \tag{9}$$

Also, we discuss about the Schur complement used in this paper. One of the basic ideas of LMI problems is that nonlinear (convex) inequalities are converted to LMI form using Schur complements.

Fact 1[10] : For the symmetric matrix $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix}$, the following are equivalent as follows:

$$\begin{aligned} &i) \ L<0 \\ &ii) \ L_{11}<0, \quad L_{22}-L_{12}^T L_{11}^{-1} L_{12}<0 \\ &iii) \ L_{22}<0, \quad L_{11}-L_{12} L_{22}^{-1} L_{12}^T<0. \end{aligned} \tag{10}$$

Lemma 1 : The discrete uncertain time delay system (1)-(3) can be transformed into the time delay system without parameter uncertainties

$$\begin{aligned} x(k+1) &= Ax(k)+A_d x(k-d_1)+B_u u(k) \\ &\quad + B_d u(k-d_2) + [B_w \ \ \gamma\lambda H_x]\begin{bmatrix} w(k) \\ \tilde{w}(k) \end{bmatrix} \\ \begin{bmatrix} z(k) \\ \tilde{z}(k) \end{bmatrix} &= \begin{bmatrix} C_z \\ \frac{1}{\lambda}E_x \end{bmatrix} x(k) + \begin{bmatrix} C_{zd} \\ \frac{1}{\lambda}E_{dx} \end{bmatrix} x(k-d_1) \\ &\quad + \begin{bmatrix} D_{zu} \\ \frac{1}{\lambda}E_u \end{bmatrix} u(k) + \begin{bmatrix} D_{zd} \\ \frac{1}{\lambda}E_{du} \end{bmatrix} u(k-d_2) \\ &\quad + \begin{bmatrix} D_{zw} & \gamma\lambda H_z \\ \frac{1}{\lambda}E_w & 0 \end{bmatrix} \begin{bmatrix} w(k) \\ \tilde{w}(k) \end{bmatrix} \end{aligned} \tag{11}$$

under preserving quadratic stability and H_∞ norm bound through some manipulations[3-5]. The system (1) can be quadratically stable with H_∞ norm bound γ of the closed-loop system if and only if there exists a $\gamma>0$ such that the system (11) can be stabilized with its H_∞ norm less than γ by a state feedback control (5). In here, $\tilde{w}(k)$ and $\tilde{z}(k)$ are additional exogenous input and controlled output, respectively.

III. Robust H_∞ state feedback control

For simplicity of manipulation, rewrite the system (11) as follows

$$\begin{aligned} x(k+1) &= Ax(k)+A_d x(k-d_1)+\hat{B}\hat{w}(k) \\ &\quad + B_u u(k) + B_d u(k-d_2) \\ \hat{z}(k) &= \hat{C}x(k)+\hat{C_d}x(k-d_1)+\hat{D}\hat{w}(k) \\ &\quad + \hat{D_u}u(k)+\hat{D_d}u(k-d_2) \\ x(k) &= 0, \ k \le 0, \end{aligned} \tag{12}$$

where

$$\begin{aligned} &\hat{B} = [B_w \ \ \gamma\lambda H_x], \quad \hat{C} = \begin{bmatrix} C_z \\ \frac{1}{\lambda}E_x \end{bmatrix}, \\ &\hat{C_d} = \begin{bmatrix} C_{zd} \\ \frac{1}{\lambda}E_{dx} \end{bmatrix}, \quad \hat{D} = \begin{bmatrix} D_{zw} & \gamma\lambda H_z \\ \frac{1}{\lambda}E_w & 0 \end{bmatrix}, \\ &\hat{D_u} = \begin{bmatrix} D_{zu} \\ \frac{1}{\lambda}E_u \end{bmatrix}, \quad \hat{D_d} = \begin{bmatrix} D_{zd} \\ \frac{1}{\lambda}E_{du} \end{bmatrix}, \\ &\hat{w}(k) = \begin{bmatrix} w(k) \\ \tilde{w}(k) \end{bmatrix}, \quad \hat{z}(k) = \begin{bmatrix} z(k) \\ \tilde{z}(k) \end{bmatrix}. \end{aligned} \tag{13}$$

When we apply the control (5) to the delay system (12), the closed-loop system from $\hat{w}(k)$ to $\hat{z}(k)$ is given by

$$\begin{aligned} x(k+1) &= A_K x(k)+A_d x(k-d_1) \\ &\quad + \hat{B}\hat{w}(k)+B_d Kx(k-d_2) \\ \hat{z}(k) &= \hat{C_K}x(k)+\hat{C_d}x(k-d_1) \\ &\quad + \hat{D}\hat{w}(k)+\hat{D_d}Kx(k-d_2) \end{aligned} \tag{14}$$

where, $A_K = A+B_u K$ and $\hat{C_K} = \hat{C}+\hat{D_u}K$.

Lemma 2 : For a given $\gamma>0$ and $\lambda>0$, the system (12) is quadratically stable with an H_∞ norm bound γ with the controller (5) if there exist positive definite matrices P, R_1, and R_2 such that

$$\begin{bmatrix} -P^{-1} & A_K & A_d & B_d & \hat{B} & 0 \\ A_K^T & -P+R_1+K^TR_2K & 0 & 0 & 0 & \widehat{C_K}^T \\ A_d^T & 0 & -R_1 & 0 & 0 & \widehat{C_d}^T \\ B_d^T & 0 & 0 & -R_2 & 0 & \widehat{D_d}^T \\ \hat{B}^T & 0 & 0 & 0 & -\gamma^2 I & \hat{D}^T \\ 0 & \widehat{C_K} & \widehat{C_d} & \widehat{D_d} & \hat{D} & -I \end{bmatrix} < 0 \quad (15)$$

holds for the time delays (2).

Proof : Firstly, we define a Lyapunov functional as

$$V(x(k)) := x(k)^TPx(k) + \sum_{i=k-d_1}^{k-1} x(i)^TR_1x(i) + \sum_{i=k-d_2}^{k-1} x(i)^TK^TR_2Kx(i). \quad (16)$$

And it is noticed that the condition (15) implies

$$\begin{bmatrix} A_K^TPA_K - P + R_1 + K^TR_2K & A_K^TPA_d & A_K^TPB_d \\ A_d^TPA_K & -R_1 + A_d^TPA_d & A_d^TPB_d \\ B_d^TPA_K & B_d^TPA_d & -R_2 + B_d^TPB_d \end{bmatrix} < 0. \quad (17)$$

Taking the difference of the Lyapunov functional (16) yields

$$\begin{aligned} \Delta V_k &= V(x(k+1)) - V(x(k)) \\ &= x(k+1)^TPx(k+1) \\ &\quad - x(k)^T(P - R_1 - K^TR_2K)x(k) \\ &\quad - x(k-d_1)^TR_1x(k-d_1) \\ &\quad - x(k-d_2)^TK^TR_2Kx(k-d_2). \end{aligned} \quad (18)$$

When assuming zero input, we have

$$\Delta V_k = \begin{bmatrix} x(k) \\ x(k-d_1) \\ Kx(k-d_2) \end{bmatrix}^T \begin{bmatrix} A_K^TPA_K - P + R_1K^TR_2K & A_K^TPA_d & A_K^TPB_d \\ A_d^TPA_K & -R_1 + A_d^TPA_d & A_d^TPB_d \\ B_d^TPA_K & B_d^TPA_d & -R_2 + B_d^TPB_d \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-d_1) \\ Kx(k-d_2) \end{bmatrix} < 0, \quad (19)$$

which ensures the quadratic stability of the closed-loop system (14). In the next place, assume the zero initial condition and introduce

$$J = \sum_{k=0}^{\infty} [\hat{z}(k)^T\hat{z}(k) - \gamma^2 \hat{w}(k)^T\hat{w}(k)]. \quad (20)$$

Noting

$$J \le \sum_{k=0}^{\infty} [\hat{z}(k)^T\hat{z}(k) - \gamma^2 \hat{w}(k)^T\hat{w}(k) + \Delta V_k] \quad (21)$$

and further substituting (18) into (21) and let $\delta(k) = [x(k)^T \; x(k-d_1)^T \; x(k-d_2)^TK^T \; \hat{w}(k)^T]^T$, then

$$J \le \sum_{k=0}^{\infty} \delta(k)^T Z \delta(k), \quad (22)$$

where Z is defined

$$Z = \begin{bmatrix} H & A_K^TPA_d + \widehat{C_K}^T\widehat{C_d} & A_K^TPB_d + \widehat{C_K}^T\widehat{D_d} & A_K^TP\hat{B} + \widehat{C_K}^T\hat{D} \\ * & -R_1 + A_d^TPA_d + \widehat{C_d}^T\widehat{C_d} & A_d^TPB_d + \widehat{C_d}^T\widehat{D_d} & A_d^TP\hat{B} + \widehat{C_d}^T\hat{D} \\ * & * & -R_2 + B_d^TPB_d + \widehat{D_d}^T\widehat{D_d} & B_d^TP\hat{B} + \widehat{D_d}^T\hat{D} \\ * & * & * & -\gamma^2 I + \hat{B}^TP\hat{B} + \hat{D}^T\hat{D} \end{bmatrix} \quad (23)$$

where $*$ means symmetric term and $H = A_K^TPA_K - P + R_1 + K^TR_2K + \widehat{C_K}^T\widehat{C_K}$. Therefore when $Z<0$, $k \ge 0$, the system (12) is quadratically stable with an H_∞ norm bound γ. Using the fact 1, $Z<0$ in (23) is transformed into (15). ■

Theorem 1 : Consider the discrete time delay system (12). For given γ and λ, if there exist positive definite matrices Q, S_1, S_2 and a matrix M such that

$$\begin{bmatrix} -Q + B_dS_2B_d^T + A_dS_1A_d^T & AQ + B_uM & \hat{B} & B_dS_2\widehat{D_d}^T + A_dS_1\widehat{C_d}^T & 0 & 0 \\ * & -Q & 0 & Q\hat{C}^T + M^T\widehat{D_u}^T & M^T & Q \\ * & * & -\gamma^2 I & \hat{D}^T & 0 & 0 \\ * & * & * & -I + \widehat{C_d}S_1\widehat{C_d}^T + \widehat{D_d}S_2\widehat{D_d}^T & 0 & 0 \\ * & * & * & * & -S_2 & 0 \\ * & * & * & * & * & -S_1 \end{bmatrix} < 0 \quad (24)$$

holds for time delays (2), then the closed-loop system (14) is quadratically stable with an H_∞ norm bound γ. In here, some variables are defined as follows:

$$\begin{aligned} M &= KP^{-1} \\ Q &= P^{-1} \\ S_i &= R_i^{-1}, \; i = 1, 2. \end{aligned} \quad (25)$$

Proof : Using the Fact 1 and some changes of variables, the proof is completed. the inequality (15) of lemma 2 is equivalent to

$$\begin{bmatrix} -P^{-1} & A_K & A_d & B_d & \hat{B} & 0 & 0 \\ * & -P+R_1 & 0 & 0 & 0 & \widehat{C_K}^T & K^T \\ * & * & -R_1 & 0 & 0 & \widehat{C_d}^T & 0 \\ * & * & * & -R_2 & 0 & \widehat{D_d}^T & 0 \\ * & * & * & * & -\gamma^2 I & \hat{D}^T & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & -R_2^{-1} \end{bmatrix} < 0 \quad (26)$$

$\Leftrightarrow$

$$\begin{bmatrix} -P^{-1} + B_dR_2^{-1}B_d^T & A_K & A_d & \hat{B} & B_dR_2^{-1}\widehat{D_d}^T & 0 \\ * & -P+R_1 & 0 & 0 & \widehat{C_K}^T & K^T \\ * & * & -R_1 & 0 & \widehat{C_d}^T & 0 \\ * & * & * & -\gamma^2 I & \hat{D}^T & 0 \\ * & * & * & * & -I + \widehat{D_d}R_2^{-1}\widehat{D_d}^T & 0 \\ * & * & * & * & * & -R_2^{-1} \end{bmatrix} < 0 \quad (27)$$

$\Leftrightarrow$

$$\begin{bmatrix} -P^{-1} + B_dR_2^{-1}B_d^T & A_K & A_d & \hat{B} & B_dR_2^{-1}\widehat{D_d}^T & 0 & 0 \\ * & -P & 0 & 0 & \widehat{C_K}^T & K^T & I \\ * & * & -R_1 & 0 & \widehat{C_d}^T & 0 & 0 \\ * & * & * & -\gamma^2 I & \hat{D}^T & 0 & 0 \\ * & * & * & * & -I + \widehat{D_d}R_2^{-1}\widehat{D_d}^T & 0 & 0 \\ * & * & * & * & * & -R_2^{-1} & 0 \\ * & * & * & * & * & * & -R_1^{-1} \end{bmatrix} < 0 \quad (28)$$

$\Leftrightarrow$

$$\begin{bmatrix} -P^{-1}+B_dR_2^{-1}B_d^T+A_dR_1^{-1}A_d^T & A_K & \hat{B} & B_dR_2^{-1}\hat{D}_d^T+A_dR_1^{-1}\hat{C}_d^T & 0 & 0 \\ * & -P & 0 & \hat{C}_K^T & K^T & I \\ * & * & -\gamma^2 I & \hat{D}^T & 0 & 0 \\ * & * & * & -I+\hat{C}_dR_1^{-1}\hat{C}_d^T+\hat{D}_dR_2^{-1}\hat{D}_d^T & 0 & 0 \\ * & * & * & * & -R_2^{-1} & 0 \\ * & * & * & * & * & -R_1^{-1} \end{bmatrix} < 0 \quad (29)$$

$\Leftrightarrow$

$$\begin{bmatrix} -P^{-1}+B_dR_2^{-1}B_d^T+A_dR_1^{-1}A_d^T & A_KP^{-1} & \hat{B} & B_dR_2^{-1}\hat{D}_d^T+A_dR_1^{-1}\hat{C}_d^T & 0 & 0 \\ * & -P^{-1} & 0 & P^{-1}\hat{C}_K^T & P^{-1}K^T & P^{-1} \\ * & * & -\gamma^2 I & \hat{D}^T & 0 & 0 \\ * & * & * & -I+\hat{C}_dR_1^{-1}\hat{C}_d^T+\hat{D}_dR_2^{-1}\hat{D}_d^T & 0 & 0 \\ * & * & * & * & -R_2^{-1} & 0 \\ * & * & * & * & * & -R_1^{-1} \end{bmatrix} < 0. \quad (30)$$

Using some changes of variables, $M=KP^{-1}$, $Q=P^{-1}$, and $S_i=R_i^{-1}$, $i=1,2$, (30) is changed to (24). ■

(24) is an LMI form in terms of Q, M, S_1, and S_2. Therefore the robust H_∞ state feedback controller gain K can be calculated from the $M=KP^{-1}$ after finding the LMI solutions, Q, M, S_1, and S_2 from the (24) and (25). Using the LMI Toolbox[9], the solutions can be easily obtained at a time because (24) is an LMI form in terms of variables.

Example : Consider a discrete uncertain time delay system

$$\begin{aligned} x(k+1) &= \left\{\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}+\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}F(k)[1 \;\; 1]\right\}x(k) \\ &+\left\{\begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix}+\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}F(k)[0.1 \;\; 0.1]\right\}x(k-d_1) \\ &+\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}+\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}F(k)\right\}u(k) \\ &+\left\{\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}+\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}F(k)0.1\right\}u(k-d_2) \\ &+\left\{\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}+\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}F(k)0.1\right\}w(k) \\ z(k) &= \{[1 \;\; 1]+0.1F(k)[1 \;\; 1]\}x(k) \\ &+\{[0.1 \;\; 0.1]+0.1F(k)[0.1 \;\; 0.1]\}x(k-d_1) \\ &+\{1+0.1F(k)\}u(k) \\ &+\{0.1+0.1F(k)0.1\}u(k-d_2) \\ &+\{0.1+0.1F(k)0.1\}w(k). \end{aligned} \quad (31)$$

If we take $\gamma=1$ and $\lambda=1$, the system (31) is changed to equivalent system as follows:

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}x(k)+\begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.1 \end{bmatrix}x(k-d_1) \\ &+\begin{bmatrix} 1 \\ 1 \end{bmatrix}u(k)+\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}u(k-d_2)+\begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}\hat{w}(k) \\ \hat{z}(k) &= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}x(k)+\begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix}x(k-d_1) \\ &+\begin{bmatrix} 1 \\ 1 \end{bmatrix}u(k)+\begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}u(k-d_2)+\begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0 \end{bmatrix}\hat{w}(k). \end{aligned} \quad (32)$$

Using LMI toolbox, all solutions are obtained at the same time as follows:

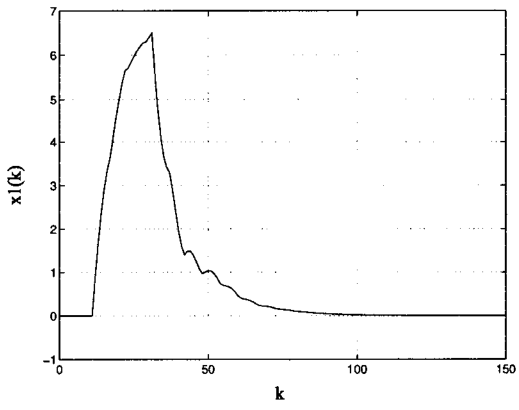
$$\begin{aligned} P &= \begin{bmatrix} 0.9383 & 0.2553 \\ 0.2553 & 0.2173 \end{bmatrix}, \\ R_1 &= \begin{bmatrix} 0.1585 & 0.0949 \\ 0.0949 & 0.0895 \end{bmatrix}, \\ R_2 &= 0.0918, \\ M &= [-0.0535 \quad -4.5303]. \end{aligned} \quad (33)$$

Therefore the final robust H_∞ state feedback gain is obtained as

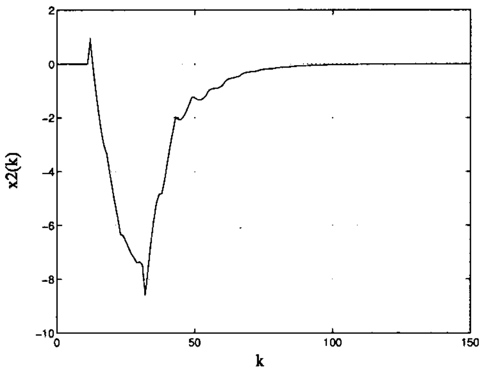
$$K=[-1.2069 \quad -0.9982]. \quad (34)$$

The simulation results are shown in Fig. 1. The trajectories of state converge to zero as time goes to infinity in (a) and (b) of Fig. 1. From this result, the obtained controller stabilizes the discrete parameter uncertain system with time delays against time delays and disturbance exogenous input. Also the H_∞ norm bound of the closed-loop system can be calculated by induced norm property between $w(k)$ and $z(k)$. Therefore we investigate that the value of γ is less than given value from the definition (9). Actually, the value of γ is 0.0905(<1) from the (c) and (d) of Fig. 1. Here, the initial value of states is zero, time delays are $d_1=5$, $d_2=10$, $F(k)=\sin k$, and the value of $w(k)$ is defined by

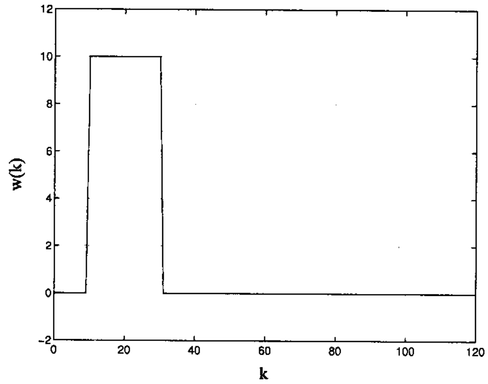
$$w(k) = \begin{cases} 10, & \text{if } 10 \le k \le 30 \\ 0, & \text{otherwise.} \end{cases} \quad (35)$$



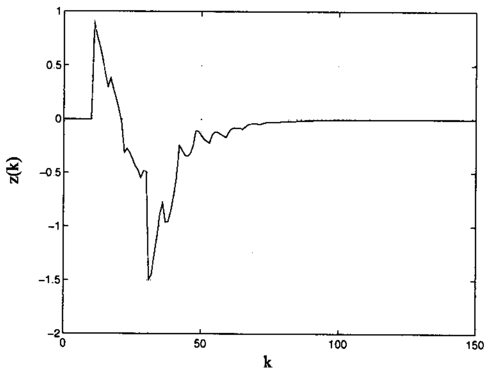
(a) $x_1(k)$



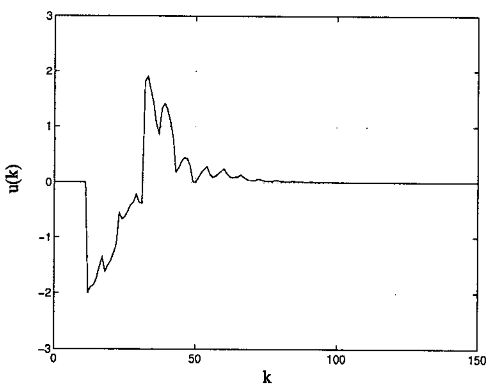
(b) $x_2(k)$



(c) $w(k)$



(d) $z(k)$



(e) $u(k)$

Fig. 1. The trajectories of states, exogenous input, controlled output, and control input.

IV. Conclusion

In this paper, we presented a design method of robust H_∞ state feedback controller of discrete parameter uncertain time delay systems. The uncertain time delay system problems were solved on the basis of LMI technique. Therefore the robust H_∞ state feedback controller was obtained at a time without pre-selection of some variables. The obtained controller guarantees the quadratic stability and H_∞ norm bound of the closed-loop system. An example showed the effectiveness of the proposed algorithm.

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