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A Pole Assignment in a Specified Disk by using Hamiltonian Properties

해밀톤 행렬의 성질을 이용한 지정된 디스크내의 극 배치법

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요   약 : 본 논문에서는 선형 시불변 시스템에 대해 상태되먹임을 이용한 폐루프계의 지정된 영역내의 극배치법을 제안한다. 본 제안된 기법은 해밀톤 행렬의 하중행렬 Q의 설정에 의해 지정된 영역 (α 중심, γ 반경)내에 극배치가 가 능함을 보인다. 먼저, Gershgorin의 이론을 적용하기 위해 해밀톤 행렬을 등가 변환시킨 후 행렬의 각 계수를 α 와 γ 의 관계를 이용하여 유도한다. 위의 관계를 만족하는 해밀톤 행렬의 각 하중행렬과 변환행렬을 이용하여 폐루프계의 상태 되먹임 제어칙을 구한다. 또한 본 기법은 해밀톤 행렬과 최적제어와의 관계를 지니고 있으므로 얻어진 폐루프계는 최적 제어법에서와 동일한 강인함을 가지게 된다. 끝으로 예제를 통하여 지정된 영역내의 극배치가 이루어짐을 보인다.

Keywords : pole assignment, specified disk, Gershgorin's theorem, Hamiltonian matrix

I. Introduction

The basic idea of pole assignment design is really quite simple. If a system is composed of n state variables and every state variable is available as a measurable output, the closed loop poles of the system to be controlled by state feedback can be arbitrarily assigned using n independent feedback gain terms. Practically, pole assignment design can be used to modify the dynamic response of the given system, such as damping ratio, overshoot, etc., and it has been of fundamental importance in the control system discipline for many years and numerous algorithms have been proposed.

Shafai and Bhattacharyya [1] transformed a given multi-input system to an upper block Hessenberg form by means of orthogonal state coordinate trans-formations. The state feedback problem is then reformulated in terms of Sylvester's equation. Therefore, the transformed system matrices along with certain assumed block forms for unknown matrices enable the Sylvester equation to be decom-posed and solved effectively. A distinct point of the proposed algorithm is that the solution procedure can be tailored to parallel implementation and is therefore fast, but this method can not be used in case of a system that has a singular input matrix. In [2], the problem in analysis of root clustering has been derived by using an extended Lyapunov equation that provides necessary and sufficient conditions for a given matrix to have all its eigenvalues in the specified regions. The conditions are formulated in terms of semi-positive definite matrices which cons-titute a parameterization of the feedback control.

Another method working with the Lyapunov equation for control law design is proposed in [3].

More recently, mathematical algebraic tests for the spectrum of the state matrix to be clustered in a desired region of the complex plane were developed. In this direction, efforts to construct a modified Lyapunov matrix equation leading to efficient syn-thesis procedures have been made [4]. Furuta and Kim [5] proposed a method for pole assignment in a specified disk by using the well known discrete Riccati equation. In their proposed algorithm, the state feedback control law is determined by using the solution of a discrete Riccati equation which can be computed directly using the design specification parameters. Also they proposed a linear fractional transformation method to solve the same types of pole assignment problems [7]. Most works in analysis of root clustering are based on Lyapunov or Riccati equations, but there is no method incorporating the Hamiltonian characteristics with the property of disks.

This paper presents a different method to assign the closed loop pole in a specified disk by a state feedback for linear continuous time-invariant systems under the condition of given system's distinct poles by incorporating Gershgorin's theorem into a Hamil-tonian matrix. Firstly, the Hamiltonian matrix is chosen such that all its eigenvalues are located in a specified disk by using Gershgorin's theorems. Secondly, a feedback control law is determined using the similarity and the properties of the chosen Hamiltonian matrix and a different Hamiltonian matrix constructed from the matrices of the given system. The obtained feedback control law not only gives the closed loop system with desired eigenvalues, but also minimizes a quadratic perfor-mance index. Also, it can be extended to assign the

poles into sub-specified regions.

II. Preliminaries

1. Problem statement

Let us consider a linear continuous time-invariant dynamic system

$$\dot{x} = Ax + Bu \tag{1}$$

where $x \in R^n$, $u \in R^m$ and $y \in R^p$ are respectively the state, control input and output vectors. The matrices in (1) are known with appropriate dimension and it is assumed that the pair (A, B) is controllable.

The problem to be considered is to determine the state feedback law

$$u = Fx \tag{2}$$

such that the closed-loop system poles lie in the disk $D(-a, r)$ with the center $(-a + j0)$ and the radius r, (a and r are any positive scalars), and this state feedback control law minimizes a quadratic performance index

$$J = \frac{1}{2} \int_0^\infty [x^T Q x + u^T R u] \, dt \tag{3}$$

where Q is a symmetric positive semi-definite matrix written as $Q = C^T C$, C is any matrix that the pair (A, C) is observable and R is a symmetric positive definite matrix.

2. Basic theory

In this subsection, we introduce some theorems which are the basic theory of the proposed method.

Theorem 1 [10] : (Gershgorin's theorem)

Let λ be a distinct eigenvalue of an arbitrary matrix

$$A = [a_{ij}] \in R^{n \times n}$$

Then for some integer i ($1 \le i \le n$), we have

$$|a_{ii} - \lambda| \le |a_{i1}| + |a_{i2}| + \cdots + |a_{ii-1}|$$
$$+ |a_{ii+1}| + \cdots + |a_{in}| \tag{4}$$

For each $i = 1, \cdots, n$, the inequality, (4) determines a closed circular disk in the complex λ plane whose center is at a_{ii} and radius is given by the expression on the right hand side of (4).

Theorem 1 states that each of the eigenvalues of A lies in one of these n disks.

Theorem 2[10] : (Extension of Gershgorin's theorem) If p Gershgorin's disks from a set S are disjoint from the $(n-p)$ other disks of the given matrix A, then S contains precisely p eigenvalues of A.

Theorem 3[8]: The optimal control law for the controllable system, (1) which minimizes the performance index, (3) is given by

$$u = -R^{-1} B^T P x = Fx \tag{5}$$

where P is the symmetric, positive definite matrix

solution of the algebraic Riccati equation

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \tag{6}$$

$$Q = Q^T \ge 0, \quad R = R^T > 0$$

III. Pole assignment

It is a well known the fact that the coefficient matrix H

$$H = \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \tag{7}$$

has n eigenvalues with negative real parts and n eigenvalues with positive real parts, and those eigenvalues are located symmetrically about the imaginary axis. The eigenvalues of the optimal feedback system

$$\dot{x} = (A + BF)x \tag{8}$$

are identical to those eigenvalues of the matrix, (7) that have negative real parts. It is, therefore, possible to assign the eigenvalues of the matrix, (7) instead of the eigenvalues of the feedback system $(A + BF)$.

Let us define an equivalent transformed matrix, $\overline{H}$ of the Hamiltonian matrix, H as the following

$$\overline{H} = T^{-1} H T \tag{9}$$

where T is a non-singular matrix.

The particular problem to be considered here is to establish the matrix $\overline{H}$ that is equivalent to the matrix H by using Gershgorin's theorem. This means that the eigenvalues of the matrices H are same with the matrix $\overline{H}$. If the eigenvalues with negative real parts of the matrices H or $\overline{H}$ are located in the disk $D(-a, r)$ in the left half complex plane, then the eigenvalues with positive real parts of the matrices H or $\overline{H}$ are located in the disk $D(a, r)$ in the right half complex plane. Those disks $D(-a, r)$ and $D(a, r)$ are symmetric about the imaginary axis.

Because the matrix H is partitioned into four blocks as shown in (7), we can separate the matrices $\overline{H}$ and T by the same way.

$$T = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix}, \quad \overline{H} = \begin{bmatrix} \overline{H_1} & \overline{H_2} \\ \overline{H_3} & \overline{H_4} \end{bmatrix}$$

where $\overline{H_1}, \overline{H_2}, \overline{H_3}, \overline{H_4}, T_1, T_2, T_3$, and T_4 are matrices with $(n \times n)$ elements.

Now (9) becomes

$$\begin{bmatrix} \overline{H_1} & \overline{H_2} \\ \overline{H_3} & \overline{H_4} \end{bmatrix} := \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix}^{-1}$$

$$\times \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} \tag{10}$$

For the sake of simplicity, let $T_3 = 0$. If the sub-matrices T_1 and T_4 are non-singular, then the

matrix T is non-singular [8], and (10) can be written in the form

$$\begin{bmatrix} \overline{H_1} & \overline{H_2} \\ \overline{H_3} & \overline{H_4} \end{bmatrix} = \begin{bmatrix} T_1^{-1} & -T_1^{-1} T_2 T_4^{-1} \\ 0 & T_4^{-1} \end{bmatrix}$$

$$\times \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} T_1 & T_2 \\ 0 & T_4 \end{bmatrix} \quad (11)$$

From (11), we obtain the following equations

$$\overline{H_1} = T_1^{-1}(A + T_2 T_4^{-1} Q) T_1 \quad (12a)$$

$$\overline{H_2} = T_1^{-1}(A + T_2 T_4^{-1} Q) T_2 -$$
$$\quad\quad T_1^{-1}(BR^{-1}B^T - T_2 T_4^{-1} A^T) T_4 \quad (12b)$$

$$\overline{H_3} = -T_4^{-1} Q T_1 \quad (12c)$$

$$\overline{H_4} = -T_4^{-1} Q T_2 - T_4^{-1} A^T T_4 \quad (12d)$$

As we can see from the above equations, there are 4 equations (12a)-(12d) with 9 unknown variables $Q, R, T_1, T_2, T_4, \overline{H_1}, \overline{H_2}, \overline{H_3}$, and $\overline{H_4}$. It means that by assigning the values of any 5 variables, we can determine 4 other variables.

Now we are going to discuss about the solution based on those equations to solve our pole assignment purpose.

Firstly, consider (12c). This equation is equivalent to

$$Q = -T_4 \overline{H_3} T_1^{-1} \quad (13)$$

In the Hamiltonian matrix, (7) the matrix Q can be given by the form

$$Q = Q^T \geq 0$$

Therefore, the following relation is satisfied

$$(-T_4) \overline{H_3} T_1^{-1} = (T_1^{-1})^T \overline{H_3}^T (-T_4^T)$$

To satisfy this condition we can choose $\overline{H_3}$ and T_4 as

$$\overline{H_3} = \overline{H_3}^T \geq 0 \quad (14)$$

and

$$-T_4 = (T_1^{-1})^T \quad (15)$$

From (13) and (15), the following yields :

$$Q = (T_1^{-1})^T \overline{H_3} T_1^{-1} \quad (16)$$

Secondly, from (12d), we can obtain T_2

$$T_2 = -Q^{-1}(A^T T_4 + T_4 \overline{H_4}) \quad (17)$$

Substituting (15), (16), and (17) into (12a), we can get

$$\overline{H_1} = T_1^{-1} A T_1 - \overline{H_3}^{-1} T_1^T A^T (T_1^{-1})^T \overline{H_3}$$
$$\quad\quad - \overline{H_3}^{-1} \overline{H_4} \overline{H_3} \quad (18)$$

From Gershgorin's theorem, we know the fact that the elements on the diagonals of the matrices $\overline{H_1}$ and $\overline{H_4}$ are denoted the centers of the circles which contain the eigenvalues of the matrix $\overline{H}$ or H. We

want that those centers are symmetric about imaginary axis. That means

$$\overline{H_1} = -\overline{H_4}^T \quad (19)$$

As a simple way to satisfy simultaneously (18) and (19), we can consider the case when the matrices $A, \overline{H_3}, \overline{H_4}$, and T_1 are transformed as diagonal form. In this case (18) becomes (19).

If the matrices A, $\overline{H_3}$, $\overline{H_4}$, and T_1 are diagonalized, then (16) and (17) become respectively

$$Q_D = T_1^{-2} \overline{H_3} \quad (20)$$

where Q_D is a state weighting matrix corresponding to a diagonal form of matrices T_1 and $\overline{H_3}$.

$$T_2 = \overline{H_3}^{-1} T_1 (A^T + \overline{H_4}) \quad (21)$$

And T_2 is a diagonal form too. Substituting (12d) into (12b) and using (15) and (21), we can get

$$\overline{H_2} = \overline{H_3}^{-1}(A^2 - \overline{H_4}^2) + T_1^{-1} BR^{-1} B^T T_1^{-1} \quad (22)$$

From now, we can determine a matrix $\overline{H}$ such that it is equivalent to the matrix H. That is, by choosing the matrices $\overline{H_1}, \overline{H_4}, \overline{H_3}, R$, and T_1, we can calculate T_4, Q_D, T_2, and $\overline{H_2}$ by (15), (20), (21), and (22) respectively.

Theorem 4 : We assume that two sets of disks with radius $r_{lef,i}$ and $r_{rig,i}$ are disjoint. If the sub-matrices $\overline{H_4}$ and $\overline{H_3}$ satisfy the conditions

$$\overline{H_4} = [\overline{h}_{4ij}] < 0, \quad (i,j = 1,\cdots,n) \quad (23)$$

and

$$\| [\overline{H_3} \quad \overline{H_4} + \alpha I] \|_\infty \leq r \quad (24)$$

then the eigenvalues with negative real parts of the Hamiltonian matrix $\overline{H}$ are located in the disk $D(-\alpha, r)$. Where $r_{lef,i}$ and $r_{rig,i}$ are respectively the radius of the disks that contain the eigenvalues with negative and positive real parts of the matrix $\overline{H}$, I is an identity matrix with appropriate dimension, and $\| \cdot \|_\infty$ denotes H_∞ norm.

Proof : It is known from Theorem 2 that if two sets of disks with radius $r_{lef,i}$ and $r_{rig,i}$ are disjoint, then every set of disks contains precisely n eigenvalues of the Hamiltonian matrix $\overline{H}$. If the elements $\overline{h}_{4ij}$, $(i,j = 1,\cdots,n)$ satisfy the condition, (23), then from Gershgorin's theorem it is yielded that the diagonal elements $\overline{h}_{4ij \mid i=j}$ of the matrix $\overline{H_4}$ are the centers of the disks that contain the eigenvalues with negative real parts of the Hamiltonian matrix $\overline{H}$, and the radius of these disks are satisfied as follows

$$r_{lef,i} = \sum_{j=1, j \neq i}^{n} |\overline{h}_{4ij}| + \sum_{j=1}^{n} |\overline{h}_{3ij}| \quad (25)$$

where

$$\overline{H}_3 = [\overline{h}_{3ij}], \qquad (i = 1,\cdots,n)$$

All of disks with radius $r_{lef.i}$ belong to the disk $D(-a, r)$ if

$$| \overline{h}_{4ij \mid i=j} + a | + r_{lef.i} \leq r \qquad (26)$$

Substituting (25) into (26), we can get

$$| \overline{h}_{4ij \mid i=j} + a | + \sum_{j=1, j \neq i}^{n} | \overline{h}_{4ij} |$$

$$+ \sum_{j=1}^{n} | \overline{h}_{3ij} | \leq r, \quad (i = 1,\cdots,n) \qquad (27)$$

Explicitly, the condition, (27) is an expression of the condition, (24). ∎

From (19) and (23), we can see

$$\overline{H}_1 = [\overline{h}_{1ij}] > 0$$

From the above equation and Gershgorin's theorem we obtain the following corollary.

Corollary 1 : The eigenvalues with positive real parts of the matrix $\overline{H}$ are located in the disks with centers at $\overline{h}_{1ij \mid i=j}$, and the radius are

$$r_{rig.i} = \sum_{j=1, j \neq i}^{n} | \overline{h}_{1ij} | + \sum_{j=1}^{n} | \overline{h}_{2ij} | \qquad (28)$$

where

$$\overline{H}_2 = [\overline{h}_{2ij}], \qquad (i = 1,\cdots,n)$$

Proof : This proof is clear from Gershgorin's theorem, and it is omitted. ∎

If the eigenvalues with negative real parts of $\overline{H}$ are located in the disk $D(-a, r)$, then to satisfy the symmetrical property of the Hamiltonian matrix $\overline{H}$, its eigenvalues with positive real parts are automatically located in the disk $D(a, r)$.

Corollary 2 : Two sets of disks with radius $r_{lef.i}$ and $r_{rig.i}$ are disjoint if the following condition with infinity norm is satisfied

$$\| [\overline{H}_1 - aI \quad \overline{H}_2] \|_\infty < 2a - r \qquad (29)$$

Proof : If the condition, (27) and the following condition

$$| \overline{h}_{1ij} - a |_{i=j} + \sum_{j=1, j \neq i}^{n} | \overline{h}_{1ij} |$$

$$+ \sum_{j=1}^{n} | \overline{h}_{2ij} | \leq r, \quad (i = 1,\cdots,n)$$

are satisfied, then we can get

$$r_{lef.i} \leq r, \quad r_{lef. max} = r$$

and

$$r_{rig.i} \leq r, \quad r_{rig. max} = r$$

Thus, the minimum distance between two sets of disks with radius $r_{lef.i}$ and $r_{rig.i}$ is $2a - 2r$. If $r_{rig. max}$ is increased by $\triangle r_{rig.i} = 2a - 2r$, i.e.,

$$r_{rig. max} = r + 2a - 2r = 2a - r$$

then the minimum distance is zero. In the other words, if

$$r_{rig.i} < 2a - r$$

then two sets of disks with radius $r_{lef.i}$ and $r_{rig.i}$ are disjoint. The above inequality is correct if the following condition is satisfied

$$| \overline{h}_{1ij} - a |_{i=j} + \sum_{j=1, j \neq i}^{n} | \overline{h}_{1ij} | +$$

$$\sum_{j=1}^{n} | \overline{h}_{2ij} | < 2a - r, \quad (i = 1,\cdots,n) \qquad (30)$$

Explicitly, the condition, (30) is an expression of the condition, (29). ∎

Remark 1 : The conditions to constrain the eigenvalues with negative real parts of the Hamiltonian matrix $\overline{H}$ in the disk $D(-a, r)$ are (23), (24) and (29).

Remark 2 : Because the row sum in the right side of (4) can be replaced by the column sum, the conditions in (24) and (29) can be expressed as follows :

$$\left\| \begin{bmatrix} \overline{H}_1 - aI \\ \overline{H}_3 \end{bmatrix} \right\|_1 \leq r \qquad (31)$$

and

$$\left\| \begin{bmatrix} \overline{H}_2 \\ \overline{H}_4 + aI \end{bmatrix} \right\|_1 < 2a - r \qquad (32)$$

where $\| \cdot \|_1$ denotes the first norm.

Theorem 5 : Assume that a matrix M transforms the matrix A to a diagonal form. Then, the state weighting matrix Q satisfying the pole assignment problem in the specified disk $D(-a, r)$ is given by

$$Q = (M^{-1})^T T_1^{-2} \overline{H}_3 M^{-1}$$

and the state feedback control law, (5) assigns the closed loop poles of the system, (1) in the specified disk $D(-a, r)$.

Proof : If a matrix M transforms the matrix A, which is located in the Hamiltonian matrix H to a diagonal form, then the other elements of H must be transformed respectively. This can be verified by considering the following similar transformation

$$\overline{H}_D := \begin{bmatrix} A & -\overline{R} \\ -\overline{Q} & -A \end{bmatrix}$$

$$= \begin{bmatrix} M^{-1} & 0 \\ 0 & M^T \end{bmatrix} \begin{bmatrix} A & -BR^{-1}B^T \\ -Q & -A^T \end{bmatrix} \begin{bmatrix} M & 0 \\ 0 & (M^{-1})^T \end{bmatrix} \qquad (33)$$

where A and M are the diagonal eigenvalue matrix and the corresponding eigenvector matrix, respectively, for the matrix A, and

$$\overline{R} = M^{-1} BR^{-1} B^T (M^{-1})^T \qquad (34)$$

and

$$\overline{Q} = M^T Q M \qquad (35)$$

As discussing before, if the state weighting matrix is

determined by (20), then the Hamiltonian matrix H or $\overline{H}$ will have the eigenvalues with negative real parts in the specified disk $D(-a, r)$. Comparing (35) to (20) we can obtain

$$Q_D = \overline{Q}$$

or

$$Q = (M^{-1})^T T_1^{-2} \overline{H_3} M^{-1} \qquad (36)$$

Now, it is known from Theorem 3 that the feedback control law, (5) which minimizes the performance index, (3) will give the system, (1) n stable eigenvalues of the Hamiltonian matrix H. As mentioned above, those n stable eigenvalues are located in the disk $D(-a, r)$, it is obviously that the feedback control law, (5) assigns the closed-loop poles in the specified disk $D(-a, r)$. ■

Corollary 3 : Suppose that the state weighting matrix obtained by Theorem 5 is used to determine the feedback control law, (5) under a pole assignment problem in the specified disk $D(-a, r)$, then the feedback control law, (5) minimizes the performance index, (3).

As the result, if we use the state weighting matrix in the Hamiltonian matrix, (7) obtained by Theorem 5 to solve the Riccati equation, (6), we can find out a feedback control law minimizing the performance index, (3), because we can also get the solution of the Riccati equation from the Hamiltonian matrix.

The different feedback matrix F can be obtained by changing the matrices $\overline{H_4}, \overline{H_3}, T_1$ or R. If the matrix T_1 is multiplied by any scalar $n^{-0.5}$, $(n > 0)$ and let

$$R_1 = nR$$

then from (22) we get

$$\overline{H_2} = \overline{H_3}^{-1}(A^2 - \overline{H_4}^2)$$
$$+ (n^{-0.5} T_1)^{-1} B(nR)^{-1} B^T (n^{-0.5} T_1)^{-1}$$
$$= \overline{H_3}^{-1}(A^2 - \overline{H_4}^2) + T_1^{-1} BR^{-1} B^T T_1^{-1}$$

The above equation shows that the matrix $\overline{H_2}$ is not changed when the matrices T_1 and R are replaced by $n^{-0.5} T_1$ and nR, respectively. In this case, the weighting matrix Q in (36) becomes

$$Q_1 = (M^{-1})^T (n^{-0.5} T_1)^{-2} \overline{H_3} M^{-1} = nQ$$

Therefore, if the matrices Q and R are multiplied by any scalar n, $(n > 0)$, then the matrix $\overline{H}$ is not changed and its eigenvalues remain at the same values.

From the above discussion, we can get the following procedure for the pole assignment problem in a specified disk.

Step 1 : Transform the Hamiltonian matrix H to the matrix $\overline{H_D}$ or diagonalize A.

Step 2 : Establish the matrices $\overline{H_4} = diag(\ \overline{h}_{4ij})$ $\overline{H_3} = diag(\ \overline{h}_{3ij})$. Choose $\overline{h}_{4ij} < 0$ and $\overline{h}_{3ij} > 0$ satisfying (24).

Step 3 : By choosing $T_1 = diag(t_{1ij})$, $t_{1ij} \neq 0$ and $R = R^T > 0$, calculate $\overline{H_2}$ from (22) under the condition that the elements $\overline{h}_{2ij}$ of $\overline{H_2}$ must satisfy the condition, (29).

Step 4 : Determine the matrix Q from (36).

Step 5 : Determine the optimal feedback F from (5)

IV. Numerical example

Let us consider a problem to assign the closed loop system poles in the disk $D(-6, 2)$ for the following system matrices

$$A = \begin{bmatrix} -7 & 0 & 1 \\ 1 & -2 & 2 \\ 1 & 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Step 1 : Diagonalize A by using

$$\Lambda = \begin{bmatrix} -2.0000 & 0 & 0 \\ 0 & -7.0990 & 0 \\ 0 & 0 & 3.0990 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & -0.9832 & 0.0912 \\ 1 & 0.1546 & -0.3791 \\ 0 & 0.0974 & -0.9209 \end{bmatrix}$$

Step 2 : Establish the matrices $\overline{H_4}$ and $\overline{H_3}$

$$\overline{H_4} = \begin{bmatrix} -6 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & -5.5 \end{bmatrix}$$

$$\overline{H_3} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}$$

Step 3 : Choose T_1, R and calculate the matrix $\overline{H_2}$

$$T_1 = \begin{bmatrix} 0.26 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\overline{H_2} = \begin{bmatrix} 1.4407 & -0.1623 & 5.8330 \\ -0.1623 & 1.4060 & -0.3575 \\ 5.8330 & -0.3575 & -0.9144 \end{bmatrix}$$

As the result, the matrix $\overline{H_2}$ satisfies the condition, (29).

Step 4 : Determine the state weighting matrix Q

$$Q = \begin{bmatrix} 1.5974 & 3.4137 & 0.3638 \\ 3.4137 & 29.5858 & -12.5171 \\ 0.3638 & -12.5171 & 24.5802 \end{bmatrix}$$

Step 5 : Determine the optimal feedback law F

$$F = \begin{bmatrix} -0.5573 & -3.7829 & 0.3797 \\ -0.8085 & 0.3797 & -8.7909 \end{bmatrix}$$

The eigenvalues of the closed loop system

$$\lambda_i(A + BF) = \{-7.1023 \quad -4.7544 \quad -6.7171\}$$

are located within $D(-6, 2)$.

V. Conclusion

This paper presents a method for assigning the closed loop poles in a specified disk by a state feedback for linear continuous time-invariant systems by applying Gershgorin's theorems and the properties of the Hamiltonian matrix. A distinct point of the proposed algorithm is that the closed loop matrix is firstly established. In this case, the sub-regions that contain the eigenvalues of the closed loop matrix can be assigned at the desired positions in the specified disk $D(-\alpha, r)$ before determining the feedback control law. This matter is important for solving the pole assignment problem in uncertain systems. Also, in this paper it is shown how to choose the different pairs of the weighting matrices Q and R such that all the poles of the optimal closed loop system remain at their positions in the condition of distinct given system's poles. Furthermore, it can be extended to assign the closed loop pole into a specified disk by output feedback.
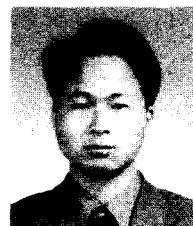
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