The Nonlinear State Estimation of the Aircraft using the Adaptive Extended Kalman Filter

적응형 확장 칼만 필터를 이용한 항공기의 비선형 상태추정

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요 약 : 비행시험을 통해 획득한 데이터의 해석과정에서 대상 항공기의 크기가 소형인 경우에는 엔진진동이나 외부의 교란에 의한 잡음이나 바이어스 등의 강도가 높기 때문에 데이터의 처리과정에서 많은 문제점을 산출하게 된다. 이와 같은 문제점을 해결하기 위해 상태추정 알고리즘이 사용되며, 본 논문에서는 항공기의 비선형 세로운동 방정식의 경우에 확장형 칼만 필터를 적용하여 항공기 세로운동의 상태변수들을 추정하였으며, 또한 확률근사과정, 이노베이션에 대한 궤환 적응 등 적응형 칼만 필터를 사용하여 수렴속도와 정확도 등을 향상시킨 알고리즘을 제안하고 그 결과를 나타내었다.

Keywords : aircraft state estimation, Kalman filtering, Cholesky factorization, stochastic approximation

I. Introduction

Data processing of the aircraft's flight test for estimation of its dynamical model includes two main stages [1][2]:

· Estimation of the state variables of the aircraft motion on the basis of the application the Kalman filtering approach to the nonlinear kinematics equations, as the systems model, for improving a measured aircraft responses from the viewpoint of their compatibility with the theoretical ones,

· Parameters identification of the longitudinal and lateral aircraft's linearized dynamics on the basis of choosing one of the methods: least square, maximum likelihood, Kalman filtering or their combinations by using the filtered data obtained at the previous stage.

· The measurement noises and biases, which are inherent to all airborne sensors, cause the biased estimates of dynamics model's parameters, so minimization of harmful results of these factors at the first stage of the data processing is the very important task. In a case of the light and ultralight aircraft its importance is essentially increasing due to the high intensity of these adverse factors. The reason is the very restricted space inside small aircraft and rigid limitations on weight of all on-board equipment, which sometimes don't permit to install more precise sensors with good vibration isolation as well as poor quality of the on-board power supply. In this situation the preliminary data processing on the basis of the extended Kalman filter (EKF) has to suppress noises with high intensity and simultaneously to detect the biases of sensors, which are weakly observable. These requirements are contradictory to some extent, so to satisfy them it is necessary to apply some innovative procedures of EKF. Here one version of such procedures is proposed, which is based on the combination of above methods:

· Regular Kalman filtering procedure, improved by the square root factorization of the covariance matrix of the state variables, for achieving the enhanced convergence of the nonlinear Kalman procedure in the presence of the intensive noises,

· Adaptive feedback, based on the innovations and Robbins-Monroe stochastic approximation procedure, for detecting some weakly observable variables, such as the sensor's biases.

The efficiency of this procedure is proved by simulation of the longitudinal motion of the aircraft and measurements with the stochastic noises.

II. The aircraft's state estimation on the basis of the extended Kalman filter in general case of the 6 degree-of-freedom motion

The main theoretical background of this approach is the nonlinear kinematics equations of 6 degree-of-freedom (6-DOF) solid body motion as the mathematical model of the dynamic system, whose state space variables must be determined on the basis of the output parameters measurements [1][2]. Measuring system which was described earlier permits to measure all parameters of flight practically, which could be divided into three categories [1][2]:

· input variables: a_x, a_y, a_z - horizontal, lateral and vertical accelerations respectively, p, q, r - angular velocities (roll, pitch, yaw respectively), which form an input (control) 6×1 vector

$$\boldsymbol{u} = [a_x, a_y, a_z, p, q, r]' = [u_1, u_2, u_3, u_4, u_5, u_6]'$$

· state-space variables: u, v, w, h - horizontal, lateral and vertical linear velocities and height, φ, θ,

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ψ - roll, pitch and yaw angles respectively, these actual state-space variables form a 7×1 dimensional vector, $\boldsymbol{x_a}$.

$\boldsymbol{x_a} = [u, v, w, h, \varphi, \theta, \psi]' = [x_1, x_2, x_3, x_4, x_5, x_6, x_7]'$

· output (measured) variables: V_m, h_m - true airspeed of aircraft and the altitude, α_m, β_m, φ_m, θ_m, ψ_m - measured angle of attack, sideslip, roll, pitch and yaw; they form an output 7×1 vector:

$\boldsymbol{y}=[V_m, \alpha_m, \beta_m, h_m, \varphi_m, \theta_m, \psi_m]'=[y_1, y_2, y_3, y_4, y_5, y_6, y_7]'$

Input and output variables are measured from the real measurement system having its own errors which could be represented as the biases, scale errors and random noises, the first two being deterministic and the third being stochastic. So in the general case each arbitrary input and output variable, z_i, could be represented as following:

$$z_i = (1 + \lambda_{zi})\,\overline{z_i} + b_{zi} + n_{zi} \tag{1}$$

where λ_{zi}, b_{zi}, n_{zi} stand for scale factor, bias and random noise respectively and $\overline{z_i}$ is a true value of parameter z_i. As it was recommended in [1][2], the scale factors has to be inputted only for V_m, α_m and β_m and in the other variables they could be omitted.

The approach for the estimation not only state space variables but the biases and scale errors simultaneously in the flight tests data processing is based on the idea of including of these errors as the dummy variables whose derivatives are equal to zero in the state-space description. These variables extend the dimension of Kalman filter and basically could be estimated during standard Kalman filtering procedure. So the actual state space vector $\boldsymbol{x_a}$ has to be extended by dummy vector $\boldsymbol{x_d}$ formed of components b_{zi} and λ_{zi}, determined from (1), and the full vector of state space variables $\boldsymbol{x}$ would be the following: $\boldsymbol{x} = [\boldsymbol{x_a}, \boldsymbol{x_d}]'$. This vector has to be defined in a Kalman filtering procedure.

In a general case state-space description of nonlinear system is described by the following equation:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= f\,[\,\boldsymbol{x}(t),\ \boldsymbol{u}(t), t] + g\,[\,\boldsymbol{x}(t),\ \boldsymbol{u}(t),\ \boldsymbol{n}(t), t] \\ \boldsymbol{y}(t) &= h\,[\,\boldsymbol{x}(t),\ \boldsymbol{u}(t), t] + \boldsymbol{\rho}(t) \end{aligned} \tag{2}$$

where f, g, h are differentiable nonlinear vector functions. It is assumed that the process and measurement noise $\boldsymbol{n}$ and $\boldsymbol{\rho}$ affect the dynamic system linearly. They are assumed to be characterized by a zero-mean white Gaussian noise and are considered to be uncorrelated. In the particular case of the nonlinear kinematics equation of aircraft, dynamic system can be derived as following differential equations:

· state equations:

$$\begin{aligned} \dot{u} &= -q\,w + r\,v + a_x - g\sin\theta \\ \dot{v} &= p\,w - r\,u + a_y + g\cos\theta\sin\phi \\ \dot{w} &= q\,u - p\,v + a_z + g\cos\theta\cos\phi \\ \dot{h} &= u\sin\theta - v\cos\theta\sin\phi - w\cos\theta\cos\phi \end{aligned}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{3}$$

· observation equations:

$$\begin{aligned} V_m &= (1+\lambda_V)\sqrt{u^2+v^2+w^2} \\ \alpha_m &= (1+\lambda_\alpha)\tan^{-1}\left[\frac{w+q x_\alpha - p y_\alpha}{u}\right] \\ \beta_m &= (1+\lambda_\beta)\tan^{-1}\left[\frac{v-r x_\beta - p z_\beta}{u}\right] \\ h_m &= h \\ \phi_m &= \phi \\ \theta_m &= \theta \\ \psi_m &= \psi \end{aligned}$$

where the notations of all variables are explained earlier. Now all input and output variables in (3) could be expressed with their errors in the general form (1). Introducing the dummy vector $\boldsymbol{x_d} = [x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}, x_{17}, x_{18}, x_{19}, x_{20}, x_{21}, x_{22}, x_{23}]' = [b_{ax}, b_{ay}, b_{az}, b_p, b_q, b_r, b_V, b_\alpha, b_\beta, b_h, b_\varphi, b_\theta, b_\psi, \lambda_V, \lambda_\alpha, \lambda_\beta]'$, where subscripts denote the biases and scaling errors of corresponding actual variables, system (3) on the basis of (1) could be rewritten in the following form. [4][5]:

$$\begin{aligned} \dot{x}_1 &= x_2u_6 - x_2x_{13} - x_3u_5 + x_3x_{12} - g\sin x_6 + u_1 - x_8 + \zeta_1 \\ \dot{x}_2 &= -x_1u_6 + x_1x_{13} + x_3u_4 - x_3x_{11} + g\sin x_5\cos x_6 + u_2 - x_9 + \zeta_2 \\ \dot{x}_3 &= x_1u_5 - x_1x_{12} - x_2u_4 + x_2x_{11} + g\cos x_5\cos x_6 + u_3 - x_{10} + \zeta_3 \\ \dot{x}_4 &= x_1\sin x_6 - x_2\sin x_5\cos x_6 - x_3\cos x_5\cos x_6 + \zeta_4 \\ \dot{x}_5 &= -x_{11} - x_{12}\sin x_5\tan x_6 - x_{13}\cos x_5\tan x_6 + u_4 + u_5\sin x_5\tan x_6 \\ &\quad + u_6\cos x_5\tan x_6 - \zeta_5 \\ \dot{x}_6 &= u_5\cos x_5 - u_6\sin x_5 - x_{12}\cos x_5 + x_{13}\sin x_5 - \zeta_6 \\ \dot{x}_7 &= -x_{12}\sin x_5\sec x_6 - x_{13}\cos x_5\sec x_6 + u_5\sin x_5\sec x_6 \\ &\quad + u_6\cos x_5\sec x_6 - \zeta_7 \\ \dot{x}_8 &= 0 \\ &\ \vdots \\ \dot{x}_{23} &= 0 \end{aligned} \tag{4a}$$

$$\begin{aligned} y_1 &= (1+x_{21})\sqrt{x_1^2+x_2^2+x_3^2} + x_{14} + \rho_1 \\ y_2 &= (1+x_{22})\tan^{-1}\left[\frac{x_3+(u_5-x_{12})x_\alpha-(u_4-x_{11})y_\alpha}{x_1}\right] + x_{15} + \rho_2 \\ y_3 &= (1+x_{23})\tan^{-1}\left[\frac{x_2-(u_6-x_{13})x_\beta-(u_4-x_{11})z_\beta}{x_1}\right] + x_{16} + \rho_3 \\ y_4 &= x_4 + x_{17} + \rho_4 \\ y_5 &= x_5 + x_{18} + \rho_5 \\ y_6 &= x_6 + x_{19} + \rho_6 \\ y_6 &= x_7 + x_{20} + \rho_7 \end{aligned} \tag{4b}$$

Here the system (4a) describes system of the state-space equations and (4b) describes nonlinear measurements, ρ_i are the noises of the output variables sensors. The constant values x_α, x_β, y_α and z_β in the 2nd and 3rd equations (4b) denotes the projections of the locations of α- and β-sensors from the center of gravity at the aircraft body axes. The process noises ζ_i are related to the noises of accelerometers and rate gyros ($n_1=n_{ax}, n_2=n_{ay}, n_3=n_{az}, n_4=n_p, n_5=n_q, n_6=n_r$) by the following system of equations:

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{bmatrix} = \begin{bmatrix} 0 & x_3 & -x_2 & 0 \\ -x_3 & 0 & x_1 & 0 \\ x_2 & -x_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} n_4 \\ n_5 \\ n_6 \\ 0 \end{bmatrix} - \begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ 0 \end{bmatrix} \tag{5a}$$

$$\begin{bmatrix} \zeta_5 \\ \zeta_6 \\ \zeta_7 \end{bmatrix} = \begin{bmatrix} 1 & \sin x_5 \tan x_6 & \cos x_5 \tan x_6 \\ 0 & \cos x_5 & -\sin x_5 \\ 0 & \sin x_5 \sec x_6 & \cos x_5 \sec x_6 \end{bmatrix} \begin{bmatrix} n_4 \\ n_5 \\ n_6 \end{bmatrix}$$

If the noises ni are not correlated, covariance matrix $\boldsymbol{Q}$ of this noise vector would have the form:

$$\boldsymbol{Q} = diag\,[\ \sigma_1^2 + x_3^2\sigma_5^2 + x_2^2\sigma_6^2;\ \sigma_2^2 + x_3^2\sigma_4^2 + x_1^2\sigma_6^2;\ \sigma_3^2 + x_2^2\sigma_4^2 + x_1^2\sigma_5^2;\ \sigma_4^2 + (\sin x_5 \tan x_6)^2\sigma_5^2 + (\cos x_5 \tan x_6)^2\sigma_6^2;\ 0;\ \cos^2 x_5\sigma_5^2 + \sin^2 x_5\sigma_6^2;\ \left(\frac{\sin x_5}{\cos x_6}\right)^2\sigma_5^2 + \left(\frac{\cos x_5}{\cos x_6}\right)^2\sigma_6^2;\ ] \quad (5b)$$

where σ_i denotes the r.m.s. of the measurement noise of the corresponding input variables sensor.

As it is known [1]-[3][5], the discrete Kalman filter for the nonlinear system (4), (5) can be represented as the following iterative form (which is known as P. Joseph form [4][5]):

$$\begin{aligned} \boldsymbol{x}(i+1/i) &= \boldsymbol{x}(i) + f\,[\,\boldsymbol{x}(i),\ \boldsymbol{u}(i),\ \boldsymbol{n}(i)=0] \cdot \triangle t \\ \boldsymbol{P}(i+1/i) &= \boldsymbol{A}(i)\,\boldsymbol{P}(i/i)\,\boldsymbol{A}(i)^T + \boldsymbol{B}(i)\,\boldsymbol{Q}(i)\,\boldsymbol{B}(i)^T \\ \boldsymbol{K}(i+1) &= \boldsymbol{P}(i+1/i)\,\boldsymbol{H}(i+1)^T \\ &\quad [\,\boldsymbol{H}(i+1)\,\boldsymbol{P}(i+1/i)\,\boldsymbol{H}(i+1)^T + \boldsymbol{R}]^{-1} \\ \boldsymbol{x}(i+1) &= \boldsymbol{x}(i+1/i) + \boldsymbol{K}(i+1)(\,\boldsymbol{y}(i+1) \\ &\quad - h[\,\boldsymbol{x}(i+1/i),\ \boldsymbol{u}(i+1),\ \rho=0)] \\ \boldsymbol{P}(i+1/i+1) &= [\,\boldsymbol{I} - \boldsymbol{K}(i+1)\,\boldsymbol{H}(i+1)]\,\boldsymbol{P}(i+1/i) \\ &\quad [\,\boldsymbol{I} - \boldsymbol{K}(i+1)\,\boldsymbol{H}(i+1)]^T + \boldsymbol{K}(i+1)\,\boldsymbol{R}\,\boldsymbol{K}^T(i+1) \end{aligned} \quad (6)$$

where the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{H}$ are the Jacobians of the nonlinear vector functions:

$$\begin{aligned} \boldsymbol{A}(i) &= \boldsymbol{I} + \left[\frac{\partial f}{\partial \boldsymbol{x}}\Big|_{x=x(i)}\right] \cdot \triangle t \\ \boldsymbol{B}(i) &= \left[\frac{\partial g}{\partial \boldsymbol{n}}\Big|_{x=x(i)}\right] \cdot \triangle t \\ \boldsymbol{H}(i+1) &= \left[\frac{\partial h}{\partial \boldsymbol{x}}\Big|_{x=x(i+1/i)}\right] \end{aligned}$$

where $\left[\frac{\partial f}{\partial \boldsymbol{x}}\Big|_{x=x(i)}\right]$ is defined from the system (4a), $\boldsymbol{I}$ is 23×23 - unit matrix, $\boldsymbol{B} = \left[\frac{\partial g}{\partial \boldsymbol{n}}\Big|_{x=x(i)}\right] \cdot \triangle t$ is defined from the system (5a), $\boldsymbol{H} = \left[\frac{\partial h}{\partial \boldsymbol{x}}\Big|_{x=x(i+1/i)}\right]$ is defined from the (4b). The matrix $\boldsymbol{P}$ is a covariance matrix of the state-space vector $\boldsymbol{x}$, the matrix $\boldsymbol{R}$ is a covariance matrix of the vector of the measurements noises and $\triangle t$ stands for the sampling interval. Although the components of Jacobian matrices are used in the data processing algorithms, these Jacobian matrices have cumbersome expressions that they are not brought here. In the isolated longitudinal motion, it is expedient to use and make the description of the filtering procedure more simple and transparent. Nevertheless this case is very important for the practical applications and from this viewpoint it has its own independent importance.

Ⅲ. The aircraft's state estimation in the isolated longitudinal motion

In this case, it is assumed that $\beta = \varphi = \psi = 0$, $a_y = 0$, $v = 0$, the kinematics equations (3) would have more simple form:

$$\begin{aligned} \dot{u} &= -q\,w + a_x - g\sin\theta \\ \dot{w} &= q\,u + a_z + g\cos\theta \\ \dot{\theta} &= q \\ \dot{h} &= u\sin\theta + w\cos\theta \end{aligned} \quad (7)$$

Introducing the vector of state-space variables, $\boldsymbol{x} = [u, w, \theta, h, bA_x, bA_z, b_q, bV, b_\alpha, b_\theta, b_h]' = [x_1, \cdots, x_{11}]'$, it is possible to write state-space nonlinear equations for longitudinal motion kinematics of the system (7) as follows:

$$\begin{aligned} \dot{x}_1 &= -x_2 u_3 + x_2 x_7 - x_5 - g\sin x_3 + u_1 - \zeta_1 \\ \dot{x}_2 &= x_1 u_3 - x_1 x_7 - x_6 + g\cos x_3 + u_2 - \zeta_2 \\ \dot{x}_3 &= -x_7 + u_3 - \zeta_3 \\ \dot{x}_4 &= x_1 \sin x_3 + x_2 \cos x_3 \\ \dot{\boldsymbol{x}}_d &= 0 \end{aligned} \quad (8)$$

where $\boldsymbol{x_d}$ is the dummy vector: $\boldsymbol{x_d} = [b_{ax}, b_{az}, b_q, bV, b_\alpha, b_\theta, b_h]' = [x_5, \cdots, x_{11}]'$.

Noises of the state space variables $\zeta_1, \zeta_2, \zeta_3$ are related to the noises of the sensors of $a_x(n_1)$, $a_z(n_2)$, $q(n_3)$ via the following equations:

$$\begin{aligned} \zeta_1 &= -x_2 \cdot n_3 + n_1 \\ \zeta_2 &= x_1 \cdot n_3 + n_2 \\ \zeta_3 &= n_3 \end{aligned} \quad (9)$$

Description of the nonlinear observations (measurements) equations have the form:

$$\begin{aligned} y_1 &= \sqrt{x_1^2 + x_2^2} + x_8 + \rho_1 \\ y_2 &= \tan^{-1}\left[\frac{x_2 - (u_3 - x_7) \cdot x_a}{x_1}\right] + x_9 + \rho_2 \\ y_3 &= x_3 + x_{10} + \rho_3 \\ y_4 &= x_4 + x_{11} + \rho_4 \end{aligned} \quad (10)$$

Equations of systems (7) - (10) are the basis for the creation of the testbed for the development and investigation of the Kalman filtering algorithms for nonlinear kinematics equations.

Ⅳ. Simulation architecture

The block diagram for the simulation of the nonlinear kinematics equations is represented at the Fig 1.

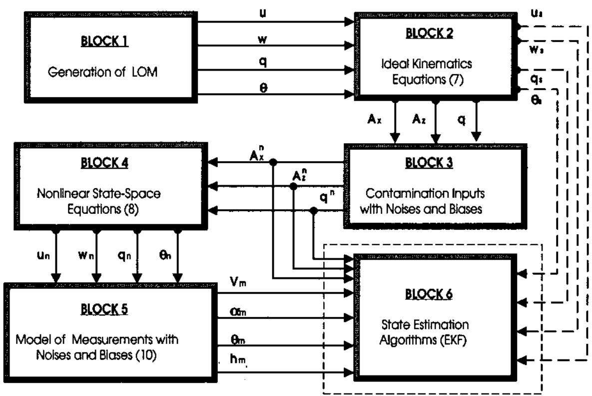


Fig. 1. Block diagram of the simulation system.

The simulation target model is the 100-seat middle range aircraft. It is equipped with 2 turbofan engines and the maximum take-off weight is 98,000 lb. In simulation the flight data has been acquired by using this model aircraft. The trimed true airspeed is 870 ft/s, altitude is 35,000 ft and time interval ($\triangle t$) is 0.01 second.

At this figure Block 1 represents the generator of the longitudinal motions of the aircraft during the flight testing. As it is known, the trapezoidal control actions on the control surfaces are the most informative, on the other hand the most acceptable from the viewpoint of the its realizability by pilots. So Block 1 is designated for the generating of the smoothed trapezoidal elevator input which are very similar to the control actions, produced by pilots during the flight test. The period of oscillations and their duration are chosen to be compatible with the short- and long- period modes of the longitudinal motion of aircraft under investigation.

Block 2 represents the simulation of the "ideal" kinematics, described by the system (7). Simulated (true) variables u_s, w_s, θ_s and hs are used later for the comparison of the predicted by Kalman filter values of variables. Other outputs of this Block 2 (a_x, a_z, q) are used for the producing of the "contaminated" variables, which represent the outputs of the corresponding "actual" sensors. The contamination of these variables with noises and biases are made in the Block 3.

Block 4 with state-space variables, which are produced by input signals alongside with their noises and biases, is described by the equations (8). Block 5 represents the output observations of components of the vector $\boldsymbol{y}$ with the noises and biases inherent to the "actual" sensors and it is described by the system (10). Outputs of Blocks 3 and 5 are those signals ,which have to be processed by the filtering algorithm in the Block 6. So "true" values of the biases ("dummy" variables) are known before the data processing as well as the "actual" ones.

Finally it is necessary to notice that in this case the scale factor errors were not included in the model of measurements. It is known that if the number of the estimated parameters ("dummy" variables) are more than the sum of the number of the measurements and "actual" state-space variables, the task would be overparametrized, thus preventing confident and stable results.

V. The extended Kalman filter for the nonlinear state estimation

As it is known, the strict mathematical proves of the optimality of Kalman filtering exist only for linear case. In nonlinear case it is possible to speak only about suboptimal estimations. Nevertheless it is expedient to use the main results of the linear theory for the substantiation of the choice of algorithms for nonlinear case.

The data processing of the real experimental signals very often requires the application of methods which possess enhanced convergence in the presence of noises with high intensity.

The problem of convergence of the Kalman filter is more hard in the nonlinear case. In this situation the experience of previous investigations [1]-[5] shows that good convergence of the square root factorizations methods. These methods guarantee the positive definiteness of the covariance matrix $\boldsymbol{P}$ at each stage of the process of filtering and good conditionality of the matrix $\boldsymbol{H}(i+1)\cdot\boldsymbol{P}(i+1/i)\cdot\boldsymbol{H}(i+1)^T$ in the third equation of the system (6), thus preventing the divergence of the Kalman filtering procedure. There are several types of these procedures, which produce practically the same results, so here it was chosen the A. Andrews algorithm, based on the Cholesky's factorization of the square symmetric matrix P, which represents matrix $\boldsymbol{P}$ as: $\boldsymbol{P} = \boldsymbol{s}\cdot\boldsymbol{s}^T$, where s is the upper triangular matrix.

In existing literature Andrews algorithm is described only for linear case [4][5], so its application for the nonlinear case is to some extent a new result. The sequence of iterations of Andrews algorithm in this case, described by the system (2), is the following:

· State-Space propagation:

$$\boldsymbol{x}(i+1/i) = \boldsymbol{x}(i) + f[\,\boldsymbol{x}(i),\ \boldsymbol{u}(i)\,]\cdot\triangle t \qquad (11a)$$

· Linearization of the nonlinear functions:

$$\begin{aligned}
\boldsymbol{A}_1(i) &= \left[\frac{\partial f}{\partial \boldsymbol{x}}\Big|_{x=x(i)}\right],\quad \boldsymbol{A}(i) = \boldsymbol{I} + \boldsymbol{A}_1(i)\cdot\triangle t \\
\boldsymbol{B}(i) &= \left[\frac{\partial f}{\partial \boldsymbol{n}}\Big|_{x=x(i)}\right]\cdot\triangle t \\
\boldsymbol{H}(i+1) &= \left[\frac{\partial h}{\partial \boldsymbol{x}}\Big|_{x=x(i+1/i)}\right]
\end{aligned} \qquad (11b)$$

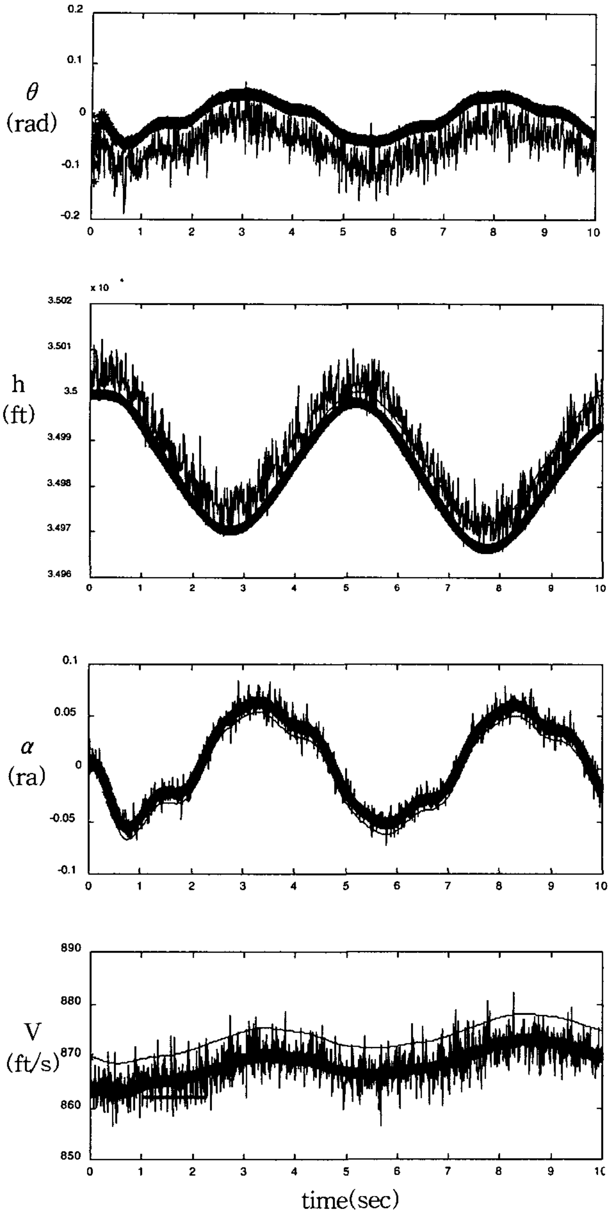
· Covariance matrix propagation, using Cholesky factorization

$$\begin{aligned}
\boldsymbol{P}(i) &= \boldsymbol{s}(i)\cdot\boldsymbol{s}(i)^T \\
\boldsymbol{\varphi}(i+1/i) &= \boldsymbol{A}(i)\cdot\boldsymbol{s}(i) \\
\boldsymbol{s}(i+1/i) &= [\,\boldsymbol{\varphi}(i+1/i)\cdot\boldsymbol{\varphi}(i+1/i)^T \\
&\quad + \boldsymbol{B}(i)\,\boldsymbol{Q}(i)\,\boldsymbol{B}(i)^T]^{\frac{1}{2}}
\end{aligned} \qquad (11c)$$

· Cholesky factorization of the matrices, which define the Kalman gain matrix $\boldsymbol{K}(i+1)$ and calculation of matrix $\boldsymbol{K}(i+1)$:

$$\begin{aligned}
\boldsymbol{F}(i+1) &= \boldsymbol{s}(i+1/i)\cdot\boldsymbol{H}(i+1)^T \\
\boldsymbol{G}(i+1) &= [\,\boldsymbol{F}(i+1)^T\cdot\boldsymbol{F}(i+1)+\boldsymbol{R}\,]^{\frac{1}{2}} \\
\boldsymbol{K}(i+1) &= \boldsymbol{s}(i+1/i)\cdot\boldsymbol{F}(i+1)\cdot\boldsymbol{G}(i+1)^{-1}\ \ \boldsymbol{G}'(i+1)^{-1}
\end{aligned} \qquad (11b)$$

· Calculation of innovation vector - $\boldsymbol{in}(i+1)$, filtered



(— true data, estimated data, measured data)

Fig. 2. Results of Kalman filteration.

state-space vector - $x(i+1)$ and the Cholesky factorization $s(i+1)$ of the updated covariance matrix:

$$\begin{aligned} in(i+1) &= y(i+1) - [h(i+1) + H(i+1)[x(i+1/i) - x(i)]] \\ x(i+1) &= x(i+1/i) + K(i+1) \cdot in(i+1) \\ s(i+1) &= s(i+1/i) \cdot [I - F(i+1) G(i+1)^{-1} \\ &\quad \cdot (G'(i+1) + R^{\frac{1}{2}})^{-1} F'(i+1)] \end{aligned} \quad (11e)$$

In this paper, above procedure is converted into the program and used for the data processing.

It is necessary to notice that for the improvement of accuracy of results this filtering procedure includes not only the first order Taylor expansion of the nonlinear vector - function $f[x(t), u(t)]$ of the state-space variables but also the quadratic approximation:

$$A(i) = I + A_1(i) \cdot \triangle t + A_2(i) \cdot \frac{1}{2}(\triangle t^2)$$

where the $A_2(i) = \left[\frac{\partial^2 f}{\partial x^2} \Big|_{x=x(i)} \right]$ is the matrix of the second derivatives. However this including didn't improve estimations essentially, the difference between more precise 2nd-order and the 1st order approximations were very small. Nevertheless this 2nd-order approximations were included in the procedure optionally.

At the Fig 2. some results of Kalman filtration based on the target model aircraft are represented: Fig 2. represents the filtration of pitch angle, the altitude, the angle of attack and the true airspeed. As it is possible to see from these pictures the predicted values are free from noises and biases and are very closed to the true values. Nevertheless there are some difficulties in the estimation of some biases: biases of the accelerations and the pitch rate could not be estimated. From the practical viewpoint it is very important to comprehend the reason of these difficulties.

Ⅵ. Observability conditions of the nonlinear Kalman filtering

From physical viewpoint the reason is the absence of these variables in the output signals model, described by the system (10). Only variable $x_7 = b_q$ is present in the observations with small coefficient of x, but the further analysis shows, that it is not sufficient.

From the mathematical viewpoint the matter of problem is the violation of the observability conditions. The strict Kalman observability conditions are valid for the linear systems, but in this case, when the nonlinearities are differentiable, it is possible to use as the first approximation the observability conditions for the linearized time-varying system. The possibility of applying of these conditions for time-varying systems was proved in [4][7]. For the i-th sampling moment, the Jacobian $H(i)$ of the nonlinear vector-function of measurements in the case (10) has the following form:

$$H(i) = \begin{bmatrix} \frac{\partial h}{\partial x_1}(i) & \frac{\partial h}{\partial x_2}(i) & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{\partial h}{\partial x_2}(i) & \frac{\partial h}{\partial x_1}(i) & 0 & 0 & 0 & 0 & \frac{\partial h}{\partial x_7}(i) & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

As it could be easily noticed that this matrix is sparse and it is the reason of the singularity of observability Gramian G_{ob} of linear discrete time-varying system [4][7]. The observability Gramian matrix of linear dynamic system model over a discrete time interval can be defined as following [4]:

$$G_{ob}(H(i), A(i), 1 \le i \le N) = \left\{ \sum_{i=0}^{N} \left[\prod_{j=0}^{i-1} A(i-j) \right]^T H(i)^T H(i) \left[\prod_{j=0}^{i-1} A(i-j) \right] \right\} \quad (13)$$

where $\boldsymbol{A}(i)$ is the state transition matrix. Our analysis indicates that the state variables $x_5=b_{ax}$, $x_6=b_{az}$, and $x_7=b_q$. are unobservable due to rank deficiency of observability gramian in (13).

Nevertheless, even for observable biases (b_V, b_α, b_θ, b_h) the convergence of Kalman procedure is weak sometimes, because the coefficients in the last 4 rows of the Kalman gain matrix $\boldsymbol{K}(i+1)$, which are related to these variables, appears to be small at each step of the filtering procedure, thus causing weak convergence of Kalman procedure.

Strictly speaking, the structure of actual filter affects the observability of system, because this structure determines aforementioned coefficients. So it is necessary to underline that the square-root factorization algorithms for Kalman filtering were chosen intentionally as the most robust algorithms, which are now widely used in these tasks, especially for practical purposes.

It is the property of square-root factorization algorithms of Kalman filtering, because their increased robustness suppresses "dummy" variables. From the other hand these algorithms now are the most wide-spread in the identification and filteration tasks especially in the presence of the intensive noises. And it is very desirable to improve them from the viewpoint of better observability of the "dummy" variables. To achieve this goal adaptive algorithm was applied.

The performance criterion of this algorithm is the maximum likelihood of the vectors $\boldsymbol{x}(i+1)$ and $\boldsymbol{y}_k(i+1) = [\boldsymbol{y}(i+1), ..., \boldsymbol{y}(i+k)]$ estimation [5]:

$$MAX \ \ln \ \boldsymbol{\Pi} [\ \boldsymbol{x}(i+1),\ \boldsymbol{Yk}(i+1) \mid \boldsymbol{y}(i),\ \boldsymbol{P}(0),\ \boldsymbol{Q}(i),\ \boldsymbol{R}] \\ \boldsymbol{x}(i),\ \boldsymbol{P}(0),\ \boldsymbol{Q}(i),\ \boldsymbol{R}$$

where $\boldsymbol{\Pi}[\cdot]$ is the conditional density of probabilities, $\boldsymbol{P}(0)$ is the initial value of the covariance matrix. The algorithm differs from the usual Kalman algorithms only by the method of the calculation of the Kalman gain matrix $\boldsymbol{K}_a$, which is defined by the following iterative procedure [5]:

$$\begin{aligned} \boldsymbol{M} &= [\ \boldsymbol{in}(i+1) \cdot \boldsymbol{in}\ (i+1)^T, \cdots, \boldsymbol{in}(i+k) \cdot \boldsymbol{in}\ (i+k)^T]^T \\ \boldsymbol{S} &= [\ \boldsymbol{H}(i+1),\ \boldsymbol{H}(i+2) \cdot \boldsymbol{A}, \cdots, \boldsymbol{H}(i+k) \cdot \boldsymbol{A}^{k-1}]^T \\ \boldsymbol{S}^+ &= (\ \boldsymbol{S}^T \boldsymbol{S})^{-1} \cdot \boldsymbol{S}^T \\ \boldsymbol{K}_a(i+1) &= \triangle t \cdot \boldsymbol{S}^+ \cdot \boldsymbol{M} \cdot [\ \boldsymbol{in}(i+1) \cdot \boldsymbol{in}\ (i+1)^T\ ]^{-1} \end{aligned} \quad (14)$$

where $\boldsymbol{S}^+$ denotes the Penrose-Moor pseudoinversion procedure and $\triangle t$ is the sampling interval. As it could be seen from this procedure the matrix $\boldsymbol{K}$ is estimated on the basis of k steps of observations, matrices $\boldsymbol{M}$ and $\boldsymbol{S}$ being filled step by step. The value of k is chosen between 4 and 6. Insofar as pseudoinversion procedure requires the full rank of the matrix-operand, this procedure in this case was applied only to truncated dummy vector with observable components, so it was essentially simplified. Moreover it is necessary to notice, that this procedure uses the stochastic values of innovations, which are averaging at the short time period, so resulting values of $\boldsymbol{K}_a$ would have stochastic components. As it is known [5][6], the estimation of the constant parameters in this case could be improved on the basis of stochastic approximation procedure. It is quite sufficient to apply here the simplest Robbins -Monroe procedure at least at the beginning of the Kalman filtering (the first 10 - 20 % of the length of all records to be processed). Using all these methods it is possible to write the expression for adapted Kalman gain matrix in the following form:

$$\boldsymbol{K}_{ad}(i+1) = \boldsymbol{K}(i+1) + \boldsymbol{K}_j \cdot \boldsymbol{K}_a(i+1) \quad (15)$$

where $\boldsymbol{K}(i+1)$ is defined from the iterative subprocedure (11d), $\boldsymbol{K}_a$ is defined from iterative subprocedure (14), $\boldsymbol{K}_j$ is the stochastic approximation gain at the restricted time interval:

$$\boldsymbol{K}_j = \frac{1}{i}\ (\text{if } i \le j),\ \text{or } \frac{1}{j}\ (\text{ if } i > j) \quad (16)$$

j stands for the working period of stochastic approximation procedure $(0.1 - 0.2) \cdot N$, N is the total length of processed signals. The value of j is used as a tuning parameter of adaptation. For short-period state variables (angular variables) this parameter appears to be several times less than the long-period (linear) variables. The adapted Kalman gain (15) together with (14), (15) is used instead of usual Kalman gain matrix $\boldsymbol{K}(i+1)$ in the second expression of the subprocedure (11e).

Table 1. Results of bias estimated.

Bias	bv	b_α	b_θ	bh
True value	-5.0	+0.01	-0.05	+5.0
Estimated value (Andrews algorithm)	-5.270	+0.0104	-0.0503	+5.460
Estimated value (Adaptive algorithm)	-5.165	+0.0103	-0.0499	+4.854

The application of this procedure to the longitudinal motion kinematics equations permits to obtain more stable and accurate results in comparison with the Andrews algorithm as shown in Table 1. Results of bias estimation, using this algorithm, are represented in the Table 1.

Fig 3. represents the behavior of corresponding bias estimates based on the adaptive algorithm during the process of Kalman filtration. Fig 4. represents the correlation function of innovations, whose mean values are equal to zero. The last graphics demonstrate that innovations could be considered as

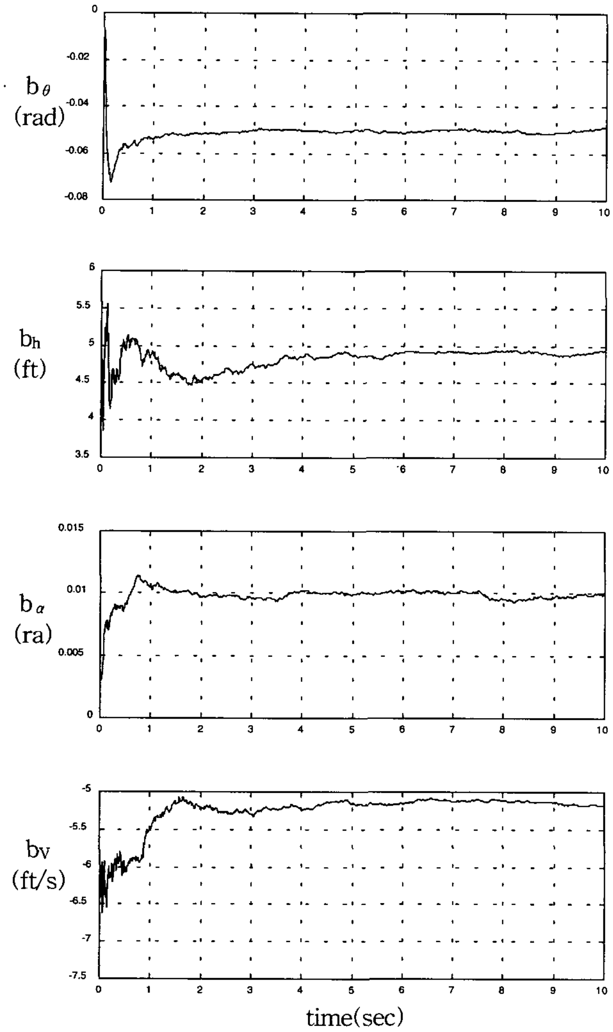


Fig. 3. Results of bias estimated by adpative Kalman filter.

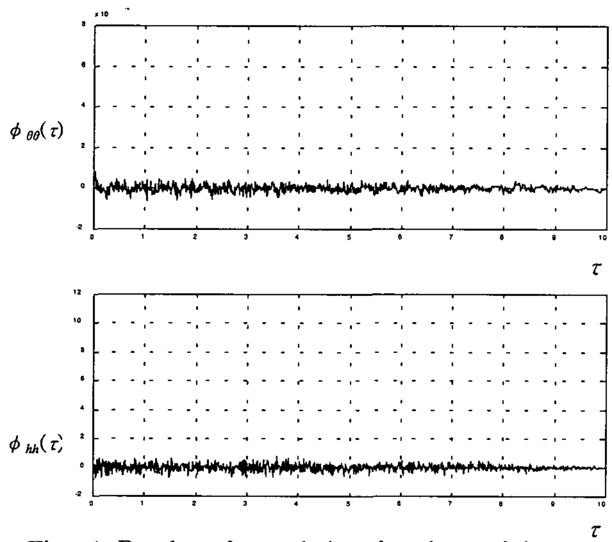


Fig. 4. Results of correlation functions of innovation.

a white noises. This fact proves the proximity of state estimates to the optimal ones.

Ⅶ. Conclusion

In the presence of intensive noises and biasing, which are inherent to the measurement process in the flight test, one of the most promising ways to obtain confident results in the parameters of aircraft's flight dynamics identification is the application of a two-stage procedures, aircraft's state estimation on the basis of nonlinear Kalman filtration and parameter identification on the basis of the maximum likelihood approach in the combination with the Kalman optimal observation.

Since the procedure of Kalman filtration in nonlinear case has essential difficulties connected with poor convergence, estimation biases etc., for improvement of its efficiency it is expedient to apply the following mathematical methods and corresponding software:

- the checking Kalman observability conditions before the running of filtering procedure to detect unobservable variables; in this case it is necessary to exclude them from this procedure;
- the application of the procedures using the factorization of the state vectors covariance matrix $\boldsymbol{P}$, such as Andrews, Bierman and others [4][5];
- the application of adaptive Kalman procedure with the feedback on innovations and stochastic approximation to improve the estimation of the slightly observable biases.

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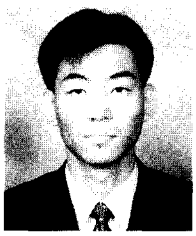

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