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Warrant 가격 결정변수에 관한 실증연구

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An Empirical Study on Variables Affecting Warrant Pricing of Japan

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요 약 Warrant란 소유자에게 일반회사채의 권리에 일정한 기간내에 일정한 가격으로 정해진 수의 발행회사 주식을 매입할 수 있는 권리인 신주인수권을 함께 부여한 사채로서 금융파생상품의 하나이다. Warrant는 콜옵션과 동일한 성격을 가지면서도 투자자가 아니라 기업에 의해서 발행된다는 점이 콜옵션과 상이하다. 이러한 warrant의 특징은 특히 배당(dividend)을 지급하는 경우 블랙·숄즈 옵션가격 모형으로 평가하는데 문제가 있다. 또한 신주인수권의 행사는 발행주식의 수를 증가시킴으로써 기업의 자산과 이익이 희석화(dilution) 된다. 본 연구는 OPM 대신에, 다변량 분석기법중의 하나인 다중회귀분석을 통하여 warrant가격에 영향을 미치는 주요 변수를 분석함으로써 warrant 가격결정 문제를 해결하고자 한다 이를 위하여 1995년과 1996년의 일본동경주식시장의 300여개 warrant 자료를 토대로 실증분석 함으로써 warrant 가격결정 주요변수와 warrant 가격 예측 모형을 검토하였다.

Abstract Warrants are often described as call potions written by firms on their own stock. Howerer, a call option is a pure side bet; i.e., none of the cash flows associated with the call's sale or exercise involves the firm. Issuing warrants on the other hand, can affect the firm's aggregate level of investment, composition of its capital structure, and the price of the stock on which warrant can be exercised. The problem of the warrant pricing can be solved by using of multivariate data analysis techniques, such as regression analysis or discriminant analysis, instead of OPM. The value of this approach is that we can evlauate the relative importance of each independent variable which affect a price of a warrant. This study empirically examines the Japanese warrant pricing by multiple regression analysis using a sample of 300 observations traded on Tokyo Stock Exchange during the periods between 1995 and 1996.

Key Words : Warrant, Finacial derivative products, Multiple regression analysis, Black-Sholes option pricing model

1. Introduction

A warrant is a security issued by the corporation in return for cash. Therefore a warrant is an option that is a liability of a corporation. The holder of a warrant has the right to buy a fixed number of shares of a specified corporation's stock (or other assets) at a specified price at any time until a given date. So, a warrant is a convertible security. A conertible security is one which at the owner's option, may be exchanged for another security with different characteristics[1].

Derivatives can be thought of as financial products that are more closely linked to the pure gains associated with the risk distribution structure inherent to all financial products. Financial derivative products such as options, warrants, futures or swap contracts have become a standard risk management tool that enables risk sharing and thus facilitates the efficient allocation of capital to productive investment opportunities. These days, with the overall volume of financial transactions and investors are looking for new and different ways to distribution structure by unbundling part of the risk that a financial institution or investors has traditionally been obliged to shoulder entirely[2]. Moreover, derivative instruments are not redundant securities once the existence of informational asymmetry and market frictions is recognized. They thus contribute to complete the financial markets and to

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본 논문의 핵심내용은 최근 중요성이 증대되고 있는 주요 금융파생상품 중 하나인 신주인수권부사채의 가격결정에 영향을 미치는 변수들에 대한 연구이다. 본 실증연구는 향후 우리나라 금융시장과 기업활동에 있어서 자본조달과 투자 및 금융상품으로서 중요성을 파악하는데 활용될 수 있을 것이다.

gather information that is not readily available from trading in the physical markets.

While the benefits stemming from the economic functions performed by derivative securities have been discussed and proven by academics, there is increasing concern within the financial community that the growth of the derivatives markets - whether standardized or not - destabilizes the economy. In particular, one often hears that the widespread use of derivatives has reduced long term investments since it concentrates capital in short term speculative transactions. Moreover, several articles stated that the derivative markets trading activity destabilizes the cash markets by increasing the volatility of its fundamentals (Such as interest rates, currency rates, etc.).

In the wake of the Barings Securities collapse and a string of massive losses by Japanese compaines through failed speculative investments in derivatives, many people wrongly believe that derivatives are fundamentally more dangerous than conventional financial products and dervative trading is a very risky activity that may ultimately lead to systemic disruptions in the financial system. Certainly, derivatives enable investors to build high-risk, high-return product schemes through risk bundling. The recent derivatives related failures were caused by not the product schemes as such but lack of proper management and control.

The purpose of this study is to examine empiricially main variables affecting the warrant pricing by multiple regressioin analysis using a sample of 300 observations during the period between 1995 and 1996 in Tokyo Stock Exchange.

The problem of the warrant pricing can be solved by using of multivariate data andalysis techniques, such as regressioin analysis or discriminant analysis, instead of OPM. The value of this approach is that we can evaluate the relative importance of each independent variable which affect a price of a warrant.

In security analysis, the starting point is to determine the historical perfomance of the firm, Undertaking this evaluation requires a heavy reliance on its published financial statements. Therefore, the independent variable employed for stock prices included various data per share and ratio can be analyzed to evaluate the pricing of warrant on stocks. Since the Black-Scholes OPM doesn't give any information about relative importance of adjustment both dividend and dilution, the multiple regression analysis was performed to evaluate the warrant prices. The objeive of this research is assessing the degree and charater of the relationship between dependent and independent variables as well as more accurately prediction of warrant prices.

2. Overview of Empirical Studies of Warrant

Unfortunately, empirical studies of warrant valuation are few because of the unavailability of warrant data in machine-readable form. However, several studies examine the use of option valuation models in the pricing of warrants. Since Black and Scholes[3] proposed that option pricing can be used for the warrant pricing, this problem has been studied by Emanuel[4], Schwartz (1977), Galai and Schneller[5], and Constantinides[6]. The simplest approach to the problem assumes a one-period model. The firm assumed to be 100% equity financed, and investment policy is not affected by its financing decisions. For example, the proceeds from issuing warrants are immediately distributed as dividends to the old shareholders. Also the firm pays no end-of- period dividends, and the warrants are assumed to be exercised as a block. These some what retrictive assumptions facilitate the estemation of the warrant value and its equilibrium rate of return. Galai and Schneller (1978) show, for the above mentioned assumptions, that the returns on a warrant are perfectly correlated with those of a call option on the same firm without warrants. Because the warrant and the call option are perectly correlated, they will have exactly the same systematic risk and therefore the same required rate of return. The expected return is the before-tax cost of capital for issuing warrants and can easily be estimated for a company that is contemplating a new issue of warrants. One problem with the above approach is that warrants are not constrained to be exercised simultaneously in one large block. Emanuel (1983) demonstrated that if all warrants were held by a single profit-maximizing monopolist, the warrants would be exercised sequentially. Constantinides (1984) has solved the warrant valuation problem for competitive warrant holders and shown that the warrant price, given block exercise. Frequently the balance sheet of a firm has several contingent claim securities, e.g., warrants and convertible bonds, with different maturity dates. This means that the expiration and subsequent exercise (or conversion) of the security can result in equity dilution and therfore early exercise of the longer maturity contingent claim securities. Firms can also forces early exercise or conversion by paying a large cash or stock dividend[7].

3. A Decision Process for Multiple Regression Analysis

3.1 Objectives of Multiple Regression

Multiple regression analysis is a multivariate statistical technique used to examine the relationship between an single dependent variable and a set of dependent variables. The necessary starting point in multiple regression, as with all multivariate statistical techniques, is the research problem. The flexibility and adaptability of multiple regression allows for its use with almost any dependence relationship. So in selecting suitable applications of multiple regression, the analyst must consider three primary issues: 1) the appropriateness of the research problem, 2) specification of a statistical relationship, and 3) selection of the dependent and independent variables[8].

3.2. Research Design of a Multiple Regression Analysis

In the design of a multiple regression analysis, sample size, the nature of the independent variables, and the possible creation of new variables to represent special realtionships between the dependent and independent variables must be considered. In doing so, the criteria of statistical and practical significance must always be maintained.

The sample size used in multiple regression analysis is perhaps the most influential single element under the control in designing the analysis. The effects of sample size are seen most directly in the statistical power of significance testing and the generalizability of the result[9].

The size of the sample has a direct impact on the appropriateness and the statistical power of multiple regression. Small samples, usually characterized as having fewer than 20 observations, are appropriate only for analysis by simple regression with a single independent variable. Even in these situations, only very strong relationships can be detected with any degree of certainty. Likewise, very large samples of 1,000 observations or more make the statistical significance tests overly sensitive, indicating that almost any relationship is statistically significant. With large samples the criteria of practical significance must be met along with statistical significance. Power in multiple regression refers to the probability of detecting as statistically significant a specific level of R^2 of a regression coefficient at a specified significance level and a specific sample size. Sample size has a direct and sizable impact on power.

In addition to sample size's role in determining statistical power. It also affects the generalizability of the results by the ratio of observations to independent variables. A general rule is that the ration should never fall below five, meaning that there should be five observations for each independent variable in the variate. While the minimum ratio is 5 to 1. the desired level is between 15~20 observations for each independent variable. In cases when the available sample does not meet these criteria, the analyst should be certain to validate of generalizability of the results.

3.3. Assumptions in Multiple Regression Analysis

The assumptions underlying multiple regression analysis apply both the variables (dependent and indepenent) and to the relationship as a whole. The basic issue is whether the assumptions of regression analysis have been met. Are the errors in prediction a result of an actual absence of a relationship among the variables, or are they caused by some characteristics of the data not accommodated by the regression model? The assumptions to be examined are as follows:

1) The linearity of the phenomenon measured
2) The constant variance of the error terms
3) The independence of the error terms
4) The normality of the error term distribution

The principal measure of prediction error for the variate is the residual - the difference between the observed and predicted values for the dependent variable[10].

Analysis of residuals provides a simple yet powerful set of analytical tools for examining the appropriateness of a regression model.

3.4. Estimating the Regression Model and Assessing Overall Fit

Specified the objectives of the regression analysis, selected the independent and dependent variables, addressed the issues of research design, and assessed the variables for meeting the assumptions of regression allow to estimate the regression model and assess the overall predictive accuracy of the independent variables. In this stage, three basic tasks should be accomplished: 1) select a method for specifying the regression model to be estimated, 2) assess the statistical significance of the overall model in predicting the dependent variable, and 3) determine whether any of the observations exert an undue influence on the results.

3.5. Interpreting the Regression Variate

The next our task is to interpret the regression variate by evaluating the estimated regression coefficients for their explanation of the dependent variable. The estimated regression coefficients are used to calculate the predicted values for each observations and to express the expected change in the dependent variable for each unit change in the independent varables. In addition to making the prediction, it is important to know which variable is more helpful in predictiong of the dependent variable. Unfofrunately, the regression coefficients do not give this information. Therefore, standardized regression coefficients (beta coefficients) are used instead of them[11].

A key issue in interpreting the regression variate is the correlation among the predictor variables. This is a data problem, not a problem of model specification. But it has substantial effects on the results of regression procedure. First, it limits the size of the substantial effects on the results of regression procedure. First, it limits the size of the coefficient of determinatioin and makes increasingly more difficult to add unique explanatory prediction from additional variables. Second, it makes determining the contribution of each independent variable difficult because the effects of the independent variables are mixed.

3.6. Validation of the Results

After identifying the best regressioin model, the final step is to ensure that it represents the general population (generalizability) and is appropriate for the situations in which it will be used (transferablility)[12].

The most appropriate empirical validation approach is to test the regrressiion model on a new sample drawn from the general population. A new sample will ensure representatveness and can be used in several ways. First, the original model can predict values in the new sample, and predictive fit can be calculated. Second, a separate model can be estimated with the new sample and then compared with the original equation. Since many pressures, or availability of respondents. When this is the case, the sample may be divied into two parts: an estimation subsample for creating the regression model and the holdout validation subsample used to test the equation.

4. An Empirical Test of Variables Affecting Warrant Price

4.1. Data and Variables Selection for Objectives of Test

The data consist of about 300 monthly observations on 15 warrants that were actively traded on Tokyo Stock Exchange (TSE) between January 1995 and August 1996. Only warrants with finite expiration dates that can be changed for the common stock of the issuer are included in the sample. All end-of-month warrant closing prices are drawn from Mnthly

Statistics Report (Tokoy Stock Exchange). Other information drawn from this source includes prices, dividends, and ex-distribution dates for the underlying common stocks that trade on the TSE.

Government bond data for the last trading day of each month are drawn from the Asian Wall Street Journal. To apply the regression procedure, a warrant price(W) was selected as the dependent variable to be predicted by independent variable. The following nine variables were included as predictor variables:

S	Stock Price
X	Exercise Price
T	Time to Maturity
n	Number of Listed Shares
m	Number of Listed Warrants
DIV	Dividends
r	Risk-Free Interest Rate(Government Bond)
PER	Price Earnings Ratio
PBR	Stock Holders Equity

The relationship among the nine predictor variables and a warrant price was assumed to be statistical, not functional, because it involved perceptions of performance and may have has levels of measurement error.

Nine variables mentioned above are related with theoretical studies, especially Dividend and Dilution adjusted Black-Scholes' Model which evaluates the price of warrant.

Since, howerver, the Black-Sholes OPM adjusted doesn't give any information about relative importance of each adjustment, the multiple regression analysis was perfromed to assess the degree and character of the relationship between dependent and independent variables as well as more accurately prediction of warrant prices.

4.2. Research Design and Assumptions of the Empirical Model

The first question to be concerning sample size is the level of relationship (R^2) that can be reliably detected with the proposed regression analysis. The sample of about 300 observations, with nine potential independent, is able to detect relationships with (R^2) values approximately 20 percent at a power of .80 with significance level set at .01. If the significance level is relaxed to .05, then the analysis will identify relationships explaining about 15 percent of the variance. The proposed regressioin analysis was deemed sufficient not only statiscally significant relationships but also relationships that had managerial significance.

The sample of about 300 observations also meets the proposed guideline for the ratio of observations to independent variables with a ratio of 15 to 1.

Meeting the assumptions of regression analysis is essential to ensure that the results obtained were truly representative of the sample and that it is possible to obtain the best results. Any serious violations of the assumptions must be detected and corrected if at all possible.

The three assumptions to be addressed for the individual variables are linearity, constant variance, and normality. First, scatterplots of the individual variables did not indicate any nonlinear relationships between the dependent and the independent variables. Second, in the tests of normality, three of the varialbes (*PER*, *r*, and *W*) were found to violate the statistical tests. In each case, transformations were indicated; for *PER* by taking logarihms, for *r* by calculating $1/r$, for *W* by calculationg square root. (Table 1.)

4.3. Estimating the Empirical Model and Assesing Overall Model Fit

With the regression analysis specified in terms of dependent and independent variables, the sample deemed adequate for the objectives of the study and the assumptions assessed for the individual variables, the process now proceeds to estimation of the regression model an assessing the overall model fit. In the present research has been used stepwise estimation (Table 2).

4.4. Interpreting the Variate

With the model completed, the regression variate specified, and the diagnostic tests administered that confirm the appropriateness of the results, the predictive equatiion, which includes *S*, *X*, *T*, can be

Table 1. Multiple regression for untransformed data

***** MULTIPLE REGRESSION *****

Listwise Deletion of Missing Data
Equation Numlber 1   Dependent Variable...   W Warrant Price   Block Number 1.
Method: Enter
Variable(s) Entered on Steep Number
1.. X Exercise Price
2.. R Risk-Free Interest Rate
3.. T Time to Maturity
4.. PER Price Earnings Ratio
5.. PBR PBR (Stock Holders Equity)
6.. N Listed Shares
7.. M Listed Warrants
8.. DIV Dividends
9.. S Stock Price

Multiple R	.86088
R Square	.74111
Adjusted R Square	.72724
Standard Error	2.85541

Analysis of Variance

	DF	Sum of Squares	Mean Square
Regression	9	3921.11017	435.67891
Residual	168	1369.76094	8.15334

F = 53.43564

------ -------------- Variables in the Equation ------- ---------- ---------

Variable	B	SE B	Beta	T
DIV	-.073712	.105180	-.082876	-.701
M	7.70913E-05	1.7450E-05	.261787	4.418
N	1.40724E-06	5.7007E-07	.261787	2.469
PBR	.008883	.611579	9.466E-04	.015
PER	-.012886	.003608	-.187356	-3.571
R	.563604	.348586	.066877	1.617
S	.012775	.001704	.891239	7.499
T	.609923	.263615	.141813	2.314
X	-.012743	9.5170E-04	-.831607	-13.390
(Constant)	4.929311	1.393995		3.536

Table 2. Multiple regression for transformed data

***** MULTIPLE REGRESSION *****

Equation Number 1   Dependent Variable..   W1

Variable(s) Entered on Step Number 3.. X Exercise Price

Multiple R	.89953
R Square	.80915
Adjusted R Square	.80586
Standard Error	.48950

Analysis of Variance

	DF	Sum of Squares	Mean Square
Regression	3	176.75656	58.91885
Residual	17	41.69153	.23961

F = 245.89840

------------- ------ Variables in the Equation ---------------------------

Variable	B	SE B	Beta	T
S	.002796	1.4003E-04	.959963	19.967
T	.348672	.029851	.398977	11.680
X	-.002816	1.4993E-04	-.904490	-18.784
(Constant)	2.217584	.147812		15.003

------- ------------ Variables in the Equation ---------------------------

Variable	Beta In	Partial	Min Toler	T
DIV	-.053966	-.058393	.220101	-.769
M	-.001533	-.002586	.467533	-.034
N	.034421	.071931	.430395	.949
PBR	-.044936	-.091705	.451370	-1.211
PER1	-.049308	-.106428	.457931	-1.408
R1	-.001838	-.004201	.472319	-.055

examined. The predictive equation would be written:

$$W = 2.217584 + 0.002796S - 0.002816X + 0.348672T \quad (1)$$

With this equation, the expected warrant prices could be calculated: In addition to providing a basis for predicting warrant prices, the regression coefficients also provide a means of assessing the relative importance of the individual variables in the ovrall prediction of warrant prices. The following beta coefficients show the relative importance of each independent variable included into the predictive equation:

$\beta_S = 0.959963$

$\beta_X = -0.904490$

$\beta_T = 0.398977$

The results of mutliple regression analysis are similar to theoretical results of the original Black-Scholes model. In the Black-Scholes option pricing model the relationships between the warrant price and variables, which affects it, can be described in functional form as:

$$C = f(S, X, T), \quad (2)$$

and the parial derivatives of the call price, c, with respect to its various arguments are:

$$\frac{\partial C}{\partial S} > 0, \quad \frac{\partial C}{\partial X} < 0, \quad \frac{\partial C}{\partial T} > 0 \quad (3)$$

5. Summary and Conclusions

The purpose of this paper is an empirical study on variables affecting the Japanese warrant pricing by multiple regression Analysis instead of OPM, during the period between 1995 and 1996 in Tokyo Stock Exchange.

This study analyzed nine variables as predictor variables for warrant prices using a sample of 300 observations. The major results of regression for warrant pricing can be summarized as the following.

First, The predictive equation would be written as following:

$$W = 2.217584 + 0.002796S - 0.002816X + 0.348672T$$

Second, The main variables which affect warrant prices in order of the degree of their importance: stock price, warrant exercise price and time to maturity. Effect of the risk-fress interest rate and the variance of return on the equity was not revealed. It seem that the time period covered by sample is very short as well as it can be characterized as stanation period of Japanese economy, so the effect of the variance of return on the equity is minimal.

Third, It is most suprise that other independent variables are not included into predictive equation, because there are some empirical evidence that financial markets in Japan as well as in the other advanced countries have semistrong-and sometimes strong-form efficiency, so all relevant information are fully reflected in prices of basic securities(bond and stocks). Therfore, in case of warrants all relevant information, such as dividends and dillution are reflected in stock prices.

Fourth, The components of dividend adjustment have less significance than dilution adjustment's ones. Therefore in the valuation of warrant prices the dilution adjustment is more important than the divi dend adjustment.

The results of Regression Analysis model can be improved by the reasonable design of samples considering industrial sector, dividends, time t maturity and so on.

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