A Novel Channel Estimation Method for OFDM under Rayleigh Fading Channel

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Abstract

Conventional channel estimation methods for orthogonal frequency division multiplexing (OFDM) system don't show the good characteristics in terms of fast fading channels. To solve this drawback in conventional methods, we propose the channel estimation method for OFDM, assisted pilot for improvement and convergence in mobile system (APIM), which has a good performance and computational complexity in consideration of other methods. APIM uses the more developed concept of conventional methods and a block frame structure within a whole channel. This concept prevents overall performance from diverging or showing a poor one. The simulation results demonstrate the APIM outperforms pilot symbol assisted modulation (PSAM) and extended symbol aided estimation (ESAE) in terms of mean square error (MSE) and bit error rate (BER) performance under all Rayleigh fading environment. Considering the simulation performance and computational complexity, we can see APIM shows better characteristics than conventional methods for OFDM and has not any error floor even in a fast Rayleigh fading environment.

I. Introduction

Multimedia and computer communications are playing an increasing role in today's society, creating new challenges to those working in the development of telecommunications system. Especially, digital multimedia environment, which requires high-speed data rates, causes an increased research on telecommunication system[3]. A broadband wireless channel is characterized both by time-variant behavior caused by a moving receiver or transmitter and by frequency-selective fading which is caused by multipath propagation[1]. In order to provide these services to mobile users, OFDM, recognized as a set of transmission techniques, has been particularly successful in numerous wireless applications, where its superior performance in multipath environments is desirable[4].

One of the main reasons to use OFDM is to increase the robustness against frequency selective fading or narrowband interference. But, if modulation method like M-ary quadrature amplitude modulation (QAM) is used, fading distortion happened in a channel should be estimated. So exact channel estimation in OFDM is a very crucial part for improvement of that system. Recently the range and applications of wireless communications are undergoing a rapid evolution. To cope with the multipath fading which is the major distortion in the mobile radio channels, a series of studies which utilize the symbol-aided techniques have been conducted to combat with fading channels. These methods use known pilot symbols which provides the receiver with observable fading reference for detection. Originally symbol-aided techniques use only pilot symbols to estimate fading process and referred to as PSAM. PSAM in a single carrier system was proposed in 1980's and linear minimum mean-squared error (LMMSE) method with pilot symbols in OFDM was proposed in 1991[6]. In 1997, Nam proposed ESAE method, which is generally superior to PSAM, for a single carrier system[2]. ESAE uses not only pilot symbol like PSAM but also an estimated fading value in a previous frame. ESAE has more robust features in the Rayleigh fading environment than those of PSAM and shows better performance when pilot intervals are so large. But ESAE has much more computational complexities than PSAM and the performance of channel estimation may be very poor or unstable if the previously estimated channels are wrong and accumulated. So we propose the APIM method which has robustness in a slow and fast fading channel and stable property. In this paper, we use the LMMSE, which has a good performance in terms of MSE, among channel estimation methods for OFDM. And the channel estimation method using a separable filter is applied instead of the 2-D Wiener filter. The separable filter can be implemented easily and shows the similar performance compared with 2-D filter. The analytical results demonstrate the APIM method outperforms PSAM and ESAE in terms of MSE and BER performance under Rayleigh fading environment. APIM shows far better MSE and BER performance than conventional methods in a high-speed mobile environment as

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well as normal speed environment. And APIM has the robustness against varying SNR while PSAM and ESAE haven't those characteristics. Furthermore we can ensure that APIM has a good performance in consideration of trade-off between computational complexity and performance.

In section II, conventional channel estimation methods are described briefly. In section III, proposed channel estimation method is presented. And the simulation results of these methods are shown and analyzed in section IV. Finally section V describes the conclusion.

Ⅱ. Conventional Channel Estimation Method for OFDM

In order to adapt with fast time-varying channel, pilot symbol should be inserted intervally and the method using combination of this idea is PSAM. PSAM was first introduced in the late 80's for single-carrier system. The use of PSAM requires a flat-fading and is based on transmission of known symbols in the data sequence, allowing channel estimation[5]. In 1997, Nam proposed ESAE method in that estimates the channel fading process in the current frame based on not only pilot symbol but also the previously estimated fading values[2]. It was shown that ESAE achieves better performance than the conventional PSAM at high signal to noise ratio (SNR) under fast fading and in case pilot intervals are large. But, at low SNR under slow fading, it was also pointed out that the performance of ESAE can be worse than that of PSAM. In addition, in case that the previously estimated channels are wrong and are accumulated, the performance of estimating may be very poor or unstable.

While 2 dimension (2-D) filter tends to have a high computational complexity in the time and frequency directions, the outer product of double 1-D filter can offer a good trade-off between performance and complexity. This is a standard technique in multidimensional signal processing and it has also been proposed for pilot-based channel estimation in OFDM systems. In OFDM, because subcarrier modulation such as QAM is used, precise Double 1-D filter can be described as 2-D because the channel correlation is separable. So, in this paper, we use a double 1-D filter.

Until now, channel estimators using least squares (LS), linear minimum mean-squared error (LMMSE) and singular value decomposition (SVD) schemes have been widely used for OFDM system[5]. The LS estimator has low complexity, but its performance is not as good as that of the LMMSE. And LMMSE estimator has good performance but high complexity. The channel estimator scheme used in this paper is LMMSE as in [5]. In (1), the minimum mean-squared error estimator of $\mathbf{h}$ is shown by arranging the available LS estimates at pilot positions in a vector $\mathbf{p}$ and the channel attenuations to be estimated in a vector $\mathbf{h}$.

$$\hat{\mathbf{h}} = \mathbf{R}_{\mathrm{hp}}\mathbf{R}_{\mathrm{pp}}^{-1}\mathbf{p} \tag{1}$$

where $\hat{\mathbf{h}}$ is the LMMSE, $\mathbf{p}$ is a vector of back-rotated observations, $\mathbf{R}_{\mathrm{hp}}$ is the cross-covariance matrix between the estimated channel attenuation $\mathbf{h}_{\mathrm{kl}}$ and $\mathbf{p}$, and $\mathbf{R}_{\mathrm{pp}}$ is the auto-covariance matrix.

Because all random variables are independent the correlation between channel attenuations separated by k subcarriers and l OFDM symbols is as (2), i.e., the channel correlation is separable.

$$E\{h_{k',l'}h^{*}_{k'-k,l'-l}\} = r_f(k)r_t(l) \tag{2}$$

And the expectations can be found from a standard Fourier transform such that

$$r_f(k) = R_f\left(\frac{k}{NT_s}\right) = \frac{(1-e^{-T_{cp}(1/\tau_{rms}+j2\pi k/N)})}{(1-e^{-L/\tau_{rms}})(1+j2\pi k\tilde{\tau}_{rms}/N)} \tag{3}$$

$$r_t(l) = R_t(l(N+L)T_s) = J_0\left(2\pi f_{D,\max}\left(1+\frac{L}{N}\right)l\right) \tag{4}$$

where $1/NT_S$ is the inter-carrier spacing, $(N+L)T_s$ is the duration of an OFDM symbol, $\tilde{\tau}_{rms}$ is the RMS-spread relative to the sampling interval, $f_{D,\max}$ is the maximum Doppler frequency relative to the inter-carrier spacing, and $J_0(\cdot)$ is the zeroth order Bessel function of the first kind.

And the other correlations can be represented by

$$E\{p_{k,l}p^{*}_{k',l'}\} = r_f(k-k')r_t(l-l') + \sigma_n^2 E\left\{\frac{1}{|x_k|^2}\right\}\delta(k-k',l-l') \tag{5}$$

$$E\{h_{k,l}p^{*}_{k',l'}\} = r_f(k-k')r_t(l-l') \tag{6}$$

As mentioned before, originally symbol-aided techniques use only pilot symbols to estimate fading process and referred to as PSAM. In that method, known symbols are inserted periodically into data sequence prior to pulse shaping, and the composite signal is transmitted through the channel characterized by non-selective Rayleigh fading and additive noise. At the receiver, after matched filter detection the pilot symbols are fed to the interpolation to provide the fading channel estimations on data symbols. On the other hands, in ESAE method, the interpolator is fed not only by the pilot symbols but also the estimated fading values on data symbols belonging to the previous frame and uses these values for estimating the fading values on data symbols in the current frame[2].

It was shown that ESAE achieves better performance than the PSAM at high SNR under a fast fading. However, at low SNR under slow fading, it was also pointed out that the performance of ESAE can be worse than that of PSAM.

Ⅲ. A Proposed Channel Estimation Method for OFDM

ESAE has better performance than PSAM especially in a fast fading environment but the former may have fatal characteristics. Specifically, it may cause the "run-away" phenomenon at fast fading since the estimation errors are accumulated up leading to the break down of the algorithm[2]. And this phenomenon can be found in PSAM if the fading environment is severe. In order to keep away that phenomenon, some additive pilots for preventing a divergence and different frame concept for improving the performance are needed. So we propose the method to avoid the unstable property and to improve the performance at the same time.

In order to solve the problems of ESAE and get the better performance than conventional methods, we propose the APIM method in Fig. 1. Fig. 1 shows the channel region to be estimated in each step by the APIM method and one block frame having 6 pilots and 2 additional pilots. 6 pilots, $p_j(1)$, $p_j(3)$, $p_j(5)$, $p_j(6)$, $p_j(7)$ and $p_j(8)$, having a regular interval provide the same function as those of PSAM. Conventional methods use the each frame structure having pilots and data but APIM utilizes the block frame structure consisted of five frames. And these two additional pilots, $p_j(2)$ and $p_j(4)$ in (7), are provided to avoid the unstable property occurred in ESAE and the technique to improve a performance. Also these two pilots are located in the middle position within a single frame, respectively. In Fig. 1, the block frame is composed of 5 separate frames, i.e., j ~ (j+4)th frame. For example, if the all fame channel size is 1024, that frame is composed of nearly 200 block frames. And, among all steps, step 4, 6 and 7 are proposed for solving the divergence problem and improving the performance in algorithm.

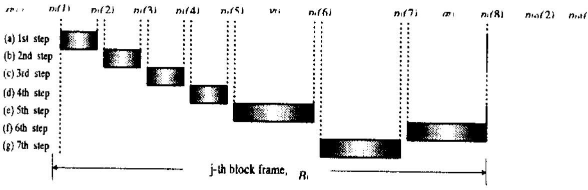

Fig. 1. Channel to be estimated in each step by APIM.

$B_{x,y}$ means the block frame where x, y represent block frame number and the frame number within a block frame, respectively. In this case, the size of y is M. Therefore M/2 in Fig. 1 means the middle point in a frame. The used channel attenuation vector for the proposed method is (7) based on [2]. Each position of eight pilots in (7) can be known by Fig. 1. In this paper, all notations written by boldface mean vector or matrix and notations written by normal face or italics represent the scalar value. Specifically, in (7), let **p** be the channel attenuation vector in which pilot is located. And let **u** be the channel vector. M represents the frame size and $n, \tilde{b}$, mean AWGN and pilot, respectively. Also j, j-1 mean the current and previous frame, separately.

$$\begin{aligned}\mathbf{p}_\mathbf{j}^\mathrm{T} &= [p_j(1), p_j(2), p_j(3), p_j(4), \cdots, p_j(8)] \\ &= [u(jM) + n(jM)/\tilde{b}, \ \cdots, \ u(j+5)M + n(j+5)M/\tilde{b}] \end{aligned} \tag{7}$$

where $p_j(2)$ and $p_j(4)$ are the additional pilots.

The value composed by 3 pilots, $p_j(1), p_j(2)$ and $p_j(3)$, and a pre-estimated channel, $q_{7,j-1}$, used for estimating channel at the 1st step is

$$\mathbf{p}_\mathbf{1,j}^\mathrm{T} = [q_{7,j-1}, p_j(1), p_j(2), p_j(3)] \tag{8}$$

where $q_{7,j-1} = \tilde{u}((j-1)M + M/2) = \tilde{u}_{6,j-1}(M/2)$ and $\tilde{u}(\cdot)$ means an estimated channel.

And the estimated channel in each step can be written by using relationship of LMMSE channel estimator in (1) as follows

$$\tilde{\mathbf{u}}_\mathbf{i,j}^\mathrm{T} = \mathbf{R}_\mathbf{up} \mathbf{R}_\mathbf{pp}^{-1} \mathbf{p}_\mathbf{i,j} \tag{9}$$

where i means step number.

And the minimum mean square error(MMSE)[5] is defined by

$$\xi_u = \mathrm{E}\left[| \mathbf{u}_\mathbf{i,j} - \tilde{\mathbf{u}}_\mathbf{i,j} |^2 \right] \tag{10}$$

Using (10), we can get the performance of APIM. Therefore the autocorrelation and crosscorrelation functions are needed to show the performance.

Also in step 4 and 6, based on ESAE method[2], (9) can be rewritten as

$$\tilde{\mathbf{u}}_\mathbf{i,j}^\mathrm{T} = \mathbf{R}_\mathbf{up} \mathbf{R}_\mathbf{pp}^{-1} \mathbf{p}_\mathbf{i,j} = \mathbf{O}^\mathrm{H} \begin{bmatrix} \mathbf{p}_\mathbf{i,j} \\ v_{i,j} \end{bmatrix} \tag{11}$$

where

$$\mathbf{O}^{H} = \begin{bmatrix}\mathbf{R}_{up} & \mathbf{R}_{uv}\end{bmatrix}\begin{bmatrix}\mathbf{R}_{pp} & \mathbf{R}_{pv}\\ \mathbf{R}_{vp} & \mathbf{R}_{vv}\end{bmatrix}^{-1} \tag{12}$$

and $v_{i,j}$ is the known value which is used for estimating the channel more efficiently.

The used vector at the 2nd step is shown by

$$\mathbf{p}_{2,j}^{T} = [p_j(1), p_j(2), p_j(3), p_j(4)] \tag{13}$$

The used vector at the 3rd step is written by

$$\mathbf{p}_{3,j}^{T} = [p_j(2), p_j(3), p_j(4), p_j(5)] \tag{14}$$

In the 2nd, 3rd and 5th step, Eq. (9) is applied for estimating the channel. The used vector at the 4th step is represented by

$$\mathbf{p}_{4,j}^{T} = [p_j(3), p_j(5), p_j(6), p_j(7)] \tag{15}$$

And by using (11) the estimated channel in the 4th step is given by

$$\tilde{\mathbf{u}}_{4,j}^{T} = \mathbf{O}^{H}\begin{bmatrix}\mathbf{p}_{4,j}\\ v_{4,j}\end{bmatrix} \tag{16}$$

where $v_{4,j} = p_j(4)$

The used vector at the 5th step is

$$\mathbf{p}_{5,j}^{T} = [p_j(3), p_j(4), p_j(5), \tilde{u}_{4,j}(M/2)] \tag{17}$$

The used vector at the 6th step is written by

$$\mathbf{p}_{6,j}^{T} = [p_j(6), p_j(7), p_j(8), p_{j+1}(3)] \tag{18}$$

And the estimated channel in the 6th step is

$$\tilde{\mathbf{u}}_{6,j}^{T} = \mathbf{O}^{H}\begin{bmatrix}\mathbf{p}_{6,j}\\ v_{6,j}\end{bmatrix} \tag{19}$$

where $v_{6,j} = p_{j+1}(2)$

The used vector at the 7th step is

$$\mathbf{p}_{7,j}^{T} = [p_j(5), p_j(6), p_j(7), p_j(8)] \tag{20}$$

In step 7, the channel vector is represented as

$$\tilde{\mathbf{u}}_{7,j}^{T} = \mathbf{R}_{up}\mathbf{R}_{pp}^{-1}\mathbf{p}_{7,j} = \mathbf{O}^{H}\begin{bmatrix}\mathbf{p}_{7,j}\\ v_{7,j}\\ q_{7,j}\end{bmatrix} \tag{21}$$

where

$$v_{7,j} = \tilde{u}_{4,j}(M/2) \tag{22}$$

$$q_{7,j} = \tilde{u}_{6,j}(M/2) \tag{23}$$

and

$$\mathbf{O}^{H} = \begin{bmatrix}\mathbf{R}_{up} & \mathbf{R}_{uv} & \mathbf{R}_{uq}\end{bmatrix}\begin{bmatrix}\mathbf{R}_{pp} & \mathbf{R}_{pv} & \mathbf{R}_{pq}\\ \mathbf{R}_{vp} & \mathbf{R}_{vv} & \mathbf{R}_{vq}\\ \mathbf{R}_{qp} & \mathbf{R}_{qv} & \mathbf{R}_{qq}\end{bmatrix}^{-1} \tag{24}$$

and $q_{7,j}$ is the pre-estimated channel in step (6). Consequently, by putting (7)~(24) processes into (10), we can obtain the performance of the proposed method .

Ⅳ. Simulation Results

In this section, we use computer simulation to show the performance of conventional methods and the proposed method. Eq. (25) is a channel model used in this paper[7].

$$G(f;t) = \frac{1}{M}\sum_{n=1}^{M} e^{j\left(\theta_n + 2\pi F_{D_n} t - 2\pi f \tau_n\right)} \tag{25}$$

where θ_n is a phase, τ_n is n-th path delay time and F_{Dn} is Doppler frequency. Rayleigh fading by Jakes spectrum model is applied to this channel model. The system in this paper has a bandwidth of 5MHz. With 1024 and 512 channels for frequency and time axis respectively, 1024 subcarriers have 0.2μs separately and effective symbol length is 205μs. In order to remove ISI, 61 cyclic prefix are used. And mobile object speed in computer simulation are 100, 125km/h. So, Doppler spread values for simulation are 0.031, 0.039. 16-QAM method is applied and the pilot interval on frequency axis is fixed at 4 under all environments. And, in this paper, all figures are shown in a mismatched environment.

In simulation results, frame size of all methods except the proposed method is set by (M-1) to evaluate fairly the performance of all methods. That is, the size of frame channel in the conventional methods is (M-1) and that of the APIM is M. In this case, the computational complexity of all methods can be known by Table 1. As we can see in Table 1, APIM method has a little larger computational complexity than PSAM. But, if the computational complexity, run-away phenomenon and performance are considered simultaneously, this gap in terms of complexity is generally of less importance . Because APIM has much better performance than PSAM in all fading environment even if the former has a little larger complexity than the latter. The proposed method uses the additional pilots in a block frame, i.e., this method uses more pilots than other methods. So the frame size of the APIM is set by M. In other words, this is for dissolving a biased problem which can be occurred by inserting two additional pilots into the APIM. But, in this case, the APIM also represents so good performance. And, in simulation results, two different ESAEs are used ; one uses two

Table 1. A comparison of computational complexity among all methods.(M=Number of channel in a frame, L_p = Number of used pilots, L_0=Number of estimated channel in a previous frame).

	General computational complexity	L_0	M	L_p	Computational complexity
PSAM	$5(M-2)L_p$		16	4	280
ESAE(2)	$5(M-2)(L_p+L_0)$	2			420
ESAE(Full)	$5(M-2)(L_p+L_0)$	$(M-2)$			1260
APIM	$(M-1)(3L_p+4L_0)+(2M-4)L_p$	1			412

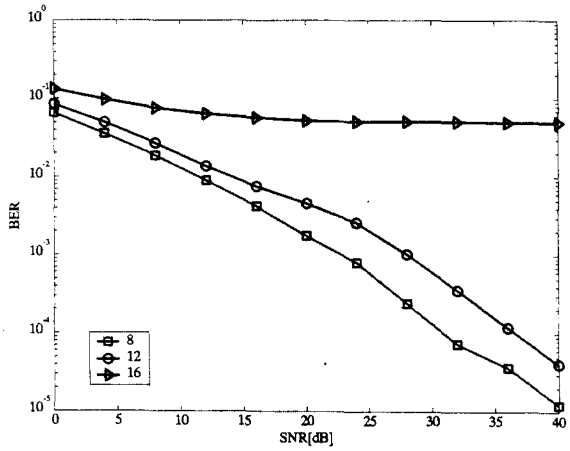

Fig. 2. BER performance of the proposed method according to pilot interval(Velocity = 125 km/h).

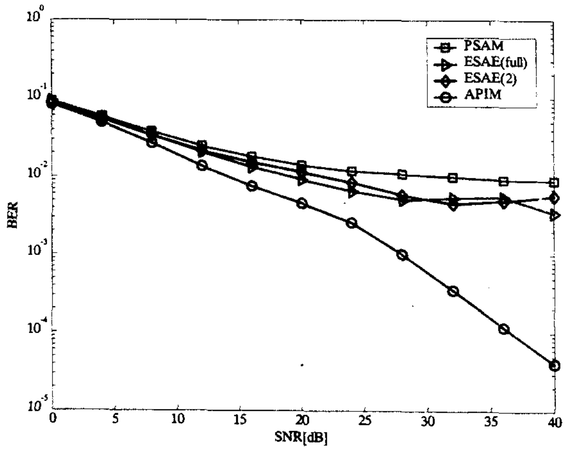

Fig. 4. BER performance according to SNR(Velocity=125 km/h, pilot interval = 12).

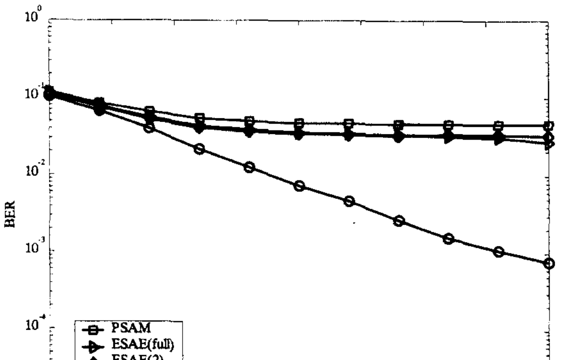

Fig. 3. BER performance according to SNR(Velocity=100 km/h, pilot interval = 16).

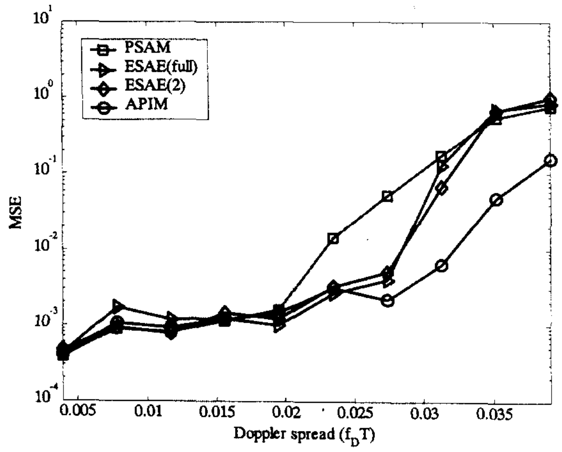

Fig. 5. MSE performance according to Doppler spread(SNR = 30 dB, pilot interval = 16).

pre-estimated channels in a previous channel, another uses entire pre-estimated channels. Fig. 2 shows the BER performance of the APIM according to pilot interval when velocity is 125 km/h. Except for pilot interval = 16, BER performance of APIM is so good and robust under all SNR even in a fast Rayleigh fading environment. Fig. 2 shows, at SNR=20 dB, BER performance in pilot interval=8 and 12 is about 10^{-2} and 10^{-1}superior to pilot interval=16 respectively. For the sake of data transmission efficiency, the large interval of pilot is preferable. And as we can know from this result, the pilot interval = 12 is suitable for data transmission efficiency and performance. In Fig. 3, when mobility velocity is 100 km/h and pilot interval is 16, only the

APIM shows stable performance and its BER is 10^{-1} superior to conventional methods at SNR=20 dB. Here, the pilot interval is so large that PSAM and ESAE methods are easily inclined to diverge, i.e., have the run-away phenomenon. But the APIM method has not only stable property but also better performance than conventional methods at SNR$\geq$5 dB. Also, Fig. 4 indicates the better performance of APIM than conventional methods even in a high-speed environment. And the performance of the APIM shows good performance at SNR = 20~40 dB. Fig. 5 shows the performance related with Doppler spread. In this figure, compared with conventional methods, the larger the Doppler spread, the better the MSE performance of APIM is. For example, when the Doppler spread is over 0.023, i.e., the mobile speed is about 75 km/h, its MSE performance is better than conventional methods. Simulation results indicate that APIM has a good performance regardless of fading condition and pilot interval. Especially, this method has so stable property in severe fading environment that proves the problem of ESAE. And we can conclude the optimal pilot interval is 12 because it has much better performance than 8, 16 in consideration of trade-off between the computational complexity and the performance.

V. Conclusions

In this paper we have proposed the method, which has good MSE and BER performance under Rayleigh fading channel, for OFDM in multimedia mobile communication environment. The proposed method dissolves an unstable characteristic, shown in ESAE, by inserting an additional pilot with large interval into block frame. APIM shows better performance than PSAM and ESAE even in severe fading channel environment and in a larger pilot interval condition. So, the APIM method is suitable for multimedia mobile communication system. Consequently, the proposed method has better MSE, BER performance than those of conventional methods. But this proposed method needs larger computational complexity than PSAM. So in order to solve this defect, the more improved method should be studied. By doing this work, we can find the channel estimation method suitable for high-speed mobile system.

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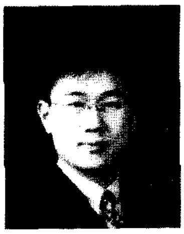

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