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Polarizability for a Circular Aperture Near a Conducting Plane

Yong Bae Park · Il Han Kim · Hyo Joon Eom

Abstract

A polarizability for a circular aperture near a conducting plane is derived. The Hankel-transform and mode-matching is used to obtain a simple series solution. The presented series solution is fast convergent so that it is very efficient for numerical computations.

Key words : circular aperture, polarizability, Hankel-transform, mode-matching

Ⅰ. Introduction

The polarizability is an useful concept to estimate low-frequency field penetration into a aperture. The polarizability for a circular aperture in a conducting plane has been studied in [1]~[4] due to its application in electromagnetic inteference and compatibility. It is of theoretical interest to investigate the behavior of field penetration into a circular aperture when a charged conducting plane is placed close to a circular aperture. In this paper, we will derive a polarizability for a circular aperture that is placed near a conducting plane. We obtain a fast-convergent series representation for the polarizability for a circular aperture by using the Hankel transform, which was used in [4].

Ⅱ. Potential Representations and Numerical Computations

A circular aperture in a thick conducting plane at zero potential is placed near a conducting plane at potential V(see Fig. 1).

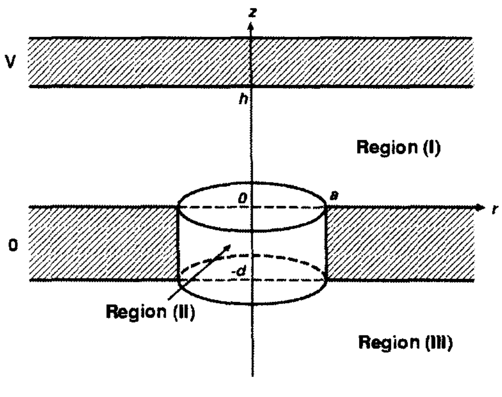

Fig. 1. Problem geometry.

The electrostatic potentials in region (I), (II), and (III) are represented as

$$\Phi^{I}(r,z)=\int_0^{\infty}\left[\tilde{\Phi}^{+}(\zeta)e^{-\zeta z}+\tilde{\Phi}^{-}(\zeta)e^{\zeta z}\right]J_0(\zeta r)\zeta d\zeta \tag{1}$$

$$\Phi^{II}(r,z)=\sum_{n=1}^{\infty}\left[b_n\sinh k_n(z+d)+c_n\cosh k_n(z+d)\right]J_0(k_n r) \tag{2}$$

$$\Phi^{III}(r,z)=\int_0^{\infty}\tilde{\Phi}^{III}(\zeta)e^{\zeta(z+d)}J_0(\zeta r)\zeta d\zeta \tag{3}$$

where the constant k_n is determined by $J_0\ (k_n\ a)=0$.

The boundary conditions on the field continuities at $z=h$, 0, and $-d$ require

$$\Phi^{I}(r,h)=V \tag{4}$$

$$\Phi^{I}(r,0)=\begin{cases}\Phi^{II}(r,0), r<a\\ 0, r>a\end{cases} \tag{5}$$

$$\Phi^{III}(r,-d)=\begin{cases}\Phi^{II}(r,-d), r<a\\ 0, r>a\end{cases} \tag{6}$$

Applying the Hankel transform to (4) through (6) yields

$$\tilde{\Phi}^{+}(\zeta)=\frac{\frac{2V}{\zeta}\delta(\zeta)-e^{\zeta h}\sum_{n=1}^{\infty}\left(b_n\sinh k_n d+c_n\cosh k_n d\right)P_n}{e^{-\zeta h}-e^{\zeta h}} \tag{7}$$

$$\tilde{\Phi}^{+}(\zeta)=\frac{-\frac{2V}{\zeta}\delta(\zeta)+e^{-\zeta h}\sum_{n=1}^{\infty}\left(b_n\sinh k_n d+c_n\cosh k_n d\right)P_n}{e^{-\zeta h}-e^{\zeta h}} \tag{8}$$

$$\tilde{\Phi}^{III}(\zeta)=\sum_{n=1}^{\infty}c_nP_n \tag{9}$$

where

Manuscript received November 5, 2001; revised November 27, 2001.
Department of Electrical Engineering, Korea Advanced Institute of Science and Technology, 373-1, Guseong-dong, Yuseong-gu, Daejeon, Korea
E-mail : hjeom@ee.kaist.ac.kr

$$P_n = -\frac{ak_n J_0(\varsigma a)J_1(k_n a)}{\varsigma^2 - k_n^2} \tag{10}$$

The continuity condition

$$\left.\frac{\partial \Phi^I(r,z)}{\partial z}\right|_{z=0} = \left.\frac{\partial \Phi^{II}(r,z)}{\partial z}\right|_{z=0} \tag{11}$$

is rewritten as

$$-\int_0^\infty \left[\tilde{\Phi}^+(\varsigma) - \tilde{\Phi}^-(\varsigma)\right] J_0(\varsigma r)\varsigma^2 d\varsigma = \sum_{n=1}^{\infty}(b_n \cosh k_n d + c_n \sinh k_n d)k_n J_0(k_n r) \tag{12}$$

Multiplying (12) by $J_m(k_q^m r)r$, integrating with respect to r from 0 to a, we get

$$\sum_{n=1}^{\infty}(b_n \sinh k_n d + c_n \cosh k_n d)a^2 k_n k_p J_1(k_n a)J_1(k_p a)I_1 = -\frac{a^2}{2}k_p[J_1(k_p a)]^2(b_p \cosh k_p d + c_p \sinh k_p d) + \frac{aV}{k_p h}J_1(k_p a) \tag{13}$$

where

$$I_1 = \int_0^\infty \coth(\varsigma h)\frac{[J_0(\varsigma a)]^2 \varsigma^2}{(\varsigma^2 - k_n^2)(\varsigma^2 - k_p^2)} \tag{14}$$

Similarly from the continuity conditions at $z = -d$ we get

$$\frac{a^2}{2k_p}[J_1(k_p a)]^2 b_p = \sum_{n=1}^{\infty} c_n a^2 k_n k_p J_1(k_n a)J_1(k_p a)I_2 \tag{15}$$

where

$$I_2 = \int_0^\infty \frac{[J_0(\varsigma a)]^2 \varsigma^2}{(\varsigma^2 - k_n^2)(\varsigma^2 - k_p^2)} \tag{16}$$

A set of simultaneous equations (13) and (15) can be solved numerically for the modal coefficient b_n and c_n. When the conducting plane at $z=h$ is removed $h \to \infty$, (12) and (15) reduces to (2.10) and (2.13) in [3]. The electric polarizability is given by

$$\chi(z) = 4\pi\int_0^a \Phi^{II}(r,z)rdr = 4\pi a\sum_{n=1}^{\infty}\left[b_n \sinh k_n(z+d) + c_n \cosh k_n(z+d)\right]J_1(k_n a)/k_n \tag{17}$$

To check the accuracy of our formulation, we plot the normalized polarizability $\chi'(z) = \ln\left[3\chi(z)/8a^3\right]$ for the circular aperture at $z=0$ and $-d$ in Fig. 2 and Fig. 3, respectively. The number of modes n used in our computation is 6, indicating that our series solution is rapidly convergent. In Fig. 2, our result for $h/a>1$ approaches the result of [3]. As the spacing between parallel conducting planes decreases to h/a=0.01, the polarizabilities become independent of d/a, approaching $\ln(3\pi h/4a)$. In Fig. 3, our result for $h/a>1$ agrees well with [3] irrespective of the size of d/a. This means that the multiple reflection between two parallel conducting planes at z=0 and h may be ignored in the polarizability computation as long as $h/a>1$.

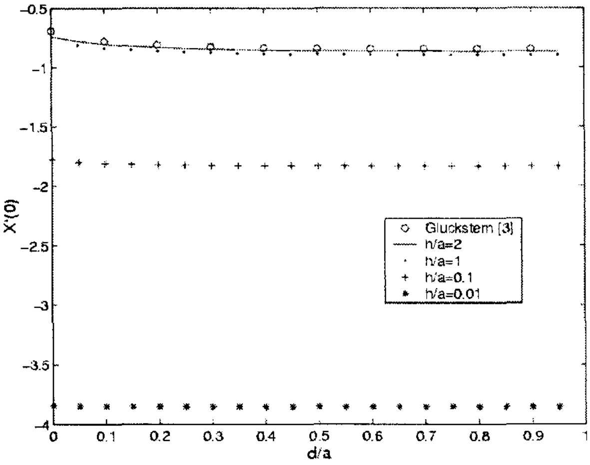

Fig. 2. Normalized polarizability as a function of d/a at $z=0$ (V/h=1).

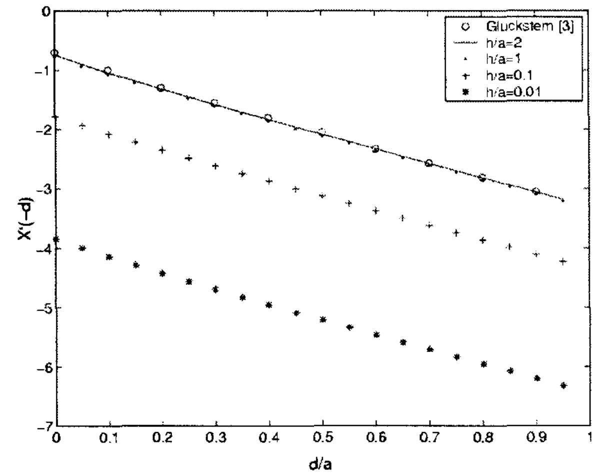

Fig. 3. Normalized polarizability as a function of d/a at $z=-d$ (V/h=1).

Ⅲ. Conclusion

The polarizability for a circular aperture near a conducting plane is investigated. The simple series solution is obtained using the Hankel transform and mode matching. The presented solution converges rapidly so that it is very efficient for numerical computation.

The effect of a spacing between two parallel conducting planes is discussed.

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Yong Bae Park

was born in Korea in 1977. He received the B.S and M.S. degrees in electrical engineering from Korea Advanced Institute of Science and Technology (KAIST), in 1998 and 2000, respectively. He is currently pursuing the Ph.D. degree in electrical engineering at KAIST. His research interests include wave scattering and EMI/EMC problems.

Il Han Kim

was born in Korea in 1980. He is now a senior in the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology.

Hyo Joon Eom

received the B.S. degree in electronic engineering from Seoul National University, Korea, and the M.S. and Ph.D. degrees in electrical engineering from the University of Kansas, Lawrence, KS, USA in 1977 and 1982, respectively. From 1981-1984, he was a Research Associate at the Remote Sensing Laboratory of the University of Kansas. From 1984-1989, he was with the faculty of the Department of Electrical Engineering and Computer Science, University of Illinois, Chicago, IL. In 1989, he joined the Department of Electrical Engineering, Korea Advanced Institute of Science and Technology where he is now a Professor. His research interests are wave diffraction and scattering.