Stable Haptic Display Based on Coupling Impedance for Internal and External Forces

Masayuki Kawai and Tsuneo Yoshikawa

Abstract: This paper discusses haptic display for grasping a virtual object by two fingers. Much research has been done on fundamental analysis for stability of haptic display. But it is difficult to apply the results immediately to grasping situations by two fingers, since the studies usually deal with a single device and a single object and the fingertip force in grasping situations has two components, internal and external components. The conventional methods, which specify the coupling impedance at each contact point separately, have no other alternative but to specify the impedance for the sum of the internal and external components. So even if only the impedance for the external force should be changed, the impedance for the internal force is also changed at the same time. In this paper, a new method, in which the coupling impedance is specified separately for the internal and external forces, is proposed and the stability of the proposed method is discussed using passivity analysis for 1-DOF(Degree-Of-Freedom) system. Finally, some experiments are performed to study the effects of the proposed method.

Keywords: virtual reality, haptic interface, grasping, passivity analysis

I. Introduction

Haptic interface is important for constructing virtual world that operators feel more real. Keeping haptic interface stable is one of the most important problems, because haptic device has contacts directly with the operators. But it is also a difficult problem to keep the system stable, since the human dynamics is always unknown.

A number of researchers have considered the analysis of stability in haptic interaction. Especially, they studied a relation between stability and coupling impedance, which is virtually set between virtual environments and a haptic display device. Minsky *et al.* [1] discussed a tradeoff relation among simulation rate, stiffness of virtual wall and viscosity of haptic display device for simple virtual environments. Colgate *et al.* [2][3] discussed passivity of haptic display of simple virtual wall and a moving object and they derived conditions under which the haptic display would exhibit passive behavior. Such fundamental stability and performance issues were well discussed by Adams and Hannaford [4] with two-port model, which is a common method of analyzing stability and performance in bilateral teleoperation. However, the object of such studies was a simple environment with a device, an operator and a virtual object.

On the other hands, we have been constructing a 3-DOF haptic display system for two fingers applying to assembling job [5][6]. In the system, the operator attaches small robotic manipulators on his/her thumb and forefinger and can grasp a virtual object with the two fingers. For developing the system, it is necessary to analyze the stability of the grasping situation but the results of the previous studies for a simple environment can not be immediately applied. This is because there are external and internal components in the fingertip force at the grasping situation. The external force is related with the dynamics of the grasped virtual object, but the internal one is not.

This paper proposes a new approach of setting the coupling impedance for grasping a virtual object by two fingers, in which the coupling impedances are divided into two components: one is for the external force and the other is for the internal force. Then, the stability of the proposed method is discussed using passivity analysis and it is shown that the stability issue can be converted to the issues for a simple environment which has been much studied in the previous researches. The coupling impedance for the external force in the proposed method is converted to the impedance of the tool-type haptic display, which is related to the dynamics of the virtual object and haptic device. The internal one is converted to the impedance of haptic display for a static environment, which is related only to the dynamics of haptic device. It is shown that the conventional method is a special case of the proposed method, which sets the internal and external components equal. Finally, experiments with two-finger type haptic display device are performed to study the effect of the proposed method.

II. Basic approach

This section describes the target of this study, the conventional method and the basic idea of the proposed method.

1. Haptic display for grasping

The purpose of this study is haptic display for grasping a virtual object with two or more haptic devices. It contains both the case of grasping with hands by multiple tool-type haptic display devices, as shown in Fig.1, and the case of grasping with a hand by multi-fingered wearable haptic display device, as shown in Fig.2. Such display systems can be generally expressed as Fig.3.

In the figure, n is the number of haptic devices, $\boldsymbol{r}_i$ is the fingertip position of the ith haptic device , $\boldsymbol{r}_{di}$ is the contact point on the surface of virtual object for the ith haptic device, $\boldsymbol{e}_i$ is

---

Manuscript received: Aug. 23, 2001., Accepted: Nov. 16, 2001.

Masayuki Kawai: Department of Mechanical Engineering, Fukui University. (mkawai@mech.fukui-u.ac.jp)
Tsuneo Yoshikawa: Department of Mechanical Engineering, Kyoto University. (yoshi@mech.kyoto-u.ac.jp)

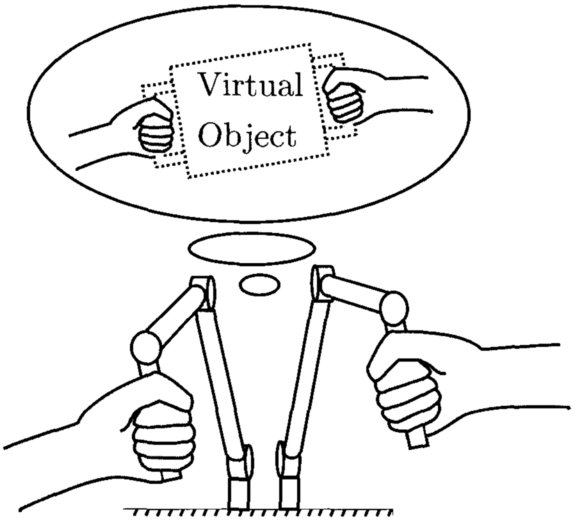


Fig. 1. Grasping by Multiple Tool-type Device.

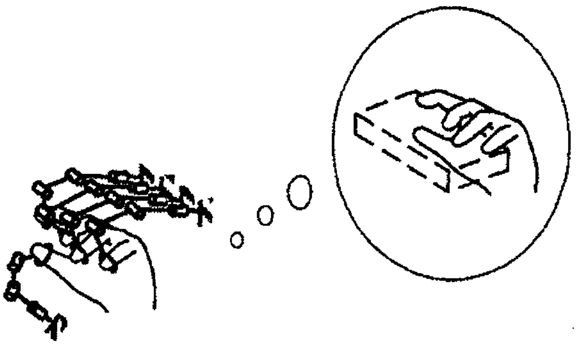


Fig. 2. Grasping by Multi-fingered Device.

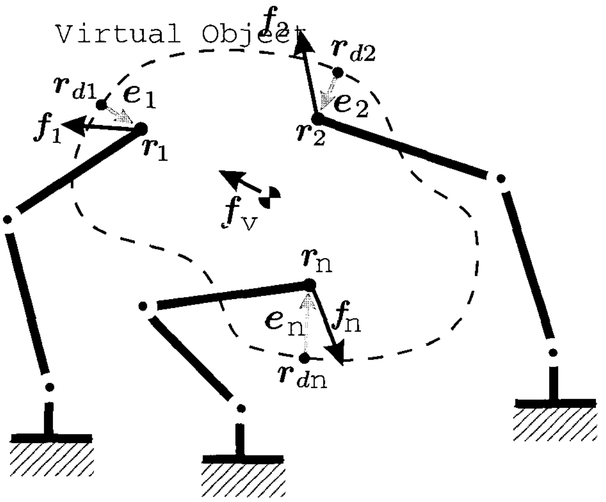


Fig. 3. Model of Grasping Situation.

the displacement from r_{di} to r_i. Conventionally, the fingertip force $f = [f_1^T f_2^T \cdots f_n^T]^T$ is specified separately for each device using the coupling impedance with $e = [e_1^T e_2^T \cdots e_n^T]^T$ described as:

$$f = K_p e + K_d \dot{e} \qquad (1)$$

where, K_p is the stiffness coefficient and K_d is the damping coefficient of the coupling impedance. This method is simple and practical and so it is relatively common. We call it "the conventional method" in this paper. But it has a problem that the internal and external forces are not distinguished.

2. Proposed method

This paper proposes a method of setting the coupling impedance separately for the internal and external force. A relation between the fingertip force f and the resultant force f_v to the grasped object is generally described as:

$$f_v = Gf \qquad (2)$$

where, G is the grasping matrix. The fingertip force f can be given with the pseudo-inverse matrix of G as:

$$f = G^+Gf + [I - G^+G]f \qquad (3)$$

where, G^+ is the pseudo-inverse matrix of G. The first term is the external force and is exerted to the grasped object and the second term is the internal force and is not related with the motion of the virtual object. The basic approach of the proposed method is setting different impedances to the internal and external force as:

$$f = G^+G[K_{ep}e + K_{ed}\dot{e}] + [I - G^+G][K_{ip}e + K_{id}\dot{e}] \quad (4)$$

where, K_{ep} and K_{ed} are the stiffness and damping coefficients for the external force and K_{ip} and K_{id} are those for the internal force.

For studying the effect of the proposed method, the case of applying the method to 1-DOF haptic display system will be discussed in the following section.

III. Analysis for 1-DOF system

1. Conventional method

This section explains the conventional method for 1-DOF system shown in Fig.4, which includes two 1-DOF haptic devices and represents displaying a virtual object moving in one-dimensional environment.

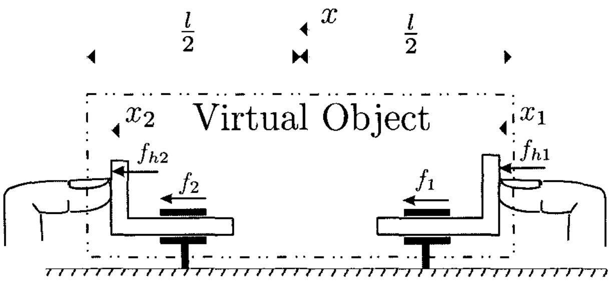


Fig. 4. 1-DOF Haptic Device for Grasping.

In the figure, x_i is the position of the ith device, f_{hi} is the force applied by the ith device to the operator, f_i is the force exerted by ith actuator, and the dynamics of each device are :

$$f_1 = m\ddot{x}_1 + b\dot{x}_1 - f_{h1} \qquad (5)$$

$$f_2 = m\ddot{x}_2 + b\dot{x}_2 - f_{h2} \qquad (6)$$

where m and b are the mass and the viscous friction of each device, respectively. It is assumed that the two devices have the same mass and the same viscous friction.

When the both devices touch a virtual object, the conventional method specifies the forces f_i, which act upon the device, with coupling impedance described by:

$$f_1 = k_p(x - x_1) + k_d(\dot{x} - \dot{x}_1) \quad (7)$$

$$f_2 = k_p(x - x_2) + k_d(\dot{x} - \dot{x}_2) \quad (8)$$

where, x is the center of mass of the virtual object, k_p is the stiffness and k_d is the damping coefficient of the coupling impedance. To make the problem simple, it is assumed that the width l of the virtual object is zero. The motion of the virtual object is simulated with the resultant force f_v calculated as :

$$f_v = [1 \;\; 1] \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad (9)$$

It is represented by the model shown in Fig.5.

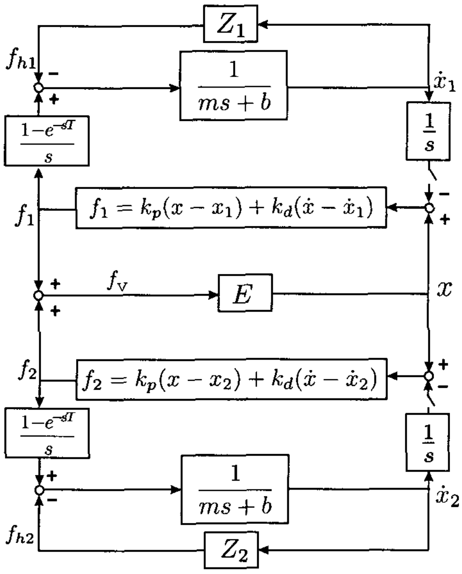


Fig. 5. Model of 1-DOF Haptic Interface of Conventional Method

In the figure, Z_i expresses the dynamics of the ith operator. E is the dynamics of the virtual object, T is the sampling rate, and $(1 - e^{-sT})/s$ is the zero order hold.

2. Proposed method for 1-DOF system

The fingertip forces f_i can be separated into the external and internal forces using (3). Since the pseudo-inverse matrix of $[1 \;\; 1]$ in (9) is $[\frac{1}{2} \;\; \frac{1}{2}]^T$, the fingertip force can be separated as:

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad (10)$$

Therefore, instead of (7) and (8), the proposed method gives the new impedance from (4) :

$$\begin{aligned} f_1 &= k_{ep}(x - \frac{x_1 + x_2}{2}) + k_{ed}(\dot{x} - \frac{\dot{x}_1 + \dot{x}_2}{2}) \\ &\quad + k_{ip}\frac{x_2 - x_1}{2} + k_{id}\frac{\dot{x}_2 - \dot{x}_1}{2} \quad (11) \\ f_2 &= k_{ep}(x - \frac{x_1 + x_2}{2}) + k_{ed}(\dot{x} - \frac{\dot{x}_1 + \dot{x}_2}{2}) \\ &\quad - k_{ip}\frac{x_2 - x_1}{2} - k_{id}\frac{\dot{x}_2 - \dot{x}_1}{2} \quad (12) \end{aligned}$$

where, k_{ep} and k_{ed} are the stiffness and damping coefficients for the external force, and k_{ip} and k_{id} are the stiffness and damping coefficients for the internal force, respectively. The method can conceptually be represented by Fig.6. The first two terms of (11) and (12) mean the coupling impedance between $(x_1 + x_2)/2$ and the center of the virtual object, and the last two terms of (11) and (12) mean the coupling impedance between the two devices.

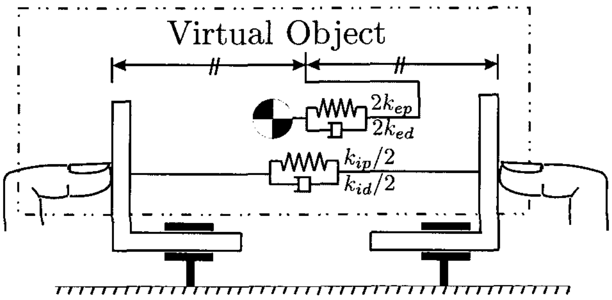


Fig. 6. Conceptual Model.

3. Stability analysis

This section shows that stability issue for the proposed method can be converted to issues for a device which has been considered in the previous works [2][3]. When the dynamics including multiple bodies are discussed, it is useful to discuss the dynamics of the center of mass and the relative motion of the each body from the center of mass. The center of mass of the devices is given by:

$$x_e = \frac{x_1 + x_2}{2} \quad (13)$$

and the dynamics equation for x_e is given by adding (5) and (6):

$$\frac{f_1 + f_2}{2} = m\ddot{x}_e + b\dot{x}_e - \frac{f_{h1} + f_{h2}}{2} \quad (14)$$

The coupling impedance for x_e is given by adding (11) and (12).

$$\frac{f_1 + f_2}{2} = k_{ep}(x - x_e) + k_{ed}(\dot{x} - \dot{x}_e) \quad (15)$$

This equation shows that only the first two terms of (11) and (12) will be effective for the center of mass and it can be represented by the model shown in Fig.7.

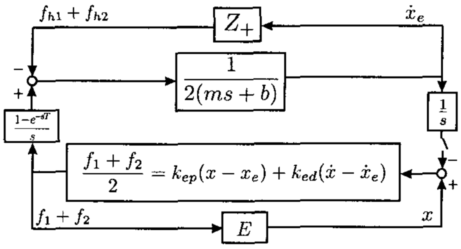


Fig. 7. Model for Center of Mass.

In the figure, Z_+ is the dynamics of the operator when the input is x_e and the output is $f_{h1} + f_{h2}$. This figure is the same model for haptic display of a dynamic virtual object with a single tool-type display device. Therefore, k_{ep} and k_{ed} which make the center of mass stable could be selected from the previous studies [3].

On the other hands, the relative positions of the two devices from the center of mass are :

$$x_{i1} = x_e - x_1 = \frac{-x_1 + x_2}{2} \tag{16}$$

$$x_{i2} = x_e - x_2 = \frac{x_1 - x_2}{2} \tag{17}$$

The dynamics equation for x_{i1} is given by subtracting (5) from (6).

$$\frac{-f_1 + f_2}{2} = m\ddot{x}_{i1} + b\dot{x}_{i1} - \frac{-f_{h1} + f_{h2}}{2} \tag{18}$$

The virtual coupling impedance is given by subtracting (11) from (12).

$$\frac{-f_1 + f_2}{2} = -k_{ip}x_{i1} - k_{id}\dot{x}_{i1} \tag{19}$$

Only the last two terms of (11) and (12) remain and are effective for the dynamics of the relative motion. It can be represented by the model shown in Fig.8.

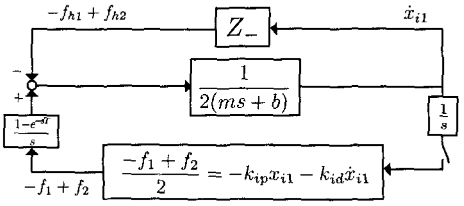


Fig. 8. Model of Relative Motion from Center of Mass.

In the figure, Z_- is the dynamics of the operator when the input is x_{i1} and the out put is $-f_{h1} + f_{h2}$. Fig.8 is the same as the model for displaying a static environment like a virtual wall with a device. Therefore, k_{ip} and k_{id} which make the relative motion stable could also be selected from the previous studies [2]. The result of analysis for x_{i2} can be derived same as x_{i1} .

It is important that the proposed method can independently specify the suitable coupling impedance for the external force and the internal force. The proposed method is considered to be useful for systems including a number of virtual objects with different dynamics because the suitable impedance for the external force changes according to the dynamics of the virtual object, but the suitable coupling impedance for the internal force does not need to be changed regardless of the dynamics of the virtual object.

Note that the assumption that the dynamics of the operator Z_+ in Fig.7 and Z_- in Fig.8 are linear time-invariant does not always correspond with the assumption that the dynamics of the operator Z_1 and Z_2 in Fig.5 are linear time-invariant. Z_1 and Z_2 are the dynamics of each finger and Z_+ and Z_- are the dynamics of a hand and an arm expressed in Fig.9.

Since the real human dynamics is not usually linear time-invariant, it is difficult to decide which assumption is better. It would be a future research topic.

4. Relation between conventional and proposed methods

In this section, the relation between the conventional method given by (7) and (8) and the proposed method by (11) and (12) is discussed. When the impedance in the proposed method is set as

$$k_{ep} = k_{ip} \tag{20}$$

$$k_{ed} = k_{id} \tag{21}$$

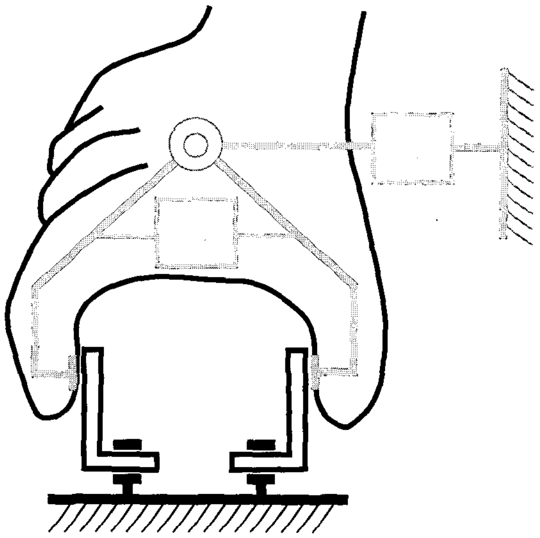


Fig. 9. Human Dynamics as Hand+Arm system.

(11) and (12) become

$$f_1 = k_{ep}(x - x_1) + k_{ed}(\dot{x} - \dot{x}_1) \tag{22}$$

$$f_2 = k_{ep}(x - x_2) + k_{ed}(\dot{x} - \dot{x}_2) \tag{23}$$

These are the same as (7) and (8). Hence the conventional method, (7) and (8), is a special case of the proposed method, in which the coupling impedances for the external and internal forces are set equal.

5. Switching between grasping and pushing

This section discusses the moment when a finger touches or leaves a virtual object and how the impedance should be chosen when only one finger touches the virtual object and the other one is away. Assume that an operator grasps a virtual object and will detach the finger i from the object and keep the jth finger touch it. It is easy to find that the force to the ith finger f_i is not zero at the moment $x_i = x$ if $k_{ep} \neq k_{ip}$. So the point $x_i = x$ should not be considered to be the contact point and a new contact point should be defined. Focusing on the force from only the stiffness k_{ep}, k_{ip}, a point where the force becomes zero is chosen as a new contact point x_{ci}, and x_{ci} can be defined as:

$$k_{ep}(x - \frac{x_{ci} + x_j}{2}) + k_{ip}\frac{x_j - x_{ci}}{2} = 0 \tag{24}$$

In other words,

$$x_{ci} = \frac{-k_{ep} + k_{ip}}{k_{ep} + k_{ip}}x_j + \frac{2k_{ep}}{k_{ep} + k_{ip}}x \tag{25}$$

When the position x_i is nearer to the virtual object than x_{ci}, the ith finger touches the virtual object and f_i is specified by (11) and (12). When x_i is farther than x_{ci}, the finger i is not in touch and $f_i = 0$. At the time when the ith finger leaves the virtual object, the force f_j for the jth finger which is touching the virtual object is given by

$$f_j = \frac{2k_{ep}k_{ip}}{k_{ep} + k_{ip}}(x - x_j) + \frac{2k_{ed}k_{id}}{k_{ed} + k_{id}}(\dot{x} - \dot{x}_j) \tag{26}$$

It is equivalent to substituting (25) into (11) and (12).

IV. Experiment

In this section, some experiments are performed with two 2-DOF haptic devices and a two-dimensional virtual environment.

1. Experimental device

The overview of the experimental set up is shown in Fig.10.

Fig. 10. Experimental device.

The system has two haptic display devices, each of which has two DC motors and encoders and two links whose length are 128mm. The operator put the fingertip of the device on the thumb and the forefinger. The position of the fingertip and a virtual object are shown on the CRT. The virtual object's shape is cube and the length of each sides is 50mm and the dynamics is described as:

$$f_v = m_v\ddot{x} + b_v\dot{x} \tag{27}$$

where, f_v is resultant force, x is the position of the virtual object, m_v is the mass and b_v is the viscosity. In all experiments, we set the virtual damping coefficients zero($k_d = k_{ed} = k_{id} = 0$), the system sampling rate 5msec and the mass of the virtual object m_v 0.1kg, to make comparison plain.

2. Experiment of human manipulation

First, an experiment of human manipulating and waving the object horizontally by two fingers is performed. The frequency of waving is about 0.5Hz and the amplitude is 50mm (Fig.11).

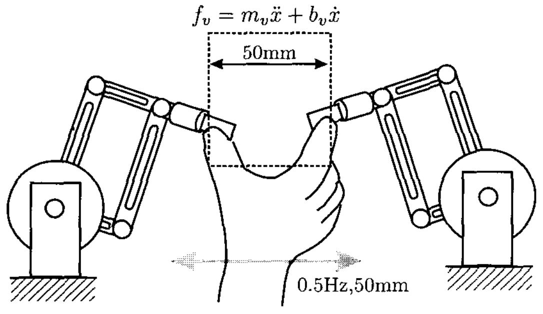
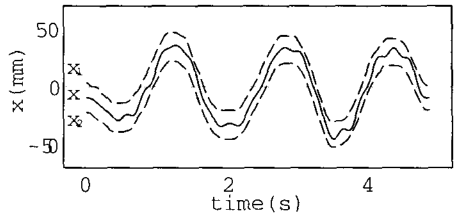
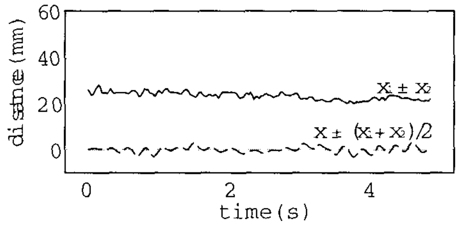


Fig. 11. Experiment of Human Manipulation.

The result of the conventional method is shown in Fig.12 when the object's viscosity is b_v = 0.8Nsec/m. The stiffness coefficient of the coupling impedance is k_p = 30N/m and it is the maximum value that the operator manage to manipulate the object.



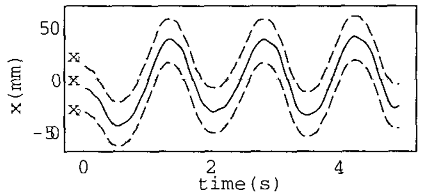
(a) Horizontal Direction



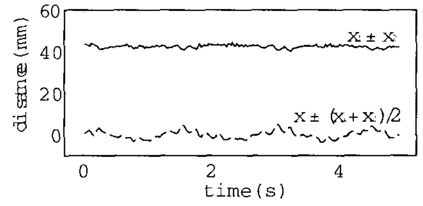
(b) Vertical Direction

Fig. 12. Result of Conventional Method.

In Fig.12(a), the solid line is the position of the virtual object, the dashed lines are the positions of each finger. In Fig.12(b), the solid line is the distance between the fingers and the dashed line is the distance between the virtual object and the center of fingers. Fig.13 shows the result of the proposed method, when the stiffness coefficient for the external force is k_{ep} = 30N/m and the internal coefficient is k_{ip} = 70N/m.



(a) Horizontal Direction



(b) Vertical Direction

Fig. 13. Result of Proposed Method.

The solid line in Fig.13(b) is larger than the solid line in Fig.12(b) and it shows that the invasion of the fingers to the object in the proposed method is more little than the conventional method though the manipulation is more stable. Other experiments are performed with some other viscosity b_v and in all experiment the operator felt that the proposed method can display with higher coupling impedance and can decrease vibration.

3. Experiment using real spring

The result of human manipulation is not objective and so another experiment are performed to obtain quantitative results. In the experiment, a real spring, which coefficient is 33N/m, is attached to the tip of linkage instead of a finger and the fingertip pushes a virtual object placed on a virtual floor, as shown in Fig.14.

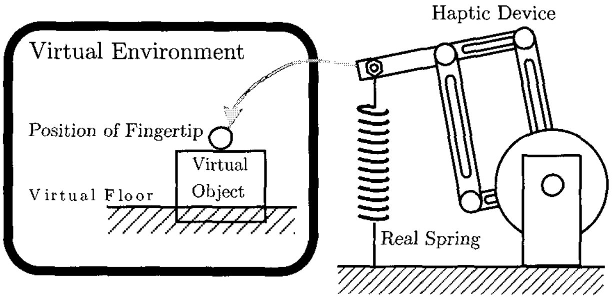


Fig. 14. Environment of Experiment Using Real Spring.

We measured the maximum value of the stiffness coefficients k_p, k_{ip}, k_{ep} that can display stably, when the viscosity b_v in (27) changes between 0.8-1.5 Nsec/m. The results are shown in Fig.15.

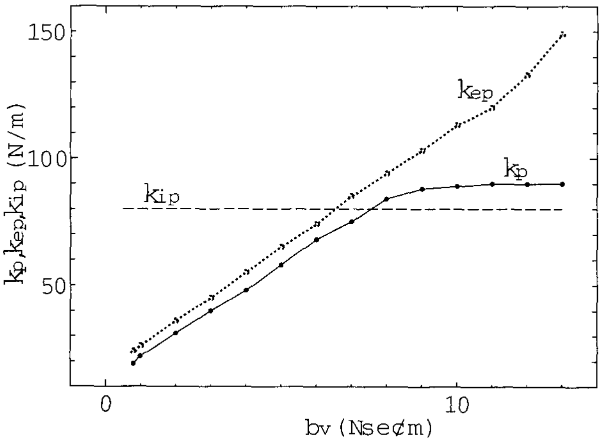


Fig. 15. Relation Between Stiffness of Coupling Impedance and Damping of Virtual Object.

In the figure, the solid line is the maximum value of k_p in the conventional method, the dashed line is k_{ip} for the internal force and the dotted line is k_{ep} for the external force in the proposed method. In the experiment, k_{ip} is constant at the value which is the maximum value for displaying a static virtual object, which has already been measured(k_{ip} = 80N/m). In the range where the viscosity of the virtual object b_v is low, k_{ip} of the proposed method can be set higher than k_p of the conventional method, though k_{ep} is almost same as k_p. In the range where the viscosity b_v is high, k_{ip} can be set higher than k_p. The results show that the proposed method can display with higher impedance in all range of b_v than the conventional method.

V. Conclusion

A new method of haptic display with the coupling impedance specified separately for the internal and external forces has been proposed for a grasping situation. For separating the coupling impedance, the proposed method can specify larger impedance than the conventional method, which does not distinguish the external and internal forces. It has been shown that the conventional method is a special case of the proposed method, in which the components for the external and internal forces are set equal. The stability of the proposed method for a 1-DOF system has been discussed and it has been shown that the stability issues can be translated in issues for displaying a simple environment that has been considered in the previous research. Finally, the effect of the proposed method has been shown by some experiments.

In the stability analysis, we have assumed that the two devices have the same dynamics. Analyzing the stability of haptic system with devices having different mass and damping coefficients will be a future research topic. Another research topic will be to analyze the stability for multi-dimensional environment.

References

[1] M. Minsky, M. Ouh-young, O. Steele, F. P. Brooks, and M. Behensky, "Feeling and seeing: Issues in force display," Comput. Graph, vol. 24,no. 2,pp. 235-243, 1990.

[2] J. E. Colgate and G. Schenkel, "Passivity of a class of sampled-data systems: Application to haptic interfaces," *Proc. of the American Control Conference*, pp. 3236-3240, 1994.

[3] J. E. Colgate, M. C. Stanley, and J. M. Brown, "Issues in the haptic display of tool use," *Proc. of IEEE/RSJ Int. Conf. of Intelligent Robotics and Systems*, 1995.

[4] R. J. Adams and B. Hannaford, "Stable haptic interaction with virtual environment," *IEEE Trans. on Robotics and Automation*, vol. 15, no. 3, pp. 465-474, 1999.

[5] T. Yoshikawa and H. Ueda, "Haptic virtual reality: Display of operating feel of dynamic virtual objects," *Robotic Research, The Seventh Symposium*, pp. 214-221, 1996.

[6] T. Yoshikawa and K. Yoshimoto, "Haptic simulation of assembly operation in virtual environment," *Proc. of the ASME, Dynamic Systems and Control Division*, pp. 1191-1198, 2000.

Masayuki Kawai

He received the B.S., the M.S. and the Ph.D. degrees in mechanical engineering from Kyoto University, Kyoto, Japan, in 1992, 1994 and 2000, respectively. From 1994 to 1997, he worked in Ishikawajima-harima Heavy Industries Co. as an mechanical engineer of factory automation and a system designer of industrial logistic system. Since 2000 he has been with Fukui University, where he is currently Research Associate of Mechanical Engineering. His current research interests include robotics and haptic virtual reality.

Tsuneo Yoshikawa

He received the Ph.D. degree in applied mathematics and physics from Kyoto University, Kyoto, Japan, in 1969. Since 1969 he has been with Kyoto University, where he is currently Professor of Mechanical Engineering. From 1973 to 1975, he was an NRC Resident Research Associate at NASA Marshall Space Flight Center, Huntsville, Alabama. He is the author of *Foundations of Robotics* (MIT Press, 1990). His current research interests include robotics, haptic virtual reality, and control of nonlinear mechanical systems. He has received several awards including the 1995 JDSMC Best Paper Award from the Dynamic Systems and Control Division of the ASME, and the 1997 ICRA Best Conference Paper Award from the IEEE Robotics and Automation Society. He is a Fellow of the IEEE.